THE ENERGY OF THE GRAVITATIONAL FIELD FOR THE SCHWARZSCHILD METRIC

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Abstract

In (viXra.org 1304.0106) there described the 3 generalizations of the First Noether theorem. As the simple example of application of this generalized theorem there is a paper "Energy-MomentumVectors for the Matter and Gravitational Field"(viXra.org 1304.0130). And the application of the last paper to the Schwarzschild metric is described in the current paper. And it gives the energy of the Earth gravitational field.

Content

1). The vector of energy-momentum for the gravitational field.

Let us take from [1] (4):

$$\theta_n{}^{k\cdot} = -L\cdot\delta_n{}^k - \left[\frac{\partial L}{\partial g_{\mu\nu,k}} - \partial_{\mathbf{l}}\left(\frac{\partial L}{\partial g_{\mu\nu,k\mathbf{l}}}\right)\right]\cdot g_{\mu\nu,n} -$$

$$-\frac{\partial L}{\partial g_{\mu\nu,\mathbf{l}k}}\cdot g_{\mu\nu,n\mathbf{l}} \qquad (1.1)$$

and from [1] (5):

$$j_n = n_k \cdot X^k{}_n = \tfrac{1}{2}\cdot g^{m\nu}\cdot b_{m\nu,k}\cdot\delta^k{}_n = \tfrac{1}{2}\cdot g^{m\nu}\cdot b_{m\nu,\,n} \quad (1.2)$$

Let us take the Lagrangian in this form:

$$L = \eta\cdot R\cdot\sqrt{-g} \qquad \eta = \frac{c^3}{16\cdot\pi\cdot\gamma} \qquad R = R^{\rho}{}_{\nu\rho\lambda}\cdot g^{\lambda\nu}$$

here $R^{\rho}{}_{\nu\mu\lambda}$ - the Riemann tensor of curvature.

From [1] (3) we have:

$$P_n = \int\left(\theta^1{}_n - \int dx^1 \cdot \tfrac{1}{2} \cdot g^{m\nu} \cdot b_{m\nu,\,n}\right) \cdot dx^2 \cdot dx^3 \cdot dx^4 \qquad (1.3)$$

2). The application of item 1 to the Schwarzschild metric.

The components of this metric tensor which are not equal zero are:

$$g_{11} = 1 - \frac{2 \cdot m}{r} \qquad g_{22} = -\frac{1}{1 - \frac{2 \cdot m}{r}} \qquad g_{33} = -r^2 \qquad g_{44} = -r^2 \cdot \sin^2\theta$$

This tensor is symmetric and so $b_{m\nu} = 0 \quad (2.1)$.

Formula (1.1) became such (at κ = 1 and n = 1) :

$$\theta_1{}^1 = -\eta \cdot R \cdot \sqrt{-g} \cdot \delta_1{}^1$$

The energy of the gravitational field is:

$$E = c \cdot P_1 = c \cdot \int \theta^1{}_1 \cdot dx^2 \cdot dx^3 \cdot dx^4$$

The curvature of this space is this:

$$R = -\frac{2 \cdot m^2}{r^4 \cdot (1 - \frac{2 \cdot m}{r})}$$

And for the energy we get:

$$E = c \cdot \eta \cdot \int_{R_1}^{R_2} dr \cdot \int_0^{\pi} d\theta \cdot \int_0^{2\pi} d\varphi \cdot r^2 \cdot \sin\theta \cdot \frac{2 \cdot m^2}{r^4 \cdot (1 - \frac{2 \cdot m}{r})} =$$

$$= 4 \cdot \pi \cdot m \cdot c \cdot \eta \cdot \ln\left|\frac{(1 - \frac{2 \cdot m}{R_2})}{(1 - \frac{2 \cdot m}{R_1})}\right|$$

3). The energy of the Earth gravitational field.

For the Earth m = 0.45 см, $R_1 = 6380\ km,\ \ R_2 = \infty$; $\eta \approx 0.8 \cdot 10^{37}\ \frac{g}{\sec}$

$E = 1.9 \cdot 10^{40}\ erg \approx m_{equ} \cdot c^2 \qquad m_{equ} = 2.1 \cdot 10^{19}\ g = 3.5 \cdot 10^{-9} \cdot M_{Earth}$

That means that the equivalent mass of the Earth gravitational field is approximately in billion times less than the Earth mass.

Literature :

1 V. Telnin “Energy – momentum Vectors for matter and gravitational field.” http://viXra.org/abs/1304.0130