Some Thoughts on Special Relativity.

Jeremy Dunning-Davies,
Institute for Theoretical Physics and Advanced Mathematics (IFM) Einstein-Galilei,
Via Santa Gonda,
14 - 59100 Prato,
Italy
and
Institute for Basic Research,
Palm Harbor, Florida, U.S.A.

e-mail: j.dunning-davies@hull.ac.uk

Abstract.

Here an introduction to Wesley’s neomechanics is presented. It is shown to produce some of the same results as Special Relativity but without both the mathematical and philosophical basis of that subject. As with other work in which results associated with General Relativity are obtained without recourse to the fundamental bases of that subject, so here too the pre-eminent place afforded Special Relativity in modern science is called into question. The opportunity is taken to extend Wesley’s ideas to the case where the mass of the body when at rest is not constant.
Introduction.

Qualms concerning at least some aspects of the Special and General Theories of Relativity seem to have existed ever since both broke into modern science. However, in more recent years, some very real queries have been raised concerning the true place those theories should occupy in science. For example, as far as General Relativity is concerned, three crucial tests are normally discussed in most, if not all, textbooks on the subject. However, it has been shown that, as far as the test associated with the gravitational red shift is concerned, although the final result draws on modern results due to Planck and Poincaré, it is fundamentally deduced from notions of Newtonian mechanics alone. Further, in a 2005\textsuperscript{2}, Lavenda showed that the deflection of light and the advance of the perihelion of Mercury may be treated as diffraction phenomena on the basis of Fermat’s principle and the modification of a Bessel function in the short wavelength limit. He has also shown, in the same article, that the problem of time delay in radar echoes may be treated by the same means. Hence, where does that leave the General Theory of Relativity? It must be admitted that that theory does seem to lead to correct results but it is now clear that those same results may be obtained by other means which do not involve any of the ideas of General Relativity. This certainly denies the General Theory of Relativity the pre-eminent position it has seemingly occupied in physics for many years.

However, this says nothing of the Special Theory of Relativity and possibly even more qualms have been harboured about that theory for many years and have led to the almost complete ostracising of such as Herbert Dingle. As far as Dingle’s particular case is concerned, the complete history is chronicled in detail in the book Science at the Crossroads\textsuperscript{3}, which may be found on the internet. It is interesting, therefore, that manipulations by J. P. Wesley\textsuperscript{4} seem to have gone unnoticed.

Although the main object is to make more people acquainted with the ideas of Wesley, the opportunity has been taken to extend that gentleman’s results to the case when the mass of the body when at rest is not a constant but may vary with time.

Wesley’s Neomechanics.

Although rarely mentioned, it is a long known fact that there is a strong relationship linking mass and energy or, in other words, the idea of mass/energy equivalence was known and used long before Einstein’s 1905 article in which, to many, it was introduced into science for the first time. In the 1800’s, the topic was investigated at length and the notion that the two were linked via a coefficient of the order of $c^2$ was established by electrodynamic considerations. One pre-1905 reference to this is to be found in J. J. Thomson’s book Electricity and Matter, which is a printing of a series of lectures Thomson gave at Yale University in May 1903, but the history of the relation goes back much further in time. Although there was some doubt for a time over the exact coefficient linking mass and energy, it became apparent that that coefficient was $c^2$ and that has been verified experimentally to a high degree of accuracy. Wesley’s so-called neo-mechanics is based on this vitally important principle of mass/energy equivalence as expressed in

$$E = mc^2$$ \hspace{1cm} (1)

and has absolutely no connection with the space-time of Special Relativity. For those unacquainted with Wesley’s work, it is worth examining his deduction.

Where Wesley diverges from traditional Newtonian mechanics is a result of his noting that, since mass/energy equivalence is an established fact, if this applies to any form of energy, it follows that there must be a mass equivalent for kinetic energy. This fact has to be
included, therefore, in traditional Newtonian mechanics as a modification. Consider a body at rest whose measured mass is \( m \). Suppose this same body then moves and when in motion, possesses a kinetic energy \( T \) then, the mass equivalent is \( T/c^2 \) and, therefore, the total momentum of the body is given by

\[
p = (m + T/c^2)v
\]

From Newton’s Second Law, it follows that the force acting, \( P \), is given by

\[
P = \frac{d}{dt}(m + T/c^2)v
\]

Since the rate of working of the force equals the rate of increase of the kinetic energy, it follows that

\[
\frac{dT}{dt} = v \cdot \frac{dP}{dt} = v \cdot \frac{d}{dt}(m + T/c^2)v
\]

Next note that, if \( \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \),

\[
\frac{d\gamma}{dt} = \frac{v^2}{c^2} \frac{dv}{dt}
\]

and so

\[
\frac{dT}{dt} = v \cdot \frac{d}{dt}(m + T/c^2)v = \left(m + T/c^2\right) \frac{dv}{dt} + (v, v) \frac{1}{c^2} \frac{dT}{dt}
\]

\[
= \left(m + T/c^2\right) \frac{c^2 d\gamma}{dt} + \frac{v^2}{c^2} \frac{dT}{dt}
\]

Rearranging leads to

\[
\frac{dT}{dt} = (mc^2 + T) \frac{1}{\gamma} \frac{d\gamma}{dt}
\]

or

\[
\frac{1}{(mc^2 + T)} \frac{dT}{dt} = \frac{1}{\gamma} \frac{d\gamma}{dt}
\]

Integrating both sides with respect to \( t \) and noting that \( T = 0 \) when \( v = 0 \) leads to

\[
T = mc^2(\gamma - 1),
\]

a result normally associated with the Special Theory of Relativity.

**A modification of Wesley’s neomechanics.**

In the above \( m \), the mass of the body when at rest, is tacitly assumed to be constant. However, it is not unreasonable to wonder what modifications would need to be made to the above if \( m \) was not constant; it can be imagined to alter with time for example. Consequently, assume \( m \) to be a function of time. Under these circumstances, the equation for \( dT/dt \) above becomes

\[
\frac{dT}{dt} = v \cdot \frac{d}{dt}(m + T/c^2)v = \left(m + T/c^2\right) \frac{dv}{dt} + (v, v) \left(\frac{dm}{dt} + \frac{1}{c^2} \frac{dT}{dt}\right)
\]

\[
= \left(m + T/c^2\right) \frac{c^2 d\gamma}{dt} + \frac{v^2}{c^2} \frac{dT}{dt} + v^2 \frac{dm}{dt},
\]

that is,

\[
\frac{dT}{dt} = \frac{(mc^2 + T) d\gamma}{\gamma} + v^2 \gamma^2 \frac{dm}{dt},
\]

\[ (2) \]
But,

\[ \frac{1}{\gamma} \left( \frac{dT}{dt} - \frac{T}{\gamma} \frac{dy}{dt} \right) = \frac{1}{\gamma^2} \left( \frac{dy}{dt} \right) = \frac{d}{dt} \left( \frac{T}{\gamma} \right). \]

Hence, the above equation (2) becomes

\[ \frac{d}{dt} \left( \frac{T}{\gamma} \right) = \frac{mc^2}{\gamma^2} \frac{dy}{dt} + v^2 \gamma \frac{dm}{dt}. \]

Then, integrating throughout with respect to \( t \), remembering that \( m \) is a function of \( t \), leads to

\[ \frac{T}{\gamma} = c^2 \int \frac{m}{\gamma^2} \, dy + \int v^2 \gamma \, dm + \text{const}. \]

\[ = c^2 \left[ \int \frac{1}{\gamma} \, dm - \int \left( \frac{m}{\gamma} \right) \, d\gamma \right] + \int v^2 \gamma \, dm + \text{const}. \]

\[ = \int \left( \frac{c^2}{\gamma} + v^2 \gamma \right) \, dm - \frac{mc^2}{\gamma} + \text{const}. \]

Substituting for \( \gamma \) and simplifying leads to

\[ \frac{T}{\gamma} = - \frac{mc^2}{\gamma} + \int c^2 \gamma \, dm + \text{const}. \]

Once again \( T = 0 \) when \( v = 0 \) and so the value of the constant is given by

\[ mc^2 - \left[ \int c^2 \gamma \, dm \right]_{v=0} \]

Hence, the final result may be written symbolically as

\[ T = mc^2(\gamma - 1) + \int_{v=0}^{v} c^2 \gamma \, dm. \]

Comments.

It should be noted that none of the above derivation in the section purely devoted to an introduction to Wesley’s neomechanics is new; it may be found, albeit in a somewhat abbreviated form, in Wesley’s book\(^4\). However, it does appear that this work is not readily available and it seems it is certainly largely unknown in scientific circles. However, although the introduction of the usual Special Relativity factor \( \gamma \) is crucial and it is not clear how or why that factor is introduced, it is seen that the expression for the kinetic energy normally associated with Special Relativity is derived without recourse to the methods or philosophy of that subject. This is basically the starting point for Wesley’s so-called neo-mechanics. True; it does have some of the features of Special Relativity but that is where any similarity ends. Neo-mechanics has a totally different base from Relativity and is without the awkward results which accompany that popularly accepted theory. Those awkward results of the mathematical theory which have caused so many problems over the years and have led to literally reams of paper devoted to numerous publications, all attempting to justify a physical interpretation for what are purely mathematical results, are missing. However, the useful, physically justified results remain.

Yet again then, results associated in most peoples’ minds exclusively with relativity are found to be derivable by alternative methods. Just as the results alluded to in the introduction raise grave questions about the place afforded General Relativity, so this discussion here,
highlighting ideas of Wesley, raises similarly serious questions about the pre-eminent place afforded Special Relativity in modern science. In the case of General Relativity, no-one was claiming that subject incorrect; they were simply pointing out that a lot, maybe all, of the results on which the various crucial tests are based were obtainable by alternative methods which relied on mathematical techniques which had been in place for many years. They did not draw on, for example, results of Riemannian Geometry. In that case though, if it became clear that a result was obtainable only via the new methods that might indicate a need for caution when using the said result until an alternative derivation is found. The position in Special Relativity is, however, possibly more serious because, in that case, there are several results – to which reference has been made already – which cause grave problems when any attempt is made to fit a seemingly feasible physical explanation to them. With Wesley’s approach, the equations of the Lorentz transformation simply do not appear and that in itself removes many awkward aspects of the currently favoured theory. However, the truly crucial result, equation (1) above, which should not really be regarded as the exclusive property of Einstein and his Special Relativity, remains and forms the cornerstone for this new approach.

In the section devoted to a modification of Wesley’s work, the mass of the body when at rest is assumed to be non-constant and to be, in fact, a function of time. It seems it cannot depend on anything else since all effects due to subsequent motion are contained in the kinetic energy. However, one can envisage effects which might cause the mass of a body while at rest to alter over time and so this slight modification is, it seems, worth considering. Once again though, it should be emphasised that, although the familiar factor \( \gamma \) is very much in evidence in all the expressions, there has been absolutely no recourse to the methods or philosophy normally associated with Special Relativity.

The crucial point to be drawn from all this is that, as with the examples mentioned associated with General Relativity, a practically important result usually regarded as being solely a consequence of Special Relativity has been obtained without utilising any of the basics of that subject. This does not even hint at Special Relativity being incorrect but does show that some of its results, at least, are obtainable by other means. This, in turn, casts doubt on the position and usefulness of those results which have led to the awkward paradoxes that have caused so much intellectual effort to be spent on attempting to explain them; in fact, to find physical explanations for results which, quite simply, may not admit such explanations since the results are themselves merely non-physical consequences of the particular mathematical techniques initially used to investigate some genuine physical phenomena.
References.

1. R. F. Evans and J. Dunning-Davies; Galilean Electrodynamics 18 No. 4 (2007), 71
   http://arxiv.org gr.qc/0403082

2. B. H. Lavenda; Three Tests of General Relativity via Fermat’s Principle and the Phase of Bessel Functions,

3. H. Dingle; Science at the Crossroads
   (Martin, Brian & P’Keefe, London, 1972)

4. J. P. Wesley; Selected Topics in Scientific Physics,
   (Benjamin Wesley, Blumberg, 2002)