Making an analogy between a multi Chain interaction in Charge Density Wave transport and Wave functionals to form S-S' Pairs

A. Beckwith[1]
Chongqing University *Department of Physics: Chongqing University*, Chongqing , PRC , 400044

E-mail: abeckwith@uh.edu

Abstract. First, we show through a numerical simulation that the massive Schwinger model used to formulate solutions to CDW transport is insufficient for transport of solitons (anti-solitons) through a pinning gap model of CDW transport. We show that a model Hamiltonian with Peierls condensation energy used to couple adjacent chains (or transverse wave vectors) permits formation of solitons (anti- solitons) which could be used to transport CDW through a potential barrier. We argue that there are analogies between this construction and the false vacuum hypothesis used for showing a necessary and sufficient condition for formation of CDW soliton – anti - soliton (S-S') pairs in wave functionals presented in a prior publication.

1.Introduction

We have prior to this paper formed an argument using the integral Bogomol'nyi inequality to present how a soliton-anti soliton (S-S') pair could form [1], [2] In addition, we also have shown how the formation of wave functionals is congruent with Lin's nucleation [3] of an electron – positron pair as a sufficiency argument as to forming Gaussian wavefunctionals. Here, we argue our wavefunctional result is equivalent to putting in a multi chain interaction term in our simulated Hamiltonian system. We found that a single chain simulation of the S-S' transport problem suffers from two defects. First, it does not answer what are necessary and sufficient conditions for formation of a soliton(anti soliton).. Our numerical simulation of the single chain problem for CDW involving solitons (anti solitons) gave a resonance condition in transport behavior over time, with no barrier tunneling. The argument here we will present is that the false vacuum hypothesis [1],[2] [3] [4] [5] is a necessary condition for the formation of soliton – anti soliton (S-S') pairs and that the multi chain term we add to a massive Schwinger equation for CDW transport is a sufficiency condition for the explicit formation of a soliton (anti soliton) in our charge density wave transport problem.

.

[1] To whom any correspondence should be addressed.

2. **Review of the numerical behavior of a single chain for CDW dynamics**

We are modifying a one chain model of Charge Density wave (CDW) transport initially pioneered by Dr. John Miller [4] which furthered Dr. John Bardeens work on a pinning gap presentation of CDW transport [5]. We start by using an extended Schwinger model [6]with the Hamiltonian set as [1],[2]

$$H = \int_x \left[\frac{1}{2 \cdot D} \cdot \Pi_x^2 + \frac{1}{2} \cdot \left(\partial_x \phi_x\right)^2 + \frac{1}{2} \cdot \mu_E^2 \cdot \left(\phi_x - \varphi\right)^2 + \frac{1}{2} \cdot D \cdot \omega_P^2 \cdot \left(1 - \cos\phi\right) \right] \qquad (1)$$

We should note in writing this that that a washboard potential with small driving term added to the main potential term of the washboard potential, is used to model transport phenomenology. We also argue that this potential permits domain wall modeling of S-S' pairs [4]. will have the value of (pinning energy) times and work with a quantum mechanically based energy

$$E = i\hbar \frac{\partial}{\partial t} \qquad (2)$$

and momentum

$$\Pi = \left(\hbar / i\right) \cdot \partial / \partial\phi \; (x) \qquad (3)$$

Here, $\Theta \equiv \omega_D t$ was used, so then we found that the Dunford-Frankel and 'fully implicit' [7], allows us to expand the time step even further. Then, the 'massive Schwinger model' is

$$\phi(j, n+1) = \frac{2 \cdot \tilde{R}}{1 + 2 \cdot \tilde{R}} \cdot \left(\phi(j-1, n) - \phi(j+1, n)\right) + \frac{1 - 2 \cdot \tilde{R}}{1 + 2 \cdot \tilde{R}} \cdot \phi(j, n-1) \; - i \cdot \Delta t \frac{V(j,n)}{\hbar} \phi(j,n) \qquad (4)$$

where $\tilde{R} = -i \cdot \Delta t \dfrac{\mathrm{h}}{2 \cdot D \cdot (\Delta x)^2}$ .The advantage of this model is that it is second order accurate, explicit, and unconditionally stable, so as to avoid numerical blow up behavior. One then gets resonance results in a runaway oscillation [7]

3. Addition of an additional term in the Massive Schwinger equation to permit formation of a S-S' pair

Initially we will present how addition of an interaction term between adjacent CDW chains will allow a soliton (anti – soliton) to form where either one used the Bogomil'nyi inequality[15] as a necessary condition to the formation of S-S' terms and that we use a multi chain simulation Hamiltonian with Peierls condensation energy used to couple adjacent chains (or transverse wave vectors) as represented by [7]

$$H = \sum_n \left[\frac{\Pi_n^2}{2 \cdot D_1} + E_1 \left[1 - \cos\phi_n\right] + E_2 \left(\phi_n - \Theta\right)^2 + \Delta' \cdot \left[1 - \cos\left(\phi_n - \phi_{n-1}\right)\right] \right] \qquad (5)$$

with 'momentum 'we define as

$$\Pi_n = \left(\hbar / i\right) \cdot \partial / \partial \phi_n \tag{6}$$

Here, we set $\Delta' >> E_1 >> E_2$, so then which then permits us to

write $U \approx E_1 \cdot \sum_{l=0}^{n+1} \left[1 - \cos \phi_l\right] + \frac{\Delta'}{2} \cdot \sum_{l=0}^{n} \left(\phi_{l+1} - \phi_l\right)^2$ (7)

$$\omega_0^2 = \frac{\Delta'}{m_{e^-} l^2} \qquad \omega_1^2 = \frac{E_1}{m_{e^-} l^2} \tag{8}$$

where we assume the chain of pendulums, each of length l, leads to a kinetic energy using a momentum space representation of soliton- anti soliton pair(***S-S***'), i.e. via a Fourier transform in momentum space of a phase we call in position space

$$\phi(x) = \pi \cdot \left[\tanh b(x - x_a) + \tanh b(x_b - x)\right] \tag{9}$$

4. Conclusion we need to improve this following current expression. Given below

Our work lead to the modulus of the tunneling Hamiltonian being proportional to[1], [2], [7]

current $I \propto \tilde{C}_1 \cdot \left[\cosh\left[\sqrt{\frac{2 \cdot E}{E_T \cdot c_V}} - \sqrt{\frac{E_T \cdot c_V}{E}}\right]\right] \cdot \exp\left(-\frac{E_T \cdot c_V}{E}\right)$ (10)

This expression due to the round off procedures is very imprecise even if it did lead to the following diagram given below. Equation (7) done above gave qualitative fits to eventually obtaining Eq. (10) above, and although there is a close fit with the experimental graphs made by Miller, and others in [5] and in [8]

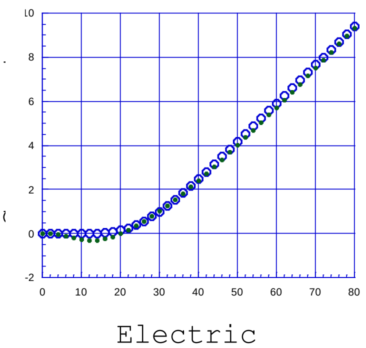

Figure 1, main result of A.W. Beckwith Dissertation.[7] Left hand axis is current.

Urgent task is to move way beyond the crude round off procedures and to get an exact solution for the current and nearest neighbor calculations for the chains. The hope is for refinement of this documents results along the lines brought up by Zee [9] and further non linear analysis. I.e. the work is important but still very crude. Future work should emphasize a precise derivation of Eq. (10) formed by round off procedures from Eq (7). The results are crude but suggestive and need a lot more work.Note that the results in Figure 1 and also Eq. (10) are an improvement over the purely form fitting result given as[5]

$$I \propto G_P \cdot (E - E_T) \cdot \exp\left(-\frac{E_T}{E}\right) \; if \, E > E_{\mathrm{T}} \qquad (11)$$

$$0 \qquad \qquad otherwise$$

They do, though need to be more rigorously explained. [7],[10] Using at a minimum the theoretical tools in [10] as well as understanding better why the following worked in order to obtain Eq. (10) is a start. In Particular, the Bardeen expression was used, Eq. (12), for a tunneling Hamiltonian, and we should find better methods in future work with wave functionals.

$$T_{if} \cong \frac{(\hbar^2 \equiv 1)}{2 \cdot m_e} \int \left(\Psi^*_{initial} \frac{\delta^2 \Psi_{final}}{\delta\phi(x)_2} - \Psi_{final} \frac{\delta^2 \Psi^*_{initial}}{\delta\phi(x)_2} \right) \vartheta(\phi(x) - \phi_0(x)) \wp\, \phi(x) \qquad (12)$$

References

[1] A. W.Beckwith, "Tunneling Hamiltonian Representation of False Vacuum Decay I .Comparison with Bogomol'nyi Inequality" arxiv: math-ph/0406048

[2] A.W. Beckwith, "Tunneling Hamiltonian Representation of False Vacuum Decay II. Application to Soliton – Anti Soliton Pair Creation", arxiv math-ph/0406053

[3] A.W. Beckwith,"An open question: Are topological arguments helpful in setting initial conditions for transport problems in condensed matter physics?", EJTP 6, No. 20 (2009) 257–268

[4] J. H. Miller,Jr. , C. Ordonez, and E. Prodan, Phys. Rev. Lett 84, 1555(2000)'

[5] H. Miller,J. Richards,R.E. Thorne,W.G.Lyons and J.R. Tucker, Phys.Rev. Lett. 55,1006 (1985)

[6] J. Schwinger ; Phys.Rev.82, 664 (1951)

[7] A.W. Beckwith, "Classical and Quantum models of Density wave transport: A comparative study' PhD dissertation , U. of Houston physics department , 2001

[8] J. H. Miller, Jr., G. Cardenas, A. Garcia-Perez, W. More, and A. W. Beckwith, *J. Phys. A: Math. Gen*. **36**, 9209 (2003)

[9] A. Zee, "*Quantum field theory in a nutshell*" ; Princeton university press , 2003 pp. 279-80

[10] E. Kreyszig, *Introductory functional analysis with Applications*, Wiley, 1978, pp 102- 102; Nirmala Prakash, *Mathematical Perspectives on Theoretical physics*, Imperial College Press, 2000, p545-6, in particular 9c.77