Formulas that generate subsets of 3-Poulet numbers and few types of chains of primes

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Abstract. A simple list of sequences of products of three numbers, many of them, if not all of them, having probably an infinity of terms that are Fermat pseudoprimes to base 2 with three prime factors.

Note
I named with “3-Poulet numbers” the Fermat pseudoprimes to base 2 with 3 prime factors, obviously by similarity with the name “3-Carmichael numbers” for absolute Fermat pseudoprimes. For a list with 3-Poulet numbers see the sequence A215672 in OEIS.

I.
Poulet numbers with three prime factors of the form p*((n+1)*p-n*p)*((m+1)*p-m*p), where p prime, m, n natural:

10585 = 5*29*73 = 5*(5*7 - 6)*(5*18 - 17);
13741 = 7*13*151 = 7*(7*2 - 1)*(7*25 - 24);
13981 = 11*31*41 = 11*(11*3 - 2)*(11*4 - 3);
29341 = 13*37*61 = 13*(13*3 - 2)*(13*5 - 4);
137149 = 23*67*89 = 23*(23*3 - 2)*(23*4 - 3).

II.
Poulet numbers with three prime factors of the form p*((n*p-(n+1)*p)*(m*p-(m+1)*p),where p prime, m, n natural:

6601 = 7*23*41 = 7*(7*4 - 5)*(7*7 - 8).

Conjecture: Any 3-Poulet number which has not a prime factor of the form 30k+23 can be written as p*((n+1)*p-n*p)*((m+1)*p-m*p) or as p*((n*p-(n+1)*p)*(m*p-(m+1)*p).

III.
Poulet numbers with three prime factors of the form p*(p+2*n)*(p+2^2*n-2), where p prime, n natural:

561 = 3*11*17
p = 3; p + 2*4 = 11; p + 2^2*4 – 2 = 17, so [p,n] = [3,4];

1105 = 5*13*17
p = 5; p + 2*4 = 13; p + 2^2*4 – 2 = 17, so [p,n] = [5,4];

IV.
Poulet numbers with three prime factors of the form p*(p+2*n)*(p+2^k*n), where p prime and n, k natural:

1729 = 7*13*19
p = 7; p + 2*3 = 13; p + 2^2*3 = 19, so [p,n,k] = [7,3,2];

2465 = 5*17*29
p = 5; p + 2*6 = 17; p + 2^2*6 = 29, so [p,n,k] = [5,6,2];

2821 = 7*13*31
p = 7; p + 2*3 = 17; p + 2^3*3 = 31, so [p,n,k] = [5,6,3];

29341 = 13*37*61
p = 13; p + 2*12 = 37; p + 2^2*12 = 61, so [p,n,k] = [13,12,2];

V.
Poulet numbers with three prime factors of the form (1+2^k*m)*(1+2^k*n)*(1+2^k*(m+n)), where k, m, n natural:

13981 = 11*31*41
1 + 2^1*5 = 11, 1 + 2^1*15 = 31, 1 + 2^1*(5 + 15) = 41, so [k,m,n] = [1,5,15];

252601 = 41*61*101
1 + 2^2*10 = 41, 1 + 2^2*15 = 61, 1 + 2^2*(10 + 15) = 101, so [k,m,n] = [2,10,15];

VI.
Poulet numbers with three prime factors of the form (1+2^k*m)*(1+2^k*n)*(1+2^k*(m+n+2)), where k, m, n natural:

561 = 3*11*17
1 + 2^1*1 = 3, 1 + 2^1*5 = 11, 1 + 2^1*(1 + 5 + 2) = 17, so [k,m,n] = [1,1,5];

VII.
Poulet numbers with three prime factors of the form p*(p+2*n)*(p+2*n+2*(n+1)), where p prime, n natural:

6601 = 7*23*41
p = 7; p + 2*8 = 31; p + 2*8 + 2*9 = 41, so [p,n] = [7,8].

VIII.
Poulet numbers with three prime factors of the form 3*(3+2^k)*(3+q*2^h), where q prime and k, h natural:

645 = 3*5*43 so [q,h,k] = [5,1,3];
1905 = 3*5*127 so [q,h,k] = [31,1,2];
8481 = 3*11*257 so [q,h,k] = [127,3,1].

Notes

The chains of primes of the form [p, p+2*n,..., p+2^k*n] seems to be a very interesting object of study; such chains are, for instance, [3,5,7,11,19] for [p,n,k] = [3,1,4] and [3,13,23,43,83,163] for [p,n,k] = [3,5,5].

Also it would be interesting to study the chains of primes formed starting from a prime p and adding 2^k*n, where n is an arbitrarily chosen natural number and k the smallest values for which p+2^k*n is prime. Such a chain is, for instance, [7,13,19,31,103,199,1543,3079] for [p,n] = [7,3] and [k1,k2,k3,k4,k5,k6,k7] = [1,2,3,5,6,9,10].

An interesting triplet of primes is [p+2*m,p+2*n,p+2*(m+n)] where p is prime and m,n natural; such triplets are [11,13,17] for [p,m,n] = [7,2,3] or [23,43,59] for [p,m,n] = [7,8,18]. Generalizing, the triplet would be [p+2^k*m,p+2^k*n,p+2^k*(m+n)]; such a triplet is [11,19,23] for [p,k,m,n] = [7,2,1,3].