J. Shim

## Abstract

The dispersion relation between the conjugate Fourier variables of position and momentum,  $\nabla p_x \nabla x \ge \hbar/2$ , when expressed as an equality,  $\nabla p_x \nabla x = \hbar/2$ , defines the relationship between the ground-state kinetic energy of a confined quantum, and its degree of containment. This containment can occur in two ways: the first is by an attractive potential, and the second by a repulsive potential. These two cases behave very differently. If the quantum is bound by an attractive potential, the ground-state kinetic energy is balanced by the containing potential in a stable state where the kinetic energy remains within the bound system. In the second type of containment, which is only possible by compression, this is not so. In this case, the quantum is contained by collisions with the bounding potential, which must result in a transfer of kinetic energy to the boundary. If the boundary is sufficiently massive, then the energy transfer will have a negligible effect on the dimensions of containment, and therefore the ground-state kinetic energy of the contained quantum will not significantly change. An electron contained within the approximate diameter of an iron atom, 250 nm, for example, if contained by the attractive potential of the atomic nucleus, would transmit no energy from the atom. The same electron, however, contained in the same volume by a repulsive potential, would transmit energy to the boundary at each collision. Because the minimum velocity of the electron determined by the dispersion relation is very great compared to the dimension of containment, the number of collisions per second with the boundary would be very high, on the order of  $10^{15}$ . For even a small energy exchange per collision, this would result in a very large rate of energy transfer. An exchange of only  $10^{-6}$  ev per collision, for example, would produce  $10^9$  ev per second of energy transmitted to the boundary.

The dispersion relation between the conjugate Fourier variables of position and momentum,

$$\nabla p_{x} \nabla x \ge \hbar/2, \tag{1}$$

as developed by [1,2], cannot be regarded as merely a statistical relationship expressing an uncertainty in repeated measurements of the same variable. For a single quantum, represented by a wave function  $\psi(x)$ , located with 100% probability within a region along the x-axis between x = 0 and x = d (that is, for  $0 \ge x \ge d$ ,  $\psi(x) = 0$ , and for 0 < x < d,  $\psi(x)$  is normalized at 100%), then (1) expressed as an equality,

$$\nabla p_x \nabla x = \hbar/2, \tag{2}$$

must represent the relationship between the ground-state rms momentum of the quantum, which defines its ground-state kinetic energy,  $(\nabla p)^2/2m$ , and the magnitude of the dimension of containment, d.

The containment can occur in two ways. The first is by an attractive potential, V(x), whose source is located at x=0, such that for  $x \ge d$ , the value of  $\psi(x)$ , is negligible. The second is by repulsive potentials at x = 0 and x = d, such that the probability of the quantum being in the region 0 < x < d is 100%, and outside of it zero. In the first case, the total energy of the system, that is, the kinetic energy of the quantum plus the potential energy in the interval d, is given by the Schrödinger expression for the Hamiltonian:

$$E\Psi(x) = (-\hbar/2m) d^2\Psi(x)/dx^2 + V(x)\Psi(x), \ x > 0.$$
(3)

But in the second case, the Schrödinger equation gives

$$E\Psi(x) = (-\hbar/2m) d^2\Psi(x)/dx^2m, \quad 0 < x < d,$$
(4)

and

$$E\Psi(x) = 0, \quad 0 \ge x \ge d.$$

Here the total energy eigenvalues are the kinetic energy eigenvalues, determined by the magnitude of the momentum,  $p = h/\lambda$ , where  $\lambda = d/n$  (n = 1,2,...) In this case, (1) and (2) are non-local and non-conservative. That is, a change in the magnitude of the containment distance from d to  $d \pm \delta d$  changes the energy eigenvalue spectrum, and in particular the ground-state kinetic energy from  $K = (\hbar/2d)^2/2m$  to  $K' = (\hbar/2(d\pm \delta d))^2/2m$  in the entire region of containment. This change of energy eigenvalues is solely a function of  $\delta d$  for a given d and m, and is independent of any energy from the region 0 > x > d required to change the position of the boundaries.

Clearly, both types of containment coexist in nature, but it seems to me that one type will generally determine the greater degree of containment, and therefore predominate. For example, in a normal solid, the atomic electrons are bound relatively strongly by the attractive potential of the nucleus, and relatively weakly by the repulsive potentials of the electron clouds of adjacent atoms. Therefore the first type of containment dominates. If however, the solid is compressed enough so that the interatomic dimensions shrink sufficiently for the adjacent electron clouds to impose a repulsive boundary less than the dimensions of the uncompressed atom, then at some point, the ground-state kinetic energy from the repulsive boundaries will exceed that from the attractive potential of the nucleus for some electrons, and one or more electrons will effectively act as free electrons bound by the adjacent electron clouds, and will begin colliding with them, thereby transmitting kinetic energy to the lattice.

It seems to me that a theoretical determination of the point of transition between the two types of containment would require a sufficiently accurate computer model in order to see the affect of the compression on the electron clouds, and their relation to individual electrons. But let us examine an idealized example to try to see what the possible energy production might be after a transition from the first to the second type of containment. Consider an electron bound by two large, narrow repulsive potentials at x = 0 and x = d, where d = 250 nm, or 2.5 x  $10^{-10}$  meter, the approximate diameter of an iron atom. Since  $\nabla p = m_e v_{rms}$ , where  $m_e$  is the mass of the electron, and  $v_{rms}$  is the rms velocity of the electron, then (2) would become

$$m_{\rm e} v_{\rm rms} d = \hbar/2, \qquad (3)$$

so that  $v_{rms} = \hbar/2m_e d$ . Using  $\hbar = 10^{-34}$  J-s, and  $m_e = 9x10^{-31}$  kg, we get  $v_{rms} = 2.22 \times 10^5$  meters/sec. Using  $d = 2.5 \times 10^{-10}$  meters, this gives approximately  $10^{15}$ 

per second for the number of transits of the bound interval, d, and therefore the rate of collision of the electron with the boundary. If the repulsive potentials were massive single objects, then the boundary would have to absorb the entire kinetic energy of the electron in order to contain it, but for a complex mass such as an atom, which is capable of transferring kinetic energy back to the electron, this is not necessarily so. But even if we assume a very small net transfer of only  $10^{-6}$  ev per collision, this gives a total transfer of energy to the boundaries of  $10^9$  ev per second.

The compression needed for the transition from attractive to repulsive containment for an atomic electron would seem to be very great, probably only occurring in nature under extreme conditions, such as intense gravitational compression. One can conjecture that the thermal energy transmitted to the structure of the contained matter would be a major source of heat in the interior of massive objects. One can also speculate that at some point, the compression would be great enough so that the nuclei would act under the second type of containment, and that eventually individual quanta could attain sufficient kinetic energy for pair-production to occur. Since the anti-particles would be re-absorbed by the surrounding matter, this would result in a net accretion of mass in the interior of sufficiently massive objects.

Although such free-electron energy production would probably be impossible to achieve in laboratory conditions, it seems to me that when an atom can act effectively as a quantum, then its transition from the first type of containment to the second might be accessible. In this case the atom would have to act as both the contained quantum and as a boundary, but if the energy transmitted per collision were sufficiently small, this would appear possible. There seem to me two main requirements. The first is that the compression must be sufficient to produce a dominant interval of repulsive bounding of the atom by adjacent atoms' electron clouds. The second is that the stability of the interatomic structure of the compressed material must be sufficient to adequately maintain the interval. For a solid or liquid, the compression would have to be enough to free the atom from the attractive containment of the atomic structure in the substance, but for a gas this would not be necessary. On the other hand, the gas would have a less stable interatomic structure, so it seems to me difficult to see what could achieve a suitable balance between the two requirements without some testing. But even with the larger mass of an atom compared to an electron, the number of collisions at interatomic distances would still be high. This suggests that another significant requirement for maintaining a stable structure would be transmitting thermal energy away from the contained matter sufficiently rapidly.

[1] Kennard, E. H. (1927), "Zur Quantenmechanik einfacher Bewegungstypen", *Zeitschrift für Physik* **44** (4–5): 326.

[2] Heisenberg, W. (1927), "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeitschrift für Physik* **43** (3–4): 172–198.