LORENTZ TRANSFORMATION AND THE RELATIVE VELOCITY

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Abstract: In previous publication, we have imagined a thought experiment, showing that the relative velocities of two observers in a uniform lineaire motion, are different, unlike the predictions of Lorentz transformation. Thanks to this experiment, we show that assuming the equality of the relative velocities, leads inevitably to contradictions. Based on the axioms of the affine space, and their implications, an explanation is provided to understand the source of these contradictions.

For more than a century, a brilliant group of physicists has adopted what's called Lorentz transformation[1,2,3,4,5,6,7,8,9], In the purpose to extend the principle of relativity to the laws of electromagnetism, non-invariant under the Galilean transformation, giving rise to special relativity, a theory introducing a new vision of space and time, deeply underlying, which together form the spacetime, generalizing the three dimensional space of the classical physics, and abolishing the concept of an absolut time. Lorentz transformation is given by,

$$x^{'} = \gamma\left(x - Vt\right) \Leftrightarrow x = \gamma\left(x' + Vt'\right), \tag{1}$$

$$y' = y, \tag{2}$$

$$z' = z, \tag{3}$$

$$t' = \gamma\left(t - \frac{V}{c^2}x\right) \Leftrightarrow t = \gamma\left(t^{'} + \frac{V}{c^2}x'\right). \tag{4}$$

The transformation provides a relation between time and the coordinates $(x(t), y(t), z(t))$, $(x'(t'), y'(t'), z'(t'))$ of a moving particle relative to the inertial frames of reference $R(O, x, y, z)$, and $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$ respectively, which are in standard configuration, when $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$, is in a linear uniform motion with a velocity V relative to $R(O, x, y, z)$, so that, at, $t = t^{'} = 0$, we have $O \equiv O^{'}$. c is the speed of light, and $\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$ is calld the Lorentz factor or the Lorentz term. Lorentz transformation tells us, that since V is the velocity of $O^{'}$ relative to $R(O, x, y, z)$, $O^{'}$ will necessarily note that O is moving away from him, with the same velocity, because of the Lorentz transformation of velocities, given by

$$v_x^{'} = \frac{v_x - V}{1 - \frac{V}{c^2}v_x} \Leftrightarrow v_x = \frac{v_x^{'} + V}{1 + \frac{V}{c^2}v_x^{'}}, \tag{5}$$
$$v_y^{'} = \frac{v_y}{\gamma\left(1 - \frac{V}{c^2}v_x\right)} \Leftrightarrow v_y = \frac{v_y^{'}}{\gamma\left(1 + \frac{V}{c^2}v_x^{'}\right)},$$
$$v_z^{'} = \frac{v_z}{\gamma\left(1 - \frac{V}{c^2}v_x\right)} \Leftrightarrow v_z = \frac{v_z^{'}}{\gamma\left(1 + \frac{V}{c^2}v_x^{'}\right)},$$

(v_x, v_y, v_z), $\left(v_x^{'}, v_y^{'}, v_z^{'}\right)$ are the components of the velocity vector of any particle relative to $R(O, x, y, z)$, and $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$ respectively. In a recent work[10], we have imagined a thought experiment that calls into question the equality $V = V'$, where V' is assumed to be the velocity of O relative to $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$; indeed, let's suppose that O is an observer equipped with a very sophisticated binoculars, giving him the opportunity to see the passage of O' exactly close to "A", which is an object at rest relative to O, located in the positive side of the x axis, at the distance $OA = l$ from O, for instance, we can imagine that "A" is a man raising a flag to announce to O, the arrival of $O^{'}$. Therefore, O will obviously conclude that O' velocity is $V = \frac{l}{T}$, if T is the time shown by his clock, exactly when $O^{'}$ reaches A. Before leaving, O' fixes at O, the leading end of a self-retracting tape measure, in a way that his motion, will automaticaly pull the retrctable ruler in the direction of motion, then just by looking to the pulled linear-measurement markings, he will be able to know exactely the distance between him and O at all times. If his clock shows $t^{'} = T^{'}$ when he reaches "A", he deduces that O velocity is $V^{'} = \frac{l}{T'}$. Since the motion is linear, the traveled distance for the ruler, is none other than the OA length. As $T' \neq T$, we have necessarly $V \neq V'$ unlike (5), providing $V = V^{'}$.

In this paper we will try to understand the reason of the discrepancy between the theoretical prediction and the outcome of the this experiment. According to Lorentz transformation (4), the moving clocks can not display the same time, so the only way ensuring the equality of the relative velocities is the discordance in the measure of the distance between O and O', ie $OO' \neq O'O$ and $T' \neq T$ such as $\frac{OO'}{T} = \frac{O'O}{T'}$. At first sight it appears that the problem is entirely solved, all the more so as we have strong reasons to believe that $OO' \neq O'O$, since the introduction of the Lorentz factor in (1) has the effect to produce the so-called length contraction phenomenon, proposed earlier by FitzGerald[11]. However two problems appear: Firstly, the thought experiment shows that any kind of lenght contraction gets impossible. Secondly assuming that it should exist a sorte of length contraction making $OO' \neq O'O$, does not guarantee the equality of V and V', due to the contradiction with Lorentz transformation, induced by the assumption $V = V^{'}$. Indeed let $x_{o'}$, $x_{o}^{'}$ be the O', and O coordinates relative to $R(O, x, y, z)$, and $R^{'}(O^{'}, x^{'}, y^{'}, z^{'})$ respectively. When O' reaches A, his clock shows $t' = T'$, while O clock shows $t = T$; as at that time, O' is located at $OO' = x_{o'} = l$ from O, according to (1),

$$OO' = x_{o'} = \gamma V T'. \tag{6}$$

At "A", O', see O far from him at a distance $O^{'}O = \left| x_{o}^{'} \right|$, and from Lorentz transformation (1), we have

$$O^{'}O = \left| x_{o}^{'} \right| = \gamma V T. \tag{7}$$

From (6) and (7), it is clear that $OO' \neq O'O$, when we suppose that Lorentz transformation is true, ie, $T' \neq T$, but if we assume that Lorentz transformation is true, (5) will necessarily be true, so $V = V^{'}$ is also true. As $V = \frac{OO'}{T}$, $V^{'} = \frac{O'O}{T'}$, and we have supposed that $V = V^{'}$, according to (6) and (7), we deduce that $T' = \pm T$, but T and T' are positives, so the only possibility is $T' = T$, thus contradicting the hypothesis that $T' \neq T$. On the other side assuming that Lorentz transformation is true, means that (4), and $V^{'} = V$ are true, so $T' = \gamma T$, and $T = \gamma T^{'}$, are true, thus $\gamma = \pm 1$, which is impossible, because by definition, $\gamma > 0$ and $V \neq 0 \Rightarrow \gamma \neq 1$.

This thought experiment constitutes a decisive test for the transformation that founds special relativity theory, it takes us straight to the heart of the paradox, since assuming that Lorentz transformation is true, means that the equality of the relative velocities is true, while the thought experiment proves precisely the opposite. The question now being asked is: What is the explanation of such contradictions?. The first thing that came to mind, is to say: Is it reasonable that $OO' \neq O'O$?. The intuition and the everyday experience, show that measuring the distance between two points a and b, is absolutely independant from whether we start the measure from a to b, or from b to a, in full accordance with the axioms that base the space. A coherent mathematical description of the space, would be to define it as a set of geometric points ε constituting an affine space associated to the three dimensional Euclidean space E so that

$$\forall (a, b) \in \varepsilon^2 \rightarrow \overrightarrow{ab} \in E, \tag{8}$$

$$\forall (a, b) \in \varepsilon^2, \overrightarrow{ab} = -\overrightarrow{ba}, \tag{9}$$

$$\forall (a, b, c) \in \varepsilon^3, \overrightarrow{ac} = \overrightarrow{ab} + \overrightarrow{bc}, \tag{10}$$

$$p \in \varepsilon, \overrightarrow{v} \in E, \exists! p' \in \varepsilon : \overrightarrow{pp'} = \overrightarrow{v}, \tag{11}$$

and it's within the framework of these axioms that the concept of position, and velocity have a real meaning. By defining the velocity of a point particle M whose position relatif to the frame of reference $R(O, x, y, z)$, is $\overrightarrow{r}$ so that $\overrightarrow{V}_M = \frac{d\overrightarrow{r}}{dt}$, and as $O^{'}$ is a moving point particle with a velocity V relative to $R(O, x, y, z)$, we have necessarily $\left\| \overrightarrow{OO'} \right\| = Vt$, therfore, according to (10),

$$\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M} \Rightarrow x' = x - Vt. \tag{12}$$

By comparing (1) and (12), we can conclude that due to Lorentz factor, Lorentz transformation is strongly in disagreement with the fundamental axioms that base the space, and also our perception of distances and velocities, which continue to be recognized by special relativity, since the definition of velocity in special relativity, remains the ratio of a distance and time interval, like in classical physics. We can thus, explain than any possible mathematical contradiction emerging when dealing with Lorentz transformation, as shows this experiment, is mainly due to the violation of these axioms. However, one can assume that the introduction of the four-dimensional spacetime, may be in support of a transformation which hasn't not to be exactly conforme to the axioms (8)(9)(10)(11), since Minkowski spacetime isn't not Euclidean. Actually, we have two good reasons to believe that the four-dimensional formulation of the theory can not be a justification for modifying (9) (10) like shown in (12). Firstly the pseudo-Euclidean Minkowski spacetime, is the set of all events described by the four-position $x^\mu := (ct, \overrightarrow{r})$, where postion $\overrightarrow{r} \equiv \overrightarrow{OM}$ obeys to all the rules of the Euclidean geometry, particularly rule (12). On the other side (8)(9)(10)(11) express the affine nature of the set of points, and are not specific to the dimension or to the structure of the space E, ie, whether it's an Euclidean space or not, these are a more genaral axioms, that can be associated to any vectorial space, whatever its dimension and structure, and Minkowski spacetime, is known to be an affine space.

Concretely it doesn't matter whether our arguments are convincing or not, the most important is the thought experiment which is of great importance, and a fundamental test for the correctness of Lorentz transformation.

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