Lorentz Violation and Modified Gravity

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Abstract

Modified torsion is proposed as an alternative to dark matter.

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1 Introduction

A modification of relativistic proper time has been propounded in a recent approach[1]. It explicitly breaks local Lorentz gauge symmetry[2], while preserving diffeomorphism invariance.

In this paper, we explore the impact of Lorentz violation on Einstein-Cartan equations, in an attempt to account for mass discrepancies in galactic systems without resorting to dark matter.

2 Gauge Theory of Gravity

In de Sitter gauge theory of gravity[3, 4], gravitational gauge field can be written as a Clifford-valued 1-form[5, 6, 1]

$$A = \frac{1}{l}e + \omega, \tag{1}$$

$$e = e^a\gamma_a = e^a_\mu dx^\mu \gamma_a, \tag{2}$$

$$\omega = \frac{1}{4}\omega^{ab}\gamma_{ab} = \frac{1}{4}\omega^{ab}_\mu dx^\mu \gamma_{ab}, \tag{3}$$

where e is vierbein, ω is spin connection, $\mu, a, b = 0, 1, 2, 3$, $\omega^{ab}_\mu = -\omega^{ba}_\mu$, and $\gamma_{ab} \equiv \gamma_a\gamma_b$. Here we adopt the summation convention for repeated indices. Clifford algebra vectors γ_a observe anticommutation relations

$$\{\gamma_a, \gamma_b\} \equiv \frac{1}{2}(\gamma_a\gamma_b + \gamma_b\gamma_a) = \eta_{ab}, \tag{4}$$

where η_{ab} is of signature $(+, -, -, -)$.

The constant l is related to Minkowskian vacuum expectation value (VEV) of gravity gauge field

$$\bar{A} = \frac{1}{l}\bar{e} + \bar{\omega} = \frac{1}{l}\delta^a_\mu dx^\mu \gamma_a. \tag{5}$$

Gravity curvature 2-form is given by

$$F = dA + A^2 = R + \frac{1}{l}T + \frac{1}{l^2}e^2, \tag{6}$$

where spin connection curvature 2-form R and torsion 2-form T are defined by

$$R = d\omega + \omega^2 = \frac{1}{4}R^{ab}\gamma_{ab} = \frac{1}{4}(d\omega^{ab} + \eta_{cd}\omega^{ac}\omega^{db})\gamma_{ab}, \tag{7}$$

$$T = de + \omega e + e\omega = T^a\gamma_a = (de^a + \eta_{bc}\omega^{ab}e^c)\gamma_a. \tag{8}$$

$$\tag{9}$$

Here exterior $\wedge$ products between forms are implicitly assumed.

One can write down the action for general relativity as[5, 6]

$$
\begin{aligned}
S_G &= \frac{c^4}{8\pi G}\int \left\langle -ie^2 F\right\rangle && (10)\\
&= \frac{c^4}{8\pi G}\int \left\langle -ie^2 (R+\frac{1}{l^2}e^2)\right\rangle && (11)\\
&= \frac{c^4}{8\pi G}\int \left\langle -ie^2 (R+\frac{\Lambda}{24}e^2)\right\rangle && (12)\\
&= \frac{c^4}{32\pi G}\int \epsilon_{abcd}e^a e^b (R^{cd}+\frac{\Lambda}{6}e^c e^d), && (13)\\
& && (14)
\end{aligned}
$$

where Λ is cosmological constant

$$\Lambda = \frac{24}{l^2}, \tag{15}$$

c is speed of light, G is Newton constant[1], i is Clifford unit pseudoscalar

$$i = \gamma_0\gamma_1\gamma_2\gamma_3, \tag{16}$$

and $\langle\cdots\rangle$ means Clifford scalar part of enclosed expression. The action of gravity is invariant under local Lorentz gauge transformations.

3 Modified Field Equations

Field equations are derived by varying total action

$$S = S_G + S_M \tag{17}$$

with gauge fields e and ω independently, where S_M is matter part of the action. The resulted Einstein-Cartan equations read

$$
\begin{aligned}
&\frac{c^4}{8\pi G}(Re+eR+\frac{\Lambda}{6}e^3) = \mathbb{T}i, && (18)\\
&\frac{c^4}{8\pi G}(Te-eT) = \frac{1}{2}\mathbb{S}i, && (19)
\end{aligned}
$$

where $\mathbb{T}$ is energy-momentum current 3-form, and $\mathbb{S}$ is spin current 3-form.

[1]See [5, 6] for how Newton constant G is related to l and VEV of gravity Higgs field.

With violation of Lorentz symmetry[1], we propose a change to equation (19) as

$$\frac{c^4}{8\pi G}(\hat{T}e - e\hat{T}) = \frac{1}{2}\mathbb{S}i. \tag{20}$$

Here modified torsion 2-form $\hat{T}$ is defined as

$$\hat{T} = T + z^{-\frac{1+\delta}{2}}(\omega_T e_S + e_S \omega_T), \tag{21}$$

where[2]

$$z = |\frac{12\alpha(\frac{e_S}{l})^2(\omega_T \frac{e_S}{l} + \frac{e_S}{l}\omega_T)}{i(\frac{e}{l})^4}| = |\frac{12\alpha l e_S^2(\omega_T e_S + e_S\omega_T)}{ie^4}|, \tag{22}$$

$$e_S = e^i\gamma_i = e^i_\mu dx^\mu \gamma_i, \tag{23}$$

$$\omega_T = \frac{1}{4}(\omega^{i0}\gamma_{i0} + \omega^{0i}\gamma_{0i}) = \frac{1}{2}\omega^{i0}_\mu dx^\mu \gamma_{i0}, \tag{24}$$

and $i = 1, 2, 3$. The modified torsion $\hat{T}$ breaks local Lorentz gauge symmetry, while preserving diffeomorphism invariance. Two free dimensionless parameters δ and α are to be determined in the following section by comparing predictions of our proposal with astronomical observations.

4 Weak Field Limit

In static weak field limit (gravity gauge field almost Minkowskian $A \approx \bar{A} = \frac{1}{l}\delta^a_\mu \gamma_a dx^\mu$), the modified Einstein-Cartan field equations (18) and (20) are reduced[3] to

$$\partial_i \omega^{i0}_0 = \frac{4\pi G}{c^2}\rho, \tag{25}$$

$$\partial_i e^0_0 - \omega^{i0}_0(1 + z^{-\frac{1+\delta}{2}}) = 0, \tag{26}$$

where

$$z = \alpha l(\omega^{i0}_0 \omega^{i0}_0)^{\frac{1}{2}}, \tag{27}$$

and ρ is mass density.

The acceleration of a non-relativistic test body moving in the gravitational field is given by[4]

$$\vec{a} = -c^2\nabla e^0_0 = -\nabla V_N[1 + (\frac{|\nabla V_N|}{a_0})^{-\frac{1+\delta}{2}}], \tag{28}$$

[2]See [5, 6] for the definition for magnitude of a Clifford multivector $|M|$.

[3]We are interested in galactic systems in the following discussion. Spin current $\mathbb{S}$ and cosmological constant Λ term are set to zero, since their effect is negligible.

[4]As opposed to [1], here we assume Lorentz violation has negligible impact on the definition of proper time and geodesics.

where

$$a_0 = \frac{c^2}{\alpha l}, \tag{29}$$

$$\nabla^2 V_N = c^2 \partial_i \omega_0^{i0} = 4\pi G\rho. \tag{30}$$

In the limit $|\nabla V_N| \gg a_0$, Newtonian dynamics is restored, provided $1 + \delta > 0$. For $|\nabla V_N| \ll a_0$, one can calculate circular orbit rotation velocity in potential

$$V_N = -\frac{GM}{r} \tag{31}$$

as

$$v^4 = a_0^{1+\delta} G M^{1-\delta} r^{2\delta}. \tag{32}$$

According to Tully-Fisher law[7] of galactic rotation curves, one has an estimation of parameter

$$\delta \approx 0. \tag{33}$$

The characteristic acceleration is approximately

$$a_0 \approx 10^{-8} cm/s^2 \approx \frac{c^2}{6}(\frac{\Lambda}{3})^{\frac{1}{2}}. \tag{34}$$

Thus the second parameter α of our model is determined as

$$\alpha = \frac{c^2}{l a_0} = \frac{c^2}{a_0}(\frac{\Lambda}{24})^{\frac{1}{2}} \approx 2. \tag{35}$$

5 Conclusion

We propose a modification of Einstein-Cartan equations. Spin current is coupled to modified torsion, which breaks local Lorentz gauge symmetry and leaves diffeomorphism invariance intact.

By setting the free dimensionless parameter δ to zero, one recovers Modified Newtonian Dynamics[8, 9, 10] (MOND) in weak field limit. Galactic rotation curves are explained without invoking dark matter. The characteristic acceleration scale a_0 is intrinsically linked to cosmological constant via Minkowskian VEV of gravity gauge field.

Further data analysis is needed for more accurate calibration of parameter δ. A positive deviation from zero (hence from MOND) would increase gravitational attractions for galaxy clusters.

References

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