An interesting and unexpected property of Carmichael numbers and a question

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Abstract. I was researching a kind of generalized Cunningham chains that generate, instead of primes, Fermat pseudoprimes to some base when purely by chance I noticed a property of absolute Fermat pseudoprimes, equally interesting and unexpected. By a childish simple operation, a new class of numbers is obtained from Carmichael numbers.

Like anyone that learned in school that digits are just a way to designate a number and to operate with it, I always looked with reluctance on the arbitrary play with digits. I personally gave credit to the method of concatenation when I saw the relation between it and Fermat pseudoprimes (see my articles, *A conjecture about a large subset of Carmichael numbers related to concatenation* and *Formulas for generating primes involving emirps, Carmichael numbers and concatenation*, posted on viXra).

The property of Carmichael numbers that I discovered now proves the extreme versatility of these numbers: by a childish simple operation, insertion of the digit 0 among the digits of these numbers, we obtain an entirely new class of numbers.

Thus we have the following numbers obtained from Carmichael numbers through the operation that I mentioned:

: 5601 (from 561)
We can see that n^5601 mod 5601 = n^3 for n from 2 to 17 (not for n = 18);

: 28021 (from 2821)
We can see that n^28021 mod 28021 = n^7 for n from 2 to 4 (not for n = 5);

: 24065 (from 2465)
We can see that n^24065 mod 24065 = n^5 for n from 2 to 7 (not for n = 8).

Note: For the number 1729, which is the known Hardy-Ramanujan number, we have p = 10729, p = 17029 and p = 17209 all three primes! (so, of course, n^p mod p = n for any value of n).

Note: For the relative Fermat pseudoprimes, to base 2 and respectivelly to base 3, we don't obtain resembling results through this operation.

Observation: By adding the digit 0 to Carmichael numbers, operation which itself it's not at all special, it's equivalent to a simple formula, the multiplication of a Carmichael number with the number 10, we obtain: n^5610 mod 5610 = n^10 for n = 2 (not for n = 3) and the same result for the numbers 1105 and 1729. Through multiplication of the first Carmichael number, 561, with the number 8, we obtain the number 4488 and also n^4488 mod 4488 = n^8 for n = 2 (not for n = 3). Through multiplication of the first Poulet number, 341, with the number 10, we obtain the number 3410 and also n^3410 mod 3410 = n^10 for n = 2 (not for n = 3). Through multiplication of the first Fermat pseudoprime to base three, 91, with the number 10 we don't obtain resembling results. Seems that this property, that 2^(P*k) mod (P*k) = 2^k, it's a property of Poulet numbers P (it can't be extended for Fermat pseudoprimes to base 3) while the property that I showed above it'a a property of Carmichael numbers (it cant' be extended for relative Fermat pseudoprimes).

Comment: The numbers m that satisfy the relation n^m mod m = n^k, where k > 1, for any consecutive integer value of n from 2 to some larger integer, numbers obtained from Carmichael numbers through this operation or not, seems to deserve further study.

Question: Are there any numbers m to satisfy the relation n^m mod m = n^k, where k > 1, for any value of n?