# Special Relativity With Apparent Simultaneity 

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#### Abstract

An interpretation of special relativity is presented with alternative principles that treat an appearance of simultaneity as physically real. This implies instantaneous propagation of incoming light, but maintains light's invariable propagation rate of $c$ by using an alternative measure of speed.

The relativistic Doppler effect is derived from physical principles based on the direct appearance of moving clocks, without relying on a delay of light. Predictions of the Lorentz transformation for time are demonstrated to be equivalent to what is apparent to a differently located observer. Changes to relative simultaneity are equivalent to a change in observer location. The second postulate of relativity is derived from a less restrictive alternative postulate.

Under this interpretation, simultaneity depends on observer location, time and distance are found to be equivalent, and measured delay of light is unified with time dilation.


## Introduction

In the standard classical interpretation of special relativity (SR), as presented by Albert Einstein in [1], the observed timing of distant events must be adjusted to account for the travel time of incoming observed light. The hypothesis presented here is that the adjustment is unnecessary, and that it is physically reasonable to measure timing of events as they immediately appear. This alternative, apparent interpretation of the physics of SR, treats events that appear simultaneous to a given observer, as physically simultaneous for the observer. Notably, the transmission and reception of a light signal appear simultaneous to the receiver of the signal, but not to the transmitter.

The motivation for considering an alternative interpretation is the relativistic idea that time and distance have little meaning beyond their measurements by observers, and might be ideally described exactly as they are observed. The main reason to dismiss this interpretation is the physically founded principle that light propagates at a constant rate $c$, whether incoming or outgoing. However, if we may drop the requirement that the speed of a light signal may be validly measured using the timing measurements of any given clock, then an alternative measure of the propagation rate of light allows an invariant rate of $c$ without requiring symmetrical timing of incoming and outgoing light, as will be shown.

Our strategy is to use the existing interpretation of SR to establish the meaning and validity of the alternative interpretation, and to then show that
the exclusively classical aspects are interpretive, unnecessary, or derivable. The classical interpretation is not invalidated, and neither interpretation is treated as exclusively real or as unreal; one may choose to consider the apparent interpretation as only a description of what SR already predicts will be seen.

## Assumptions

Essentially we are assuming the results of SR without its definition of simultaneity for spacelike separated events. Measurable predictions of events in SR are assumed to be accurate, including their causal relationships, observations, and proper times. These aspects of SR are considered to be independent of interpretation. The first postulate of relativity - that all physical laws are the same regardless of inertial reference frame - is also assumed.

The classical interpretation of SR is assumed to be valid and consistent with reality, but not exclusively so. The fundamental assumption of SR that is disputed in order to allow reinterpretation, is that incoming light objectively propagates over a given distance in the same amount of time as outgoing light, when measured using the observer's local measuring devices. This assumption and any derived results-including the Lorentz transformations, the specifics of relativistic effects, and the property of synchronization of separated clocks-are proposed to be particular to the classical interpretation.

Only proper times, and speed measured using proper times, are assumed to be valid measurements independent of interpretation. Specifically, one-way propagation of incoming or outgoing light measured by a stationary observer is not presumed to represent a meaningful measure of speed except by interpretation. The second postulate - that the speed of light is $c$ in all inertial frames of reference - then depends on interpretation. Consistent with SR, no proper time is assumed to be defined for light, leaving no immediate meaning that applies to non-classical interpretations. It may suffice as an alternative postulate that applies to all interpretations, to assume that any observer which approaches the speed of light will approach a velocity of $c$, measured using proper time.

The scope of this paper is limited to flat Minkowski spacetime, and to direct light transmissions in vacuum. Relative velocities are restricted to movement along the observer's $x$-axis. The term apparent refers only to the appearance of direct observations in agreement with SR. The term classical is used only in reference to and in accordance with Einstein's interpretation of SR.

## 1 Formulation of Apparent Simultaneity

The prototypical physical system used throughout this paper involves observers $O$ and $P$. Observer $P$ 's coordinate system is analogous to the primed frame of the Lorentz transformation, except that the coordinate systems are configured symmetrically, with each observer's positive $x$-axis pointing toward the other's location. The $x$-axis then corresponds to a measure of separation distance; positive velocity corresponds with increasing separation. Unless otherwise specified, all measurements are assumed to be according to $O$.

Generally the variable $\tau$ is used for proper times at $P$, and $t$ is used for local proper times at $O$. Any relation between the two times, or interpretation thereof, will depend on context and will be explicitly stated.

Letting $t$ be the proper time between some events $E_{O 1}$ and $E_{O 2}$, and $\tau$ be the proper time between $E_{P 1}$ and $E_{P 2}$, durations $t$ and $\tau$ are simultaneous if $E_{O 1}$ is simultaneous with $E_{P 1}$, and $E_{O 2}$ is simultaneous with $E_{P 2}$, with respect to a given definition of simultaneity.

A pair of events are apparently simultaneous for a given observer if they appear simultaneous in direct observations of light from the events.

### 1.1 Apparent Time

The following variables define the two main interpretations of simultaneity of distant events relative to local events:

Definition 1.1. The classical time $\tau_{\mathrm{C}}(t)$ at $P$ is the proper time at $P$ that is classically simultaneous, according to observer $O$, with local duration $t$.

Definition 1.2. The apparent time $\tau_{\mathrm{A}}(t)$ at $P$ is the proper time at $P$ that is apparently simultaneous, according to observer $O$, with local duration $t$.

This is the time that can be seen to elapse on $P$ 's clock while directly observed for a local duration of $t$.

If and only if $P$ 's clock appears to be zero when $O$ 's clock is zero, then at the time of an observation by $O$, the apparent time at $P$ coincides with the time that appears on the directly observed image of its clock. Since separated clocks are not typically set up to appear synchronized to zero this way, we define a measure of time that has this property:

Definition 1.3. The apparent local time $t_{\mathrm{A}}$ is the local time that elapses at $O$ while a time of $\tau_{\mathrm{A}}$ appears to elapse at $P$.

### 1.2 Coapparent Distance

Because distance in SR applies to a given instant defined by classical simultaneity, the apparent interpretation requires an alternative measure of distance using apparent simultaneity. To correspond with measurements of apparent time, this metric is given the following corresponding features: a) it is a measurement made at $P$ using $P$ 's measuring devices, and $b$ ) the measurement event and its observation at $O$ appear simultaneous to $O$. The following definition achieves these features:

Definition 1.4. The coapparent distance $x_{\mathrm{A}}$ to $P$, according to $O$ at time $t$, is the location on $O$ 's $x$-axis that $P$ appears to be when observed at time $t$, measured using $P$ 's ruler in $P$ 's inertial frame.

The term coapparent is used because the measurement doesn't fit any intuitive meaning of the word apparent to observer $O$, and attempts to convey that the measurement is not simply based on appearances at a single location, but a combination of what both locations observe. Conversely, apparent time is intuitively synonymous with the appearance of a clock, which implies proper time - a measurement of time made at the observed location.

Consider a physical object at rest relative to $O$ and marking a distance of $x$. If an event occurs at $P$ as it coincides with this location, even if $P$ is only passing, all observers will agree that the event occurred a distance of $x$ according
to $O$ 's ruler. If $P$ is moving, and the distance to $O$ is length contracted by a factor of $\epsilon$, then the distance to $O$ as measured by $P$ is

$$
\begin{equation*}
x_{\mathrm{A}}=\epsilon x \tag{1}
\end{equation*}
$$

According to $\mathrm{SR}, \epsilon$ equals the reciprocal of the Lorentz factor.
Coapparent distance is completely determined in the single instant of observation. Due to the invariance of $c$ and the uniformity of flat space at the moment of observation, the classical distance of an event is also completely determined in the single instant of observation, in agreement with SR. Classically, according to $O$, the entire distance to $P$ is length-contracted by the same factor that $P$ 's standard ruler is length-contracted, at every moment, including the instant of observation. Even if $P$ 's ruler changes (i.e. due to relative acceleration) during the flight time of incoming photons, the change in length contraction is accompanied by a change in relative classical simultaneity that ensures the consistency of $P$ 's ruler along the entire length of separation at any given instant. This suggests reasoning in support of the apparent interpretation; even the classical distance traveled by an incoming photon depends on only the instant of observation rather than on what happens over any extended duration of inbound travel.

### 1.3 Transformation Between Classical and Apparent Time

For a given time $t$, we can convert between $\tau_{\mathrm{C}}$ and $\tau_{\mathrm{A}}$ simply by compensating for the classical travel time of incoming light.

Lemma 1.5. If $P$ is moving at a fixed velocity relative to $O$, the apparent time $\tau_{\mathrm{A}}$ at $P$ corresponding to an apparent local time $t_{\mathrm{A}}$ is given by:

$$
\begin{equation*}
\tau_{\mathrm{A}}=\tau_{\mathrm{C}}\left(t_{\mathrm{A}}\right)-\frac{x_{\mathrm{A}}\left(t_{\mathrm{A}}\right)}{c} \tag{2}
\end{equation*}
$$

Proof. At time $t_{\mathrm{A}}$, observer $P$ appears a distance of $x_{\mathrm{A}}$, measured using $P$ 's ruler. Light from $P$ now reaching $O$ has traveled a distance of $x_{\mathrm{A}}$ according to the ruler, taking a time of $x_{\mathrm{A}} / c$ according to $P$ 's clock and the classical interpretation. Therefore, $P$ 's clock must have ticked an additional time of $x_{\mathrm{A}} / c$ during the light's journey, and the classical time $\tau_{\mathrm{C}}$ must be that much later than the time $\tau_{\mathrm{A}}$ that appears to have elapsed in direct observations.

Lemma 1.6. If $P$ is moving at a fixed velocity relative to $O$, the apparent time $\tau_{\mathrm{A}}$ at $P$ corresponding to a local time $t$ is given by:

$$
\begin{equation*}
\tau_{\mathrm{A}}=\tau_{\mathrm{C}}(t)-\frac{d_{\mathrm{A}}}{c} \tag{3}
\end{equation*}
$$

where $d_{\mathrm{A}}$ is the change in coapparent distance to $P$ that occurs during time $t$.
Proof. Let $\tau_{\mathrm{A} 0}$ be the time that appears on $P$ 's clock in observations made by $O$ at time 0 , and let $\tau_{\mathrm{A} 1}$ be the time that appears at time $t$. Then the time $\tau_{\mathrm{A}}$ that appears to elapse during time $t$ is equal to $\tau_{\mathrm{A} 1}-\tau_{\mathrm{A} 0}$. Using Eqn (2), with additional variables set accordingly,

$$
\begin{equation*}
\tau_{\mathrm{A}}(t)=\left(\tau_{\mathrm{C}}\left(t_{\mathrm{A} 1}\right)-\frac{x_{\mathrm{A} 1}}{c}\right)-\left(\tau_{\mathrm{C}}\left(t_{\mathrm{A} 0}\right)-\frac{x_{\mathrm{A} 0}}{c}\right) \tag{4}
\end{equation*}
$$

and the proof follows, where $d_{\mathrm{A}}$ equals $x_{\mathrm{A} 1}-x_{\mathrm{A} 0}$.

The condition of fixed velocity might not be necessary, however the restrictive Lemmas are sufficient for this paper.

## 2 Relativity

Using the preceding definitions and a prototypical observation of a moving clock, relativistic effects will now be derived, corresponding with the Lorentz transformation in the case of proper time. In order to derive the effects from first principles, we'll pretend that the Lorentz transformations and factor are unknown.

### 2.1 Apparent Changes in Time and Distance

The first measurement of light's finite speed, by Ole Rømer in 1676, was not a direct measurement of the timing of light signals, but rather an interpretation of observations of a clock appearing to tick at different rates. ${ }^{[2]}$ The clock in this case is Io, whose orbital period appears from Earth to respectively decrease or increase depending on whether we are moving toward or away from it. This was interpreted as a changing delay of light proportional to the changing distance; the prevailing assumption of the universality of time precluded the consideration of a physical variation in the rate of time.

Letting these observations be representative of any moving clock, the following Lemmas establish principles based on how clocks directly appear, without interpreting the observations.

Lemma 2.1. Neglecting change in proper time, a change in coapparent distance of $d_{\mathrm{A}}$ will correspond to a change in apparent time of $-d_{\mathrm{A}} / c$.

Proof. This follows directly from Eqn (3) when neglected duration $\tau_{\mathrm{C}}$ is zero.
Lemma 2.2. Neglecting induced relativistic effects, a change in apparent time $\tau_{\mathrm{A}}$ of a clock that is moving at a fixed velocity of $v$ relative to the observer, will correspond to a change in coapparent distance of $v \tau_{\mathrm{A}}$.

Proof. We will prove this with a thought experiment involving a physical coupling between measurements of time and distance. The experiment employs a rack and pinion, with the pinion gear attached to the axle of $P$ 's clock hand, and the rack fixed to observer $O$ 's $x$-axis as a ruler, such that when the clock hand turns, the gear and clock are moved along the ruler. Letting the cogs of the rack be spaced one unit of length apart, and letting the clock hand turn one revolution in one unit of proper time, then a gear with $|v|$ cogs will move a distance of $v$ units along the ruler per unit of time, with the clock oriented appropriately depending on the sign of $v$. Assume a choice of units to make $v$ whole, and that the clock is essentially weightless, and ignore relativistic effects which may deform the gear.

As the teeth of the rack and pinion will mesh properly, clearly they will also appear to mesh properly. Now, if we suppose that the gear appears to rotate $\tau_{\mathrm{A}}$ times, we must concede that the clock must also appear to move $v \tau_{\mathrm{A}}$ units along $O$ 's ruler.

By neglecting relativistic effects, we have that $P$ 's ruler is the same as $O$ 's, and so the clock is moving at a velocity of $v$ according to $P$ 's measurements.

Generally, any system where $P$ moves at a fixed velocity $v$ will behave identically to the experiment, regardless of how the fixed velocity is maintained.

The preceding Lemmas illustrate that for an object $P$ with finite velocity $v$, it may not appear to change distance without a corresponding change in time, and vice versa. This suggests a real physical basis to apparent time rather than an illusive one.

### 2.2 Relativistic Apparent Time

According to Lemma 2.1 and Lemma 2.2, a change in coapparent distance results in a change in apparent time, which while velocity is maintained is matched by additional change in distance, which must correspond to additional change in apparent time, and so on. It is therefore unrealistic to neglect secondary effects as we have. Fortunately, the Lemmas can be used to calculate these effects.

Let us suppose that the previously described device is continuously running for some period, sufficiently long that we may avoid start and stop events or their observation. Assume also that $O$ and $P$ remain sufficiently separated to avoid collision. While the device is running, let $O$ observe it for a duration of $t$. Classically, the device's clock rotates $\tau_{\mathrm{C}}$ times during that period, but appears to $O$ to rotate $\tau_{\mathrm{A}}$ times.

If the gear is disengaged from the rack, such that the velocity of $P$ relative to $O$ is zero, then $\tau_{\mathrm{A}}$ will equal $\tau_{\mathrm{C}}$. Only when there is an apparent change in distance to the observer do the two differ. So let us consider $\tau_{\mathrm{A}}$ to be the sum of the apparent change in time when no change in distance is considered, plus the change in apparent time due to movement. Letting $\tau_{\mathrm{A} 0}$ be the change in apparent time when movement is ignored:

$$
\begin{equation*}
\tau_{\mathrm{A} 0}=\tau_{\mathrm{C}} \tag{5}
\end{equation*}
$$

Considering now the motion of the device when the gear is engaged, the device will appear to move during the partial apparent duration $\tau_{\mathrm{A} 0}$. By Lemma 2.2, the corresponding change in coapparent distance is:

$$
\begin{equation*}
d_{\mathrm{A} 0}=v \tau_{\mathrm{A} 0} \quad=v \tau_{\mathrm{C}} \tag{6}
\end{equation*}
$$

By Lemma 2.1, the change in coapparent distance will correspond to an additional change in apparent time:

$$
\begin{equation*}
\tau_{\mathrm{A} 1}=-\frac{d_{\mathrm{A} 0}}{c} \quad=-\frac{v}{c} \tau_{\mathrm{C}} \tag{7}
\end{equation*}
$$

Which again corresponds to an additional change in coapparent distance, which corresponds to additional apparent time, and so on.

$$
\begin{array}{ll}
d_{\mathrm{A} 1}=v \tau_{\mathrm{A} 1} & =-\frac{v^{2}}{c} \tau_{\mathrm{C}} \\
\tau_{\mathrm{A} 2}=-\frac{d_{\mathrm{A} 1}}{c} & =\frac{v^{2}}{c^{2}} \tau_{\mathrm{C}} \tag{9}
\end{array}
$$

Each successive $\tau_{\mathrm{A}}$ term is a factor of $-v / c$ times the previous term. Taking $\tau_{\mathrm{A}}$ to be the sum of all the individual terms, we have

$$
\begin{align*}
\tau_{\mathrm{A}} & =\sum_{n=0}^{\infty} \tau_{\mathrm{A} n}  \tag{10}\\
& =\sum_{n=0}^{\infty}\left(\frac{-v}{c}\right)^{n} \tau_{\mathrm{C}} \tag{11}
\end{align*}
$$

which is a geometric series ${ }^{[3]}$ that evaluates to

$$
\begin{equation*}
\tau_{\mathrm{A}}=\frac{1}{1-\frac{-v}{c}} \tau_{\mathrm{C}} \tag{12}
\end{equation*}
$$

Similarly,

$$
\begin{align*}
d_{\mathrm{A}} & =\sum_{n=0}^{\infty} d_{\mathrm{A} n}  \tag{13}\\
& =\sum_{n=0}^{\infty} v\left(\frac{-v}{c}\right)^{n} \tau_{\mathrm{C}}  \tag{14}\\
& =v \tau_{\mathrm{A}} \tag{15}
\end{align*}
$$

Defining coapparent velocity as $d_{\mathrm{A}} / \tau_{\mathrm{A}}$, and taking it to be an appropriate measure of speed in the interpretation, we find that it is equal to $v$. Thus the conversion between interpretations preserves velocity, and suggests that the coapparent speed of light will be equal to $c$, which we will confirm later.

### 2.3 Principle of Relativity in the Apparent Interpretation

In order to express $\tau_{\mathrm{A}}$ in terms of $t$ without using $\tau_{\mathrm{C}}$, let us define

$$
\begin{equation*}
\epsilon=\frac{\tau_{\mathrm{C}}}{t} \tag{16}
\end{equation*}
$$

which is the reciprocal of the Lorentz factor defined in [4].
We will also define

$$
\begin{equation*}
\epsilon_{\mathrm{A}}=\frac{\tau_{\mathrm{A}}}{t} \tag{17}
\end{equation*}
$$

which is the ratio of the rate of apparent time of a remote clock relative to an observer's local clock.

With $\tau_{\mathrm{C}}=\epsilon t$ from Eqn (16), Eqn (12) becomes

$$
\begin{equation*}
\tau_{\mathrm{A}}=\frac{1}{1+\frac{v}{c}} \epsilon t \tag{18}
\end{equation*}
$$

and Eqn (17) gives

$$
\begin{equation*}
\epsilon_{\mathrm{A}}=\frac{1}{1+\frac{v}{c}} \epsilon \tag{19}
\end{equation*}
$$

Note that while time-dilation effects in SR are identical for $v$ as for $-v$, they don't appear as identical; receding clocks appear to tick slower, while approaching clocks appear to tick faster.

We can calculate $\epsilon$ via the following thought experiment, in which we'll have two objects $O$ and $P$ execute an identical manoeuvre relative to a third observer $Q$, and consider how $O$ and $P$ appear to each other. Assume negligible duration of acceleration.

Let $O, P$, and $Q$ initially be coincident, with clocks synchronized to zero. In the first phase of the experiment, $P$ moves with a velocity of $v$ to a location of $Q+x$, then stops. Object $P$ 's movement occurs during a proper time of $t_{\mathrm{m}}$.

In the second phase, $O$ begins to move at local time $t_{1}$, coincident with the observation of $P$ reaching the destination. Let $O$ also travel at velocity $v$ to $Q+x$, then stop, ending the experiment. This movement phase also occurs during a proper time equal to $t_{\mathrm{m}}$.

We know that since $O$ and $P$ begin and end the experiment at shared locations, and since their movement phases are symmetrical, their clocks will be synchronized at the end of the experiment. However, they will each observe the other's clock behaving differently. Specifically, $O$ will observe $P$ 's clock visually appearing to run slowly during the latter's receding journey, while $P$ will observe $O$ 's clock visually appearing to run fast during the latter's approaching journey.

The apparent times of $P$ according to $O$ for two phases are respectively labeled $\tau_{\mathrm{A} 1}$ and $\tau_{\mathrm{A} 2}$, while the apparent times of $O$ as observed by $P$ for the same phases are $\tau_{\mathrm{A} 1}^{\prime}$ and $\tau_{\mathrm{A} 2}^{\prime}$.

## What $O$ sees:

Object $P$ recedes and appears to reach its destination at proper (and apparent) time $t_{\mathrm{m}}$. The corresponding local time at $O$, as $P$ appears to stop, is by definition $t_{1}$. The relationship between these two values is given by Eqn (18), as

$$
\begin{equation*}
\tau_{\mathrm{A} 1}=t_{\mathrm{m}}=\frac{1}{1+\frac{v}{c}} \epsilon t_{1} \tag{20}
\end{equation*}
$$

Then as $O$ moves to join $P$, the latter appears to approach at $-v$. Again by Eqn (18), we know that the approaching phase appears to take

$$
\begin{equation*}
\tau_{\mathrm{A} 2}=\frac{1}{1-\frac{v}{c}} \epsilon t_{\mathrm{m}} \tag{21}
\end{equation*}
$$

where $t_{\mathrm{m}}$ is the local time of $O$ 's journey, and $\epsilon$ is known to be the same for $-v$ as for $v$. This is an assumption that is taken from SR, without an alternative explanation provided herein, and may be considered an unresolved problem with the apparent interpretation.

## What $P$ sees:

Object $P$ completes its journey and stops, and it experiences a delay between its own stopping and the starting of $O$ 's approach. We can ignore the length of the delay by including it in the timing of the first phase, noting that $O$ must appear to start its journey at proper time $t_{1}$. Then $O$ will appear to approach,
with a proper duration of $t_{\mathrm{m}}$. The durations that appears to elapse on $O$ 's clock over the two phases of the experiment are thus

$$
\begin{align*}
& \tau_{\mathrm{A} 1}^{\prime}=t_{1}  \tag{22}\\
& \tau_{\mathrm{A} 2}^{\prime}=t_{\mathrm{m}} \tag{23}
\end{align*}
$$

## On what they agree:

Since the two clocks end synchronized, we know that the total time that $O$ sees passing at $P$ must equal the total time that $P$ sees passing at $O$. By combining values from Eqns (20) through (23), this equality is expressed as

$$
\begin{equation*}
t_{\mathrm{m}}+\frac{1}{1-\frac{v}{c}} \epsilon t_{\mathrm{m}}=t_{1}+t_{\mathrm{m}} \tag{24}
\end{equation*}
$$

Solving for the ratio $\epsilon_{\mathrm{A}}$ (of $P$ 's movement phase according to $O$ ), we get

$$
\begin{equation*}
\frac{t_{\mathrm{m}}}{t_{1}}=\frac{1-\frac{v}{c}}{\epsilon} \tag{25}
\end{equation*}
$$

We also have from Eqn (20) that

$$
\begin{equation*}
\frac{t_{\mathrm{m}}}{t_{1}}=\frac{\epsilon}{1+\frac{v}{c}} \tag{26}
\end{equation*}
$$

Equating Eqns (25) and (26), and solving for positive $\epsilon$, we have

$$
\begin{equation*}
\epsilon=\sqrt{1-\frac{v^{2}}{c^{2}}} \tag{27}
\end{equation*}
$$

This is of course the correct reciprocal of the Lorentz factor, defined by Eqn (16), according to [4].

We now have a solution for $\epsilon_{\mathrm{A}}$ by substituting Eqn (27) into Eqn (19).

$$
\begin{equation*}
\epsilon_{\mathrm{A}}=\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \tag{28}
\end{equation*}
$$

This corresponds to the apparent change in observed clock frequencies predicted by SR, which is the reciprocal relativistic Doppler factor given in [5].

Although we have used knowledge of SR to establish the value of classical time $\tau_{\mathrm{C}}$, we now have an equation for $\tau_{\mathrm{A}}$ from which the variable is removed. Note that $\tau_{\mathrm{C}}$ does not span any specific events that are referenced in any of these equations. Effectively, it has become only an intermediate variable with no independent physical meaning. The appearance of relativistic effects is thus explained without the need for hidden relativistic effects (notwithstanding the unjustified assumption that $\epsilon$ is the same for $v$ and $-v$ ).

Relativistic Doppler effects can thus be interpreted as direct consequences of observations in accordance with the principles of relativity, without needing delayed incoming light as an explanation.

### 2.4 Correspondence With the Lorentz Transformation

The classical simultaneity of $t$ and $\tau_{\mathrm{C}}$ is applicable throughout the inertial frame of $O$, while the apparent simultaneity of $t$ and $\tau_{\mathrm{A}}$ is location-dependent. Correspondence with the Lorentz transformation is demonstrated here, limited to location $P$, i.e. only in the case of proper time.

In the Lorentz transformation given in [6], the time $t^{\prime}$ corresponds to the proper time at $x$ when

$$
\begin{equation*}
x=t v \tag{29}
\end{equation*}
$$

In this case, Eqn (3), with $\tau_{\mathrm{A}}=\epsilon_{\mathrm{A}} t$ and letting $d_{\mathrm{A}}=\epsilon_{\mathrm{A}} x$ by definition of coapparent velocity, gives us

$$
\begin{equation*}
\tau_{\mathrm{C}}=\epsilon_{\mathrm{A}}\left(t+\frac{x}{c}\right) \tag{30}
\end{equation*}
$$

Incorporating Eqns (27) and (28) we get

$$
\begin{equation*}
\tau_{\mathrm{C}}=\frac{1}{\epsilon}\left(1-\frac{v}{c}\right)\left(t+\frac{x}{c}\right) \tag{31}
\end{equation*}
$$

which, given Eqn (29), simplifies to

$$
\begin{equation*}
\tau_{\mathrm{C}}=\frac{1}{\epsilon}\left(t-\frac{v}{c} \frac{x}{c}\right) \tag{32}
\end{equation*}
$$

which is the Lorentz transformation of time.
Since we do not require any specifically coordinated clocks, and in fact have been using clocks that may have been set in agreement with classical SR, we can assert correspondence with the Lorentz transformation at any location $P^{\prime}$ on $O$ 's $x$-axis, by defining in accordance with SR a proper clock at $P^{\prime}$ that is classically synchronized with $P$ 's clock.

With an acceptance of a physical reality of apparent time, the Lorentz transformation for time becomes a consequence of the conversion to classical time. The "mixing of space and time" inherent in the Lorentz transformation is not apparent in the Doppler equations, but only emerges from the conversion, indicating that it is only an interpretation-dependent aspect of SR.

### 2.5 Composition of Velocities

Velocities along the $x$-axis do not change when converting between interpretations, so we should expect that the composition of velocities defined in [1] would apply here. We can show this is true for velocities along a common $x$-axis.

Suppose that $P$ is moving at velocity $v_{P}$ relative to an observer $Q$ which is moving collinearly at velocity $v_{Q}$ relative to $O$, with $Q$ in between the other two. Let $v$ be the velocity of $P$ relative to $O$. While a time of $t$ passes at $O$, let a time of $\tau_{\mathrm{A}}(t)$ appear to pass at $Q$, and let $\tau_{\mathrm{A}}^{\prime}\left(\tau_{\mathrm{A}}(t)\right)$ appear to pass at $P$. Both $O$ and $Q$ see the same proper time appear to pass at $P$ relative to a given proper time at $Q$. This can be confirmed by considering that a signal from an event at $P$ sent to $O$ must arrive at the same time as a similar signal that is relayed through $Q$ without additional delay. Let

$$
\begin{equation*}
\epsilon_{\mathrm{A}}\left(v_{Q}\right)=\frac{\tau_{\mathrm{A}}(t)}{t} \tag{33}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon_{\mathrm{A}}^{\prime}\left(v_{P}\right)=\frac{\tau_{\mathrm{A}}^{\prime}\left(\tau_{\mathrm{A}}(t)\right)}{\tau_{\mathrm{A}}(t)} \tag{34}
\end{equation*}
$$

in accordance with Eqn (28). Then

$$
\begin{equation*}
\epsilon_{\mathrm{A}}(v)=\frac{\tau_{\mathrm{A}}^{\prime}\left(\tau_{\mathrm{A}}(t)\right)}{t}=\epsilon_{\mathrm{A}}\left(v_{Q}\right) \epsilon_{\mathrm{A}}^{\prime}\left(v_{P}\right) \tag{35}
\end{equation*}
$$

Solving Eqn (28) for $v$ yields

$$
\begin{equation*}
v=c \frac{1-\epsilon_{\mathrm{A}}(v)^{2}}{1+\epsilon_{\mathrm{A}}(v)^{2}} \tag{36}
\end{equation*}
$$

Substituting Eqn (35), then using Eqn (28), and simplifying results in

$$
\begin{equation*}
v=\frac{v_{P}+v_{Q}}{1+v_{P} v_{Q} / c^{2}} \tag{37}
\end{equation*}
$$

which is the correct composition of velocities, according to [1].

### 2.6 Coapparent Speed of Light

Measuring from $O$ 's frame, incoming light appears to arrive instantly while outgoing light appears to propagate at a rate of $c / 2$. However, this involves comparing the timing of one event at the location of the clock, with one that is remote.

Coapparent velocity $d_{\mathrm{A}} / \tau_{\mathrm{A}}$ is based on the proper time of a moving object. Though proper time is not defined for light, in limit form we find

$$
\begin{equation*}
\lim _{v \rightarrow \pm c} \frac{\left|d_{\mathrm{A}}\right|}{\tau_{\mathrm{A}}}=\lim _{v \rightarrow \pm c} \frac{\left|\epsilon_{\mathrm{A}} d\right|}{\epsilon_{\mathrm{A}} t}=c \tag{38}
\end{equation*}
$$

Thus, measuring the speed of light as it appears, while also using coapparent distance and proper times, allows a coapparent speed of light equal to $c$, adhering in essence to the intention of the second postulate of relativity.

### 2.7 Significance of the Lorentz Factor

Relativity of simultaneity is an aspect of both interpretations, and requires that a pair of events that one observer measures as simultaneous, another observer may measure as separated by a delay. While it is classically more intuitive to attribute apparent delays to the propagation light, relativistic effects are needed whether or not this is done. Conversely, the apparent interpretation shows that all measured delay can be treated as a relativistic effect alone. The classical delay of light can be treated as a first-order apparent relativistic effect, and time dilation as higher-order effects. Taken together, the separate classical effects of delayed light and the Lorentz transformation for time are unified as the Doppler effect, an apparent effect.

The reciprocal Lorentz factor $\epsilon$ still shows up in the apparent interpretation as the ratio of rest distance measured by $O$ to coapparent distance, and in the proper velocity of $P$. For example, when $P$ appears to $O$ to reach a distance of $x$ measured in $O$ 's rest frame, the apparent distance $d_{\mathrm{A}}$ equals $\epsilon x$. Note that
$\epsilon$ here is a ratio of a length measured at $P$ to one measured at $O$, involving two different inertial frames. An alternative measure is the ratio of where $P$ appears on $O$ 's ruler, to where $O$ appears on $P$ 's ruler, with both measurements made in the inertial frame of $O$. In this case the ratio is $\epsilon_{\mathrm{A}}$. The ratio of $\epsilon_{\mathrm{A}}$ to $\epsilon$, equivalent to compensating for movement at velocity $v$ for the classical duration that light takes to cross a change in distance between $O$ and $P$, may in the apparent interpretation represent a translation of different observers' experiences, namely from that of $P$ to that of $O$.

## 3 Equivalence With the Classical Interpretation

Lorentz and Doppler equations give different results, and though they correspond, they seem to be fundamentally different descriptions of reality. However, they can produce identical results if used to describe the same events from different observer locations; a pair of events which are classically simultaneous at $O$ appear simultaneous at another location $S$. Without a privileged frame of reference in SR , the measurements of events by either observer are equally valid. The transformation between the interpretations is then equivalent to a transformation between observer coordinates.

We will determine, for different cases, the location of a "special observer" $S$ for which pairs of events classically simultaneous according to $O$, are apparently simultaneous. Specifically, if $t$ and $\tau$ are classically simultaneous according to $O$, they will appear simultaneous to some observer $S$.

For the following Lemmas, let $t$ refer to the time of an event at $O$, and $\tau$ refer to the time of an event at $P$, or equivalently the proper times since appropriately set clocks.

Lemma 3.1 (Local equivalence of classical and apparent interpretation). In the case that $O$ is collocated with $P, \tau$ is classically simultaneous with $t$ if and only if $\tau$ is apparently simultaneous with $t$.

Proof. Let $P$ be collocated with $O$. Then the coapparent distance $x_{\mathrm{A}}$ between $O$ and $P$ is zero. By Lemma 1.5, $\tau_{\mathrm{A}}$ and $\tau$ are equal.

Lemma 3.2 (Rest frame correspondence of classical and apparent interpretation). Let $O$ and $P$ be relatively at rest. If $\tau$ and $t$ are classically simultaneous according to $O, \tau$ and $t$ are apparently simultaneous according to $S$, where $S$ is located at $(O+P) / 2$.

Proof. Let $S$ equal $(O+P) / 2$. Location $S$ is then equidistant to $O$ and $P$, and shares their inertial frame. According to classical SR, a pair of signals that leave $P$ and $O$ simultaneously will each take the same amount of time to reach $S$, thus appearing to occur simultaneously at $S$.

For the following Lemma, velocities are expressed as ratios of $c$.
Lemma 3.3 (Inertial motion correspondence of classical and apparent interpretation). Given $P$ moving at a constant finite velocity $\beta$ relative to $O$, where $\tau$ and $t$ are classically simultaneous according to $O$, there is a reference point $S$ such that $\tau$ and $t$ are apparently simultaneous from $S$.

Proof. Let $P$ be moving inertially at a velocity $\beta$ relative to observer $O$. We will construct $S$ in such a way that the Lemma is satisfied.

First, we may assume that $O$ and $P$ are coincident at some instant, since $P$ is moving collinear with $O$ at a fixed non-zero velocity. Let us set both of their clocks to zero at that instant. Let us also choose to coincide $S$ with $O$ at the same instant and set its to zero then as well. By Lemma 3.1, $S$ would observe events at $P$ and $O$ as simultaneous at that instant.

Let $\epsilon$ be the reciprocal of the Lorentz factor, i.e.

$$
\begin{equation*}
\epsilon=\sqrt{1-\beta^{2}}=\frac{\tau}{t} \tag{39}
\end{equation*}
$$

If $S$ is satisfactory, it will have the following properties:

1. $S$ observes $P$ and $O$ consistent with what $O$ observes.
2. By Property (1), $S$ observes that $P$ is moving with a relative velocity $\beta$ relative to $O$.
3. The apparent rate of time passing at $P$ relative to that at $O$, according to $S$, is equal to the classical rate of time at $P$ relative to that at $O$, according to $O$. Letting $\tau_{\mathrm{A}}^{\prime}(\tau)$ be apparent time at $P$, and $\tau_{\mathrm{A}}^{\prime}(t)$ the apparent time at $O$, each as observed by $S$, we must have that

$$
\begin{equation*}
\frac{\tau_{\mathrm{A}}^{\prime}(\tau)}{\tau_{\mathrm{A}}^{\prime}(t)}=\epsilon \tag{40}
\end{equation*}
$$

From $S$ 's perspective, $S$ remains stationary while $O$ and $P$ move at fixed velocities. Let $\beta_{O}$ and $\beta_{P}$ be the respective velocities of $O$ and $P$ relative to $S$. By Property (2), we know that the composition of velocities of $\beta_{O}$ and $\beta_{P}$ will equal $\beta$.

Since we're using a symmetrical configuration, the velocity of $O$ relative to $S$ is the same as the velocity of $S$ relative to $O$, which is given by the equation for composition of velocities. This value, from Eqn (37) modified for velocities as ratios of $c$, is

$$
\begin{equation*}
\beta=\frac{\beta_{O}+\beta_{P}}{1+\beta_{O} \beta_{P}} \tag{41}
\end{equation*}
$$

Solving for $\beta_{P}$, we get

$$
\begin{equation*}
\beta_{P}=\frac{\beta-\beta_{O}}{1-\beta \beta_{O}} \tag{42}
\end{equation*}
$$

Let $\epsilon_{P}$ equal $\sqrt{1-\beta_{P}^{2}}$ and $\epsilon_{O}$ equal $\sqrt{1-\beta_{O}^{2}}$. Then, expanding Eqn (40) using Eqn (18), we have

$$
\begin{gather*}
\epsilon=\frac{\epsilon_{P}\left(\frac{1}{1+\beta_{P}}\right) t_{S}}{\epsilon_{O}\left(\frac{1}{1+\beta_{O}}\right) t_{S}}  \tag{43}\\
\sqrt{1-\beta^{2}}=\frac{\sqrt{1-\beta_{P}^{2}}\left(1+\beta_{O}\right)}{\sqrt{1-\beta_{O}^{2}}\left(1+\beta_{P}\right)} \tag{44}
\end{gather*}
$$

Solving Eqn (44) for $\beta_{P}$, we have

$$
\begin{equation*}
\beta_{P}=\frac{\beta_{O}+\frac{\beta^{2}}{22-\beta^{2}}}{\frac{\beta^{2}}{2-\beta^{2}} \beta_{O}+1} \tag{45}
\end{equation*}
$$

We then equate Eqns (45) and (42), and solve for $\beta_{0} .{ }^{[7]}$ Discarding the solution which gives $\left|\beta_{O}\right|>1$, we have

$$
\begin{equation*}
\beta_{O}=\frac{\beta}{\beta+2} \tag{46}
\end{equation*}
$$

For the sake of completeness: Substituting Eqn (46) into Eqn (42) and simplifying, ${ }^{[8]}$ we have

$$
\begin{equation*}
\beta_{P}=-\frac{\beta}{\beta-2} \tag{47}
\end{equation*}
$$

We now know the velocity with which $S$ must travel relative to $O$ and $P$ in order to keep $P$ 's clocks appearing to tick at a rate equal to a factor of $\epsilon$ times $O$ 's clock rate. Given that we have a location for $S$ when its clock is reset to zero, and a constant velocity at which $O$ moves relative to $S$, which is equal to the velocity that $S$ moves relative to $O$, we now have a means of determining $S$ 's location at any time relative to $O$.

In accordance with SR , we know that the observer $S$ configured as above will observe that $\tau$ and $t$ appear simultaneous.

We see that for $|\beta|<1,\left|\beta_{O}\right|$ will be less than $|\beta|$. Thus $S$ will always be located between $O$ and $P$. Specifically, $S$ will be closer to $O$ when $\beta$ is positive, and closer to $P$ when $\beta$ is negative.

As $\beta$ approaches zero, $\beta_{O}$ and $\beta_{P}$ approach equality (at zero). This means that a location $S$ will approach equidistance to each of $O$ and $P$ as $\beta$ approaches zero, so that Lemma 3.2 and Lemma 3.3 together provide a formula for location $S$ as a continuous function of $\beta$.

Lemma 3.3 on its own does not work when $P$ and $O$ are separated but relatively at rest, because the means of synchronizing the clocks described in the proof does not apply. However, in such cases, observers at any location will see that the pair's clocks appear to tick at the same rate, as SR predicts. The matched clock rates in an inertial frame may give a false sense of universal time.

Lemma 3.4 (A change in relative simultaneity corresponds to a change in observer coordinates). Let events $E_{O 1}$ and $E_{O 2}$ be events at $O$, occurring respectively before and after an instantaneous change in relative simultaneity at $P$, and with a negligible proper time between the two events. Let $E_{P 1}$ and $E_{P 2}$ be events at $P$ that are classically simultaneous with $E_{O 1}$ and $E_{O 2}$ respectively, according to $O$. Let $\tau$ be the proper time between $E_{P 1}$ and $E_{P 2}$. Let event $E_{S}$ at $S$ appear to $S$ to be simultaneous with both $E_{O 1}$ and $E_{P 1}$, and event $E_{S^{\prime}}$ at $S^{\prime}$ appear to $S^{\prime}$ to be simultaneous with both $E_{O 2}$ and $E_{P 2}$.

The change in classical simultaneity measured by $O$ as a change in time at $P$ of $\tau$ is equivalent to the change in apparent time at $P$ measured by an abstract observer undergoing a change of coordinates from $E_{S}$ to $E_{S}^{\prime}$.

Note that $\tau$ may be negative.
Proof. This follows immediately from SR.
Any observer that is coincident with $E_{S}$ would observe events consistent with what $S$ observers at that moment, i.e. that $E_{S}$ and $E_{P 1}$ appear simultaneous. Similarly, any observer coincident with $E_{S}^{\prime}$ would observe that $E_{S}^{\prime}$ and $E_{P 2}$ appear simultaneous. Thus, in accordance with SR , any observer that is transformed from $E_{S}$ to $E_{S}^{\prime}$ will correspond with the appearance of a change in time at $P$ from $E_{P 1}$ to $E_{P 2}$, which is $\tau$.

Note that the clocks of $S$ and $S^{\prime}$ are irrelevant. A potential real observer that moves from $E_{S}$ to $E_{S}^{\prime}$, regardless of the path taken and proper time elapsed, will observe a change in apparent time at $P$ of $\tau$. Moreover, $S$ and $S^{\prime}$ are abstract, and there is no requirement that a real observer is able to move from $E_{S}$ to $E_{S}^{\prime}$, which should be impossible if $\tau$ is negative. A change in relative classical simultaneity is equivalent to switching from observation at $S$ to those at $S^{\prime}$; since the former causes no real physical effect independent of $O$, it is not required that a physical observer is able to switch from $S$ to $S^{\prime}$.

With the ability of an abstract observer to switch locations to suit any momentary simultaneity between $O$ and an observed location, any set of classically simultaneous events according to $O$ is consistently apparently simultaneous to this abstract observer.

Theorem 3.5 (The classical interpretation reduces to the apparent). Given a set of pairs of events, where each pair consists of one event at $P$ that is classically simultaneous (according to $O$ ) with the other event at $O$, each pair will be apparently simultaneous according to some observer $S$, where $S$ an abstract observer with unrestricted mobility (not limited to physically possible movement through space or time).

Proof. This follows for any case of discrete velocities and instantaneous changes between them, from repeated application of Lemmas 3.1 through 3.4. The continuous case can be made by integration (not provided) of the discrete case.

Extending this to all events, pairing each with all possible other simultaneous events according to any observer, the preceding Theorem implies that reality described in terms of classical simultaneity corresponds to reality described by apparent simultaneity, as seen by an abstract observer. Since observations from different locations are all mutually consistent, this explains how both the classical and apparent interpretations provide mutually consistent models of the same reality. Both interpretations describe direct (apparent) observations of the same events, but with different apparent simultaneity as measured at different observer locations.

### 3.1 Third-person Observers and Multiple Spatial Dimensions

Any observer $S$ described in this section is essentially a third-person perspective, not directly involved in the signal transmission from $P$ to $O$. Observer $S$ can neither observe the photons comprising the signal sent from $P$, nor can it
directly measure the reception of the signal at $O$. Observation of either requires indirect or secondary signals sent from each of the primary signal's transmission and reception events, to $S$. This happens to be the typical way that events are experienced in the universe, as information from an event is broadcast to multiple observers, while the reception of information (in the form of energy) causes a change in the observer which effects additional broadcasting of information. Typically there is much information available to an observer about what other observers see. Indirect information about signals sent between others may of course be expressed in terms of only those signals which eventually make their way directly to a given observer, calculated from the perspective of each intermediate observer who relays any information, however this is consistent with the simplification of treating any unobserved signals as propagating at a rate of $c$, regardless of direction.

Expressed using Hans Reichenbach's $\varepsilon$-notation, the apparent timing of a signal across a distance of $d$, according to a third-person observer, will appear to take a time of $2(1-\varepsilon) d / c$, for some $\varepsilon$ in $[0,1]$ which depends on the positioning of the observer relative to the signal's origin and end point. In the case of an observer coincident with the recipient of a signal, $\varepsilon$ is one, corresponding to the apparent interpretation. In the case of an observer coincident with the sender, $\varepsilon$ is zero. Each $\varepsilon$ value corresponds to a set of observer positions and to an interpretation similar to the apparent interpretation, except applied to indirectly observed signals. The apparent interpretation is a specific case of a more general interpretation that considers the experiences of all possible observers, including any special observer $S$, whose observations are predicted by classical SR equations. An interpretation generalized for all possible indirect observer locations essentially includes the classical and apparent interpretations. Any such interpretations are equivalent exactly as observations made from different locations are equivalent

Note that in the rest case, a special observer $S$ with its $\varepsilon$ value of $1 / 2$ corresponds both to an "unbiased" symmetrical observer that is equidistant to both $P$ and $O$, as well as to the average of all possible other observers and their corresponding $\varepsilon$ values. The same $\varepsilon$ value can be arrived at by assuming equal timing of inbound and outbound light, without ever referring to the specific observer location $S$. Without needing to specify $S$, the classical interpretation thus provides a simplification by abstracting some location-dependent aspects of observations. The cost of the simplification is the classically interpreted delay of light, an adjustment to direct measurements of time, which translates an observer $O$ 's measurements into what would be measured by $S$. The simplification is useful in cases where no single observer is especially important, such as the interaction of multiple observers that are relatively at rest, including most human-scale experiences. It is possibly misleading in cases where the observer or its location is important and is not $S$, such as with quantum mechanics and astronomy.

The conceptual differences between the interpretations then become a matter of metaphysics, and of convenience, and are irrelevant because the interpretations are interchangeable.

### 3.2 Correspondence of Standard Second Postulate

The range of observer experiences means a range of possible apparent simultaneities within a single inertial frame. This is fine for the apparent interpretation because simultaneity is defined per observer, not per frame.

The standard second postulate avoids this problem by narrowing to one the possible choices of simultaneity within an inertial frame, thereby also precluding the apparent interpretation, because differently located observers do not generally observe a given light signal appearing to propagate at a fixed rate in all directions. However, correspondence of the interpretations allow the standard second postulate to be derived.

Theorem 3.6. A measurement of the classical speed of any light signal measured from any inertial frame, is equivalent to an apparent measurement of the same signal at some location in the same inertial frame, from where the signal appears to propagate at a rate of $c$.

Proof. Consider the inertial frame of an arbitrary observer $Q$, and a light signal from $P$ to $O$, as indirectly measured by $Q$.

Let points $P^{\prime}$ and $O^{\prime}$ share the inertial frame of $Q$, and let $P^{\prime}$ be coincident with $P$ at the time of transmission, and $O^{\prime}$ be coincident with $O$ at the time of observation. It follows from SR that the signal from $P$ to $O$ is equivalent to an identically measured signal from $P^{\prime}$ to $O^{\prime}$.

Considering this equivalent signal, there is by Lemma 3.2 a point $S$ from which the signal appears to have propagated at a rate of $c$, and which is also at rest relative to $Q$. Thus the classical speed of the signal, measured by $Q$ to be $c$, is the same as the apparent speed of the signal measured by $S$.

In any given inertial frame of reference, for any set of $n$ light signals (i.e. observations of events), there is a set $\left\{S_{1}, \ldots, S_{n}\right\}$, where each $S_{i}$ observes the corresponding $i$ 'th signal appear to propagate at $c$. This set of observers can be used by all observers in the inertial frame, as delegate measurement references, since what is measured at each $S_{i}$ is consistent with what is measured by any observer $Q$. And since the apparent time and classical time of an event at $S_{i}$ as observed by $Q$ differ by $d / c$, where $d$ is the distance between the observers at rest, the use of $S_{i}$ as a measurement reference is equivalent to adjusting a direct observation by $Q$ for a delay of light proportional to distance. And finally, because the set $\left\{S_{1}, \ldots, S_{n}\right\}$ is specifically chosen to agree with the classical speed of light, any observer which uses a measurement reference equivalent to one from that set will measure the corresponding signal as having a speed of $c$.

Thus, an observer in an inertial frame doesn't have to experience the appearance of a signal traveling at $c$, because it can equivalently delegate its measurements to a set of alternate observers, which do experience it. Classical SR is the equivalent of using the observations from one location to define the measurements at another, which is consistent due to the the agreement of different observers. With this perspective, the validity of the standard second postulate need not be either a physical law, nor a mere coincidence.

### 3.3 Equivalence of Time and Distance

Lemma 2.1 and Lemma 2.2 imply the equivalence of time and distance, whereby anything moving through a given distance will experience a proportional pass-
ing of time, and anything experiencing a given passing of time will cover a proportional distance, if for $v<c$ any additional experienced passing of time is expressed in terms of light or information traveling at $c$ across well-defined distances and subject to the same Lemmas.

The equivalence is interpretation-dependent, however I will attempt to make it independent with the following:

Theorem 3.7 (Equivalence of time and distance). Given the consistency of the classical interpretation of the predictions of $S R$, another consistent interpretation can be constructed by converting a measure of distance into time or vice versa.

Proof. It is demonstrated herein for the case of converting between $\tau_{\mathrm{A}}$ and $\tau_{\mathrm{C}}$, having established that a physical simultaneity of $\tau_{\mathrm{A}}$ and $t$ does not violate the alternative postulates of SR , and that the classical second postulate can be derived from the alternative postulate.

This theorem only weakly implies an equivalence of time and distance, arguing the case for only one interpretation (representing the experience of $O$ ) without ruling out coincidence. It is nevertheless possible to construct similar interpretations to represent the experiences of other arbitrarily located observers, and their measurement of the apparent timing of signals from $P$ to $O$.

## 4 Interpretation and Speculation

The apparent interpretation essentially follows from SR with an additional principle that time is different if measured from different locations.

### 4.1 Equivalence of Time and Distance

The equivalence of time and distance makes more sense if we consider temporal analogies to the distinct spatial notions of cumulative distance, and displacement. By analogy we would respectively have the conventional notion of cumulative time (i.e. elapsed time or ageing), and a new notion of temporal displacement. There is no distinct classical notion of temporal displacement; its equivalent is the travel time of incoming light, which is expressed in terms of ordinary elapsed time.

Ageing and temporal displacement have different properties. Ageing, like traveled distance, is a strictly increasing value, producing the characteristic arrow of time. Temporal and spatial displacement can either increase or decrease, and might provide a preferable notion of time for describing quantum mechanical interactions that allow paths of particle that go backward in time.

Equivalence of time and distance implies that any change in one must be accompanied by a change in the other. Lemma 2.1 already establishes that any change in relative distance corresponds to a change in relative time. If any change in time involves a proportional change in distance, then anything moving at subluminal speeds must be decomposable into some form of information that is traveling at $c$. This might manifest as oscillation (accumulation of a relatively large distance within a relatively small displacement), or perhaps as space-time
curvature (with light traveling in a straight line which to the observer appears to curve into a compact space, possibly implying local singularities).

A consequence of equivalence is that velocity can be expressed as a ratio of distances, or of times. We already see that it is natural for velocities to be expressed as a ratio of $c$. Velocity might also have meaning as a ratio of displacement to the distance covered by information propagating at $c$.

Also implied is that time cannot be defined at a single point. A point particle cannot measure a specific duration except relative to some other location. This predicts that there must be some corresponding measure of distance in any particle that can transition between states in a well-defined time.

If time and distance have no meaning independent of given observations, then time and distance could be considered emergent properties that are nothing more than a consistent ordering of events in space and time. Though aspects of the same measurements, displacements are experienced as length, and cumulative space-time distances are experienced as time.

### 4.1.1 Sign of Time

Lemma 1.6, converting time between classical and apparent interpretations, is a conversion between distance and time. The constant $-1 / c$ is the conversion factor.

Using natural units and reversing the sign of time, the conversion factor becomes 1, intuitively conveying the equivalence of time and distance. A negation of conventional time values assigns a greater time value to past events relative to later events. Since we are able to arbitrarily reset clocks, we can conceive of a convention whereby an observer's time remains fixed at 0 , establishing a temporal coordinate origin. In the observer's coordinates, the observer location remains at its origin in both location and time. A perfect local clock would again be meaningless, as it would require some distance to define a relative time. While the observer's time is considered to be fixed rather than passing through increasing values relative to some fixed past event, the relative times of past events continuously increase relative to an observer's "now". This provides an intuitive analog to length, where the temporal distance to a past event increases as though it is receding from the observer.

By analogy, the difference in conventions is similar to the difference between fixing the zero mark of a tape measure to a location and then moving away while carrying the tape measure, versus leaving the tape measure and taking the zero end of the tape with you. Either way, a positive distance is measured. While leaving a running clock in the past makes no physical sense, this is only a problem if one were to argue that the absolute time of a clock or event has physical meaning, rather than being arbitrary and strictly relative.

### 4.2 Apparent Properties of the Present

A signal sent to $O$ appears instantaneous to $O$, but generally not instantaneous to other observers. This may be described as apparent relativity of instantaneity. Considering events to be not extended in time, simultaneity of a set of events entails the sharing of a single instant among the events, thus relativity of instantaneity implies relativity of simultaneity, and vice versa.

Defining the apparent present as all events apparently simultaneous with an arbitrarily chosen event at $O$ labeled Now, the apparent present is the past light
cone of Now. This defines a moment in time according to what is immediately observed at $O$.

The apparent instantaneity of a signal from $P$ to $O$ is only an interpretation based on what $O$ measures, and what is apparent to $P$ provides an equally valid interpretation. A signal sent from $O$ to $P$ appears instantaneous to $P$, yet appears to $O$ to take time to propagate. This signal, on the future light cone of the transmission event at $O$, and conveying what $P$ can observe of $O$, also represents what $O$ can "now" affect at $P$. The future light cone can define for $O$ a moment in time in terms of the set of events that $O$ can effect in a given instant.

Using the apparent interpretation, no event that occurs outside of the past light cone of Now would be considered to have happened yet. The event may only be predicted to later happen, and is a part of $O$ 's apparent future.

Similarly, any event that occurs at $P$ as an effect of a signal sent from $O$ coincident with Now, cannot be affected by $O$ after Now. According to SR, the effected event cannot be brought into the future light cone of Now. Classically, we might say that if a change in inertia is made to attempt this, a corresponding change in relative simultaneity would prevent it. In the apparent interpretation, if such an attempt were made, we would see that time at $P$ appears to pass quickly, so that any reduction in the temporal offset between $O$ and $P$ is accompanied by the appearance of $P$ receiving $O$ 's signals at an advanced rate. This means that anything outside of the future light cone of Now cannot be changed. Thus any event outside of the future light cone of Now is a part of O's "effective" past, and Now's future light cone can define an effectible present for $O$.

Classically, the present may be thought of as an instantaneous "knife edge" between past and present, defined per inertial frame but occurring at every spatial location within a given frame. Instead, with this alternative interpretation, the present may be defined causally, including all events lying between a cause of Now and an effect of Now. The past is then anything that can have already been observed, and the future is anything that can yet be affected; everything in between - anything outside of Now's light cone - is the present, which is only locally instantaneous.

This idea of a time-extended present at spatially remote locations, incorporates the apparent present and the effectible present of $O$, or equivalently we might say that it incorporates the apparent presents of both $O$ and $P$ (for all possible remote points $P$ ). It allows events in any observer's present to be universally in all observers' presents, by considering the present to be not universally instantaneous. However, such a definition of present incorporating relative instantaneity would be defined by causality and not by simultaneity of events.

In other words, if we allow a present that is not instantaneous, we are able to have a universal shared present, which has relative extent. This must also acknowledge that extreme distances correspond with extreme relativity of age.

### 4.3 Behaviour of Light

The classical interpretation implies that photons travel progressively through all intervening space between its source and destination, which does not fit well with an interpretation in which the journey can be measured as a single instant.

Rather, it is noted that the effects of a photon are apparent only at its source and destination, and so it appears to leap across the intervening space.

Considering this from third-person points of view, we can describe a signal as a transfer of energy or information from one location and time, to another location and time, where the distance covered is consistently $c$ times the difference in time. There is no need to define light's behaviour except at the point of transmission and that of reception. This opens the question of whether light transmissions without definite receivers are possible, and if so whether it involves a loss of energy or if conservation laws require that photons must exist independent of observation.

## 5 Objections to Apparent Simultaneity

### 5.1 Validity of Einstein Clock Synchronization

There is an apparent error in Einstein-synchronized clocks if they are viewed asymmetrically. Classically, the error corresponds to a proportional delay of light, which is considered separately from synchronization errors. Second-order apparent relativistic effects can be minimized by minimizing relative velocity, but first-order effects are proportional to separation distance, making separated clocks impossible to generally appear synchronized.

Einstein synchronization is valid if the first-order relativistic effects are disregarded, which implies the assumption of an interpretation which disregards them. Thus this method assumes, rather than proves, the exclusivity of the classical interpretation.

### 5.2 Non-Conventionality of Simultaneity

The arguments presented in this paper do not require conventionality of simultaneity. Rather, they require conventionality of interpretation. Each interpretation may in turn have its own unique definition of simultaneity, allowing non-conventionality of simultaneity within that interpretation.

The apparent interpretation indeed seems to favour non-conventionality. Assuming that a quantum event may appear to a given observer at no more than one local observation time, there is for that observer a unique apparent simultaneity of events. The uniqueness is per observer, but not per inertial frame. Conventionality of interpretation is equivalent to conventionality - or choice - of observer, which is better represented as relativity of simultaneity, rather than conventionality thereof. These differences seem to be a matter of philosophy, without objective physical meaning.

### 5.3 Action At a Distance

The behaviour of light described herein fits the definition of action at a distance as characterized in [9] as "without there being a process that carries this influence contiguously in space and time." While instantaneous action at a distance would imply violation of causality if interactions are symmetric, apparent interaction between given distant objects is not symmetric according to any observer that measures an apparently instantaneous action between the objects.

Two-way instantaneous interaction certainly violates causality, but one-way instantaneous action does not necessarily.

### 5.4 Common Sense Objections

The classical assumption of equal timing of incoming and outgoing light is consistent with SR and so tends to be taken for granted, however it is demonstrated not a necessary assumption. The apparent interpretation offers an "alternative common sense" that presumes the principles of relativity, but abandons any assumption of universality of time.

While there is no requirement that a theory obey common sense, there remain issues with the apparent interpretation that nevertheless clash with it. One problem is that the interpretation essentially implies that things are exactly as they appear, yet they don't always behave as though they are. For example, massive objects behave with respect to gravity as though their distance is different than it appears. This in turn suggests the classical interpretation's view that apparent distances are illusory while there is an underlying "real" distance governing physical processes.

A possible resolution to this problem is that there may be different distances for different interactions or fields, just as there are different distances for different inertial frames, and different coapparent distances for different observers. Electromagnet and gravitational force vary differently as functions of distance; it may be possible to express these differences using different measures of distance.

Such conceptual difficulties of the apparent interpretation do not go against the philosophy of relativity, but rather take it to an extreme.

If it is a physical reality that photons objectively travel slowly, then the apparent interpretation is a delusion. The definition of coapparent distance may be hiding a two-way measure of light. The infinite sum of Eqn (11) also applies to the classical interpretation as a sum of terms of the time that it takes light to travel the change of distance that occurs during the previous term. However, the argument stands that slow traveling light is based on assumptions, and that empirical evidence does not disprove either interpretation.

## Conclusions

SR has always predicted the appearance of events, and incorporating the apparent interpretation is as easy as no longer distinguishing between what is seen and what is real. This implies accepting Doppler shift analysis as representative of reality, and accepting that the reality of the Lorentz transformations may require an abstract substituted observer.

Assuming that the apparent interpretation could be formulated without using the classical interpretation, but from appropriate alternative postulates alone, the derivation of the classical second postulate implies that the postulates of relativity alone do not produce a unique interpretation. Instead, uniqueness is provided by choice of measurement definitions, or alternatively by choice of observer position. Evidence of the existence of photons independent of observation would likely invalidate the interpretation.

The validity of different interpretations implies that neither describes a fundamental underlying physical reality, but each seems instead to only describe
an observer's experience of the physical universe. Classical SR, describing an average of observers and their interactions, more intuitively models the human experience, which combines many individual observations into one consistent understanding of the world. The apparent interpretation better models the experience of an individual point observation. The different types of experience, of course, remain mutually consistent.

Conversely, if the different interpretations are equally physically real, the results that are specific to one interpretation must not be. This includes many unseen effects of classical SR, such as modifiable simultaneity of events for a given observer, as with the notion of events that have already happened yet remain temporarily unmeasurable, and may be undone by a change of inertial frame. Without this notion, there may be less justification for block universe or multi-world interpretations.

The apparent interpretation allows much conceptual simplification of SR, where many of its unintuitive aspects can be explained or avoided through a simple delegation of observers.

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