Few types of chains of primes arising in the study of pseudoprimes

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Abstract. While studying Fermat pseudoprimes I met few interesting generic forms of numbers that have the property to generate chains of primes and pseudoprimes. I list in this paper few such types of chains.

I. Recurrent chains

I.1.
Chains of primes of the form P_0, $P_1 = P_0*n - n + 1$, $P_2 = P_1*n - n + 1$, ..., $P_k = P_{k-1}*n - n + 1$, where P_0, P_1, ..., P_k are primes and n is a positive integer, n > 1.

Note: For n = 2, we obtain the Cunningham chain of the second kind, i.e. $P_{i+1} = 2*P_i - 1$.

For instance:

A chain of primes of length 6, for n = 3, should have the form: p, 3p – 2, 9p – 8, 27p – 26, 81p – 80, 243p - 242. It can be seen that p must be of the form 10k + 1 for a chain of length bigger than 3. Such a chain, of length 5, is: 61, 181, 541, 1621, 4861.

A chain of primes of length 5, for n = 4, should have the form: p, 4p – 3, 16p – 15, 64p – 63, 256p - 255. It can be seen that p can't be of the form 10k + 7. Such a chain, of length 4, is: 23, 89, 353, 1409.

Notes:
The formula $P_{i+1} = P_i*n - n + 1$ can lead to the formation of chains of Fermat pseudoprimes, for instance for Carmichael numbers $[P_0,P_1]$ = [1729,46657] and n = 27.

The formula $P_{i+1} = P_i*(P_i*n - n + 1)$ can also lead to the formation of chains of Fermat pseudoprimes; for instance, for n = 2, $P_{i+1} = P_i*(2*P_i - 1)$ leads to the formation of the following Poulet numbers $[P_0,P_1]$: [645,831405], [1729,5977153] etc.; for n = 3, $P_{i+1} = P_i*(3*P_i - 2)$ leads to the formation of the following Poulet numbers $[P_0,P_1]$: [341,348161], [645,1246785] etc. (see the sequence A215343 in OEIS).

I.2.

Chains of primes of the form P_0, $P_1 = P_0*n - n - 1$, $P_2 = P_1*n - n - 1$, ..., $P_k = P_{k-1}*n - n - 1$, where P_0, P_1, ..., P_k are primes and n is a positive integer, n > 1.

For instance:
A chain of primes of length 6, for n = 2, should have the form: p, 2p – 3, 4p – 9, 8p – 21, 16p – 45, 32p - 93. It can be seen that p must be of the form 10k + 3 for a chain of length bigger than 3. Such a chain, of length 4, is: 113, 223, 443, 883.

A chain of primes of length 5, for n = 3, should have the form: p, 3p – 4, 9p – 16, 27p – 52, 81p - 160. It can be seen that p must be of the form 10k + 7 for a chain of length bigger than 3. Such a chain, of length 4, is: 7, 17, 47, 137.

Note: I met this type of numbers in the study of Fermat pseudoprimes to base 2 with three prime factors (see the sequence A215672 in OEIS). Most of them can be written as p*(p*n – n + 1)*(p*m – m + 1) or as p*(p*n – n – 1)*(p*m – m – 1).

I.3.

Chains of primes of the form P_0, $P_1 = P_0*n + n - 1$, $P_2 = P_1*n + n - 1$, ..., $P_k = P_{k-1}*n + n - 1$, where P_0, P_1, ..., P_k are primes and n is a positive integer, n > 1.

Note: For n = 2, we obtain the Cunningham chain of the first kind, i.e. $P_{i+1} = 2*P_i + 1$.

For instance:
A chain of primes of length 6, for n = 3, should have the form: p, 3p + 2, 9p + 8, 27p + 26, 81p + 80, 243p + 242. It can be seen that p must be of the form 10k + 9 for a chain of length bigger than 3. Such a chain, of length 4, is: 29, 89, 269, 809.

I.4.

Chains of primes of the form P_0, $P_1 = P_0*n + n + 1$, $P_2 = P_1*n + n + 1$, ..., $P_k = P_{k-1}*n + n + 1$, where P_0, P_1, ..., P_k are primes and n is a positive integer, n > 1.

For instance:
A chain of primes of length 6, for n = 2, should have the form: p, 2p + 3, 4p + 9, 8p + 21, 16p + 45, 32p + 93. It can be seen that p must be of the form 10k + 7 for a chain of length bigger than 3. Such a chain, of length 6, is: 47, 97, 197, 397, 797, 1597.

I.5.
Chains of primes of the form P_0, $P_1 = P_0*n - d*n + d$, $P_2 = P_1*n - d*n + d$, ..., $P_k = P_{k-1}*n - d*n + d$, where P_0, P_1, ..., P_k are primes, d is also a prime number and n is a positive integer, n > 1.

For instance:

A chain of this type of primes of length 6, for n = 2 and d = 7, should have the form: p, 2p – 7, 4p – 21, 8p – 49, 16p – 105, 32p - 217. It can be seen that p must be of the form 30k + 7 for a chain of length bigger than 3.

A chain of this type of primes of length 6, for n = 2 and d = 13, should have the form: p, 2p – 13, 4p – 39, 8p – 91, 16p – 195, 32p - 403. It can be seen that p must be of the form 30k + 13 for a chain of length bigger than 3. Such a chain, of length 4, is 163, 313, 613, 1213.

Note: I met this type of numbers in the study of Fermat pseudoprimes to base 2 with two prime factors (see the sequence A214305 in OEIS); for instance, for n = 3 and d = 73, $P_{i+1} = 3*P_i - 2*73$ leads to the formation of the following Poulet numbers [P_0,P_1]: [2701,7957] etc.; for n = 4 and d = 73, $P_{i+1} = 4*P_i - 3*73$ leads to the formation of the following Poulet numbers [P_0,P_1]: [2701,10585] etc.

II. Non-recurrent chains

II.1.
Chains of primes of the form 30*a*n - (a*p + a - 1), where p and a*p + a – 1 are primes and n has succesive values of integers.

For instance:

For p = 11, a = 2, n from -1 to 3 we have, in absolute value, the following chain of primes of length 5: 83, 23, 37, 97, 157.

For p = 23, a = 2, n from -3 to 2 we have, in absolute value, the following chain of primes of length 6: 227, 167, 107, 47, 13, 73.

For p = 7, a = 3, n from -1 to 2 we have, in absolute value, the following chain of primes of length 4: 113, 23, 67, 157.

Note: I met this type of numbers in the study of Carmichael numbers of the form C = ((30*a*n - (a*p + a - 1))*((30*b*n - (b*p + b - 1))*((30*c*n - (c*p + c - 1)), where p, a*p + a - 1, b*p + b - 1 and c*p + c - 1 are all primes. Many Carmichael numbers can be written in this form (see the sequence A182416 in OEIS).

II.2.

Chains of primes of the form 30*a*n + (a*p + a - 1), where p and a*p + a – 1 are primes and n has succesive values of integers.

For instance:

For p = 19, a = 3, n from -1 to 2 we have, in absolute value, the following chain of primes of length 4: 31, 59, 149, 239.

Note: I met this type of numbers in the study of Carmichael numbers of the form C = ((30*a*n + (a*p – a + 1))*((30*b*n + (b*p – b + 1))*((30*c*n + (c*p – c + 1)), where p, a*p + a - 1, b*p + b - 1 and c*p + c – 1 are all primes. Many Carmichael numbers can be written in this form (see the sequence A182416 in OEIS).

II.3.

Chains of primes of the form 2*p*n – 2*n + p, where p and 2p - 1 are primes and n has succesive values of integers.

For instance:

For p = 7, n from -5 to 3 we have, in absolute value, the following chain of primes of length 9: 53, 41, 29, 17, 5, 7, 19, 31, 43.

Note: I met this type of numbers in the study of Carmichael numbers of the form C = p*(2*p – 1)*(2*p*n – 2*n + p). I conjecture that all Carmichael numbers divisible with p and 2p – 1, where p and 2p – 1 are primes, can be written in this form (see the sequence A182207 in OEIS).