

Ideals Inescapable

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$$\overline{\mathbb{C} \otimes \mathbb{O}}$$

$$a_0, a_1 \in \mathbb{R} \quad a \equiv a_0 + ia_1 \in \mathbb{C}$$

$$ii = -1 \quad * : a \mapsto a^* = a_0 - ia_1$$

$$f_0, f_1, \dots, f_7 \in \mathbb{C}$$

$$f \equiv f_0e_0 + f_1e_1 \dots + f_7e_7 \in \mathbb{C} \otimes \mathbb{O}$$

$$ie_j = e_ji \quad \forall j = 0 \dots 7$$

$$e_0 \equiv 1, \quad e_1e_1 = -1, \quad \dots \quad e_7e_7 = -1$$

$$e_je_k = -e_ke_j \quad \forall j, k = 1 \dots 7, \quad j \neq k$$

$$e_je_k = e_l \Rightarrow e_{j+1}e_{k+1} = e_{l+1} \quad \forall j, k = 1 \dots 7$$

$$e_je_k = e_l \Rightarrow e_{2j}e_{2k} = e_{2l} \quad \forall j, k = 1 \dots 7$$

$$e_1e_2 = e_4$$

$$f^\dagger \equiv f_0^*e_0 - f_1^*e_1 - f_2^*e_2 \dots - f_7^*e_7$$

$$\overline{\exists \mathbb{C} \otimes \overleftarrow{\mathbb{O}}}$$

$$h_1 \in \mathbb{C} \otimes \mathbb{O}$$

$$h_1() : \mathbb{C} \otimes \mathbb{O} \rightarrow \mathbb{C} \otimes \mathbb{O}$$

$$h_1() : f \mapsto h_1f \in \mathbb{C} \otimes \mathbb{O}$$

$$h_1, h_2 \in \mathbb{C} \otimes \mathbb{O}$$

$$h_2(h_1()) : \mathbb{C} \otimes \mathbb{O} \rightarrow \mathbb{C} \otimes \mathbb{O}$$

$$h_2(h_1()) : f \mapsto h_2(h_1f) \in \mathbb{C} \otimes \mathbb{O}$$

$$h_1, h_2, h_3 \in \mathbb{C} \otimes \mathbb{O}$$

$$h_3(h_2(h_1())) : \mathbb{C} \otimes \mathbb{O} \rightarrow \mathbb{C} \otimes \mathbb{O}$$

$$h_3(h_2(h_1())) : f \mapsto h_3(h_2(h_1f)) \in \mathbb{C} \otimes \mathbb{O}$$

$$\mathbb{C} \otimes \overleftarrow{\mathbb{O}} \equiv \left\{ m = \sum_u h_u() + \sum_{v,w} h_w(h_v()) \right.$$

$$\left. + \sum_{x,y,z} h_z(h_y(h_x())) \right\} \sim Cl_6(\mathbb{C})$$

$$m, n, p \in \mathbb{C} \otimes \overleftarrow{\mathbb{O}}$$

$$nm = p \Leftrightarrow n(m(f)) = p(f) \quad \forall f \in \mathbb{C} \otimes \mathbb{O}$$

$$\overline{\exists \omega \omega^*}$$

$$\alpha_1 \equiv \frac{-e_5 + ie_4}{2} \quad \alpha_2 \equiv \frac{-e_3 + ie_1}{2} \quad \alpha_3 \equiv \frac{-e_6 + ie_2}{2}$$

$$\omega \equiv \alpha_1(\alpha_2(\alpha_3())) \in \mathbb{C} \otimes \overleftarrow{\mathbb{O}}$$

$$\omega\omega = 0$$

$$\omega\omega^*\omega\omega^* = \omega\omega^*$$

$$\begin{array}{c} \text{---} \\ \exists I \\ \text{---} \end{array}$$

$$I \equiv \left\{ s \in \mathbb{C} \otimes \overleftarrow{\mathbb{O}} \mid s = s\omega\omega^* \right\} \subset \mathbb{C} \otimes \overleftarrow{\mathbb{O}}$$

$$\nu \equiv \omega\omega^* \quad e^- \equiv \alpha_1\alpha_2\alpha_3\omega^*\omega = \omega \equiv e^{+*}$$

$$d^R \equiv -\alpha_1\omega^*\omega \quad d^G \equiv -\alpha_2\omega^*\omega \quad d^B \equiv -\alpha_3\omega^*\omega$$

$$u^R \equiv \alpha_2^*\alpha_3^*\omega\omega^* \quad u^G \equiv \alpha_3^*\alpha_1^*\omega\omega^* \quad u^B \equiv \alpha_1^*\alpha_2^*\omega\omega^*$$

$$I = \left\{ \begin{aligned} &\nu\nu + \bar{\mathcal{D}}^R d^{R*} + \bar{\mathcal{D}}^G d^{G*} + \bar{\mathcal{D}}^B d^{B*} \\ &+ U^R u^{R*} + U^G u^{G*} + U^B u^{B*} + \mathcal{E}^+ e^+ \end{aligned} \right\},$$

$$\nu, \bar{\mathcal{D}}^R, \bar{\mathcal{D}}^G, \bar{\mathcal{D}}^B, U^R, U^G, U^B, \mathcal{E}^+ \in \mathbb{C}$$

$$I^* \equiv \left\{ s \in \mathbb{C} \otimes \overleftarrow{\mathbb{O}} \mid s = s\omega^*\omega \right\} \subset \mathbb{C} \otimes \overleftarrow{\mathbb{O}}$$

$$I^* = \left\{ \bar{\nu}\nu^* + \mathcal{D}^R d^R + \mathcal{D}^G d^G + \mathcal{D}^B d^B \right.$$

$$\left. + \bar{U}^R u^{R*} + \bar{U}^G u^{G*} + \bar{U}^B u^{B*} + \mathcal{E}^- e^- \right\},$$

$$\bar{\nu}, \mathcal{D}^R, \mathcal{D}^G, \mathcal{D}^B, \bar{U}^R, \bar{U}^G, \bar{U}^B, \mathcal{E}^- \in \mathbb{C}$$

$$\text{---} \\ \mathcal{L}(U_{em}(1))I \ \& \ \mathcal{L}(SU_c(3))I \\ \text{---}$$

$$b \equiv b_1\alpha_1 + b_2\alpha_2 + b_3\alpha_3$$

$$b_1, b_2, b_3 \in \mathbb{C} \quad b^\dagger b \in \mathbb{C} \otimes \overleftarrow{\mathbb{O}}$$

$$b^\dagger b = (b_1^*b_1 + b_2^*b_2 + b_3^*b_3) \mathcal{E}$$

$$-\frac{1}{2}(b_1^*b_2 + b_2^*b_1) \Lambda_1 + \frac{i}{2}(b_1^*b_2 - b_2^*b_1) \Lambda_2$$

$$+\frac{1}{2}(b_2^*b_2 - b_1^*b_1) \Lambda_3 - \frac{1}{2}(b_3^*b_1 + b_1^*b_3) \Lambda_4$$

$$+\frac{i}{2}(b_1^*b_3 - b_3^*b_1) \Lambda_5 - \frac{1}{2}(b_2^*b_3 + b_3^*b_2) \Lambda_6$$

$$+\frac{i}{2}(b_2^*b_3 - b_3^*b_2) \Lambda_7 - \frac{1}{2\sqrt{3}}(b_1^*b_1 + b_2^*b_2 - 2b_3^*b_3) \Lambda_8$$

$$\overleftarrow{e}_{jk}f \equiv e_j(e_k(f))$$

$$\mathcal{E} \equiv \# / 3 \equiv \frac{1}{2} - \frac{i}{6}(\overleftarrow{e}_{13} + \overleftarrow{e}_{26} + \overleftarrow{e}_{45})$$

$$\Lambda_1 \equiv \frac{i}{2}(\overleftarrow{e}_{15} - \overleftarrow{e}_{34}) \quad \Lambda_2 \equiv \frac{i}{2}(-\overleftarrow{e}_{14} - \overleftarrow{e}_{35})$$

$$\Lambda_3 \equiv \frac{i}{2}(-\overleftarrow{e}_{13} + \overleftarrow{e}_{45}) \quad \Lambda_4 \equiv \frac{i}{2}(\overleftarrow{e}_{25} + \overleftarrow{e}_{46})$$

$$\Lambda_5 \equiv \frac{i}{2}(-\overleftarrow{e}_{24} + \overleftarrow{e}_{56}) \quad \Lambda_6 \equiv \frac{i}{2}(\overleftarrow{e}_{16} + \overleftarrow{e}_{23})$$

$$\Lambda_7 \equiv \frac{i}{2}(\overleftarrow{e}_{12} + \overleftarrow{e}_{36}) \quad \Lambda_8 \equiv \frac{i}{2\sqrt{3}}(\overleftarrow{e}_{13} + \overleftarrow{e}_{45} - 2\overleftarrow{e}_{26})$$

$$I \mathcal{L}(SU_L(2)) \rightarrow I^* \dots$$

