Time dependent Schrödinger equation for black hole evaporation: no information loss

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Abstract

In 1976 S. Hawking claimed that "*Because part of the information about the state of the system is lost down the hole, the final situation is represented by a density matrix rather than a pure quantum state*"[1]. This was the starting point of the popular "black hole (BH) information paradox".

In a series of papers, together with collaborators, we naturally interpreted BH quasi-normal modes (QNMs) in terms of quantum levels discussing a model of excited BH somewhat similar to the historical semi-classical Bohr model of the structure of a hydrogen atom. Here we explicitly write down, for the same model, a *time dependent Schrödinger equation* for the system composed by Hawking radiation and BH QNMs. The physical state and the correspondent *wave function* are written in terms of an *unitary* evolution matrix instead of a density matrix. Thus, the final state results to be a *pure* quantum state instead of a mixed one. Hence, Hawking's claim is falsified because BHs result to be well defined quantum mechanical systems, having ordered, discrete quantum spectra, which respect 't Hooft's assumption that Schröedinger equations can be

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[1]Verbatim from ref. 2

used universally for all dynamics in the universe. As a consequence, information comes out in BH evaporation in terms of pure states in an unitary time dependent evolution.

In Section 4 of this paper we show that the present approach permits also to solve the entanglement problem connected with the information paradox.

To the memory of the latter IFM Secretary Franco Pettini.

1 Introduction: the black hole information paradox

One of the most famous and intriguing scientific controversies in the whole history of Science is the so called "BH information paradox". In classical gravity, a BH is the definitive prison. Nothing can escape from it. Thus, when matter disappears into a BH, the information encoded is considered as preserved inside it, although inaccessible to outside observers. The situation radically changed when Hawking discovered that quantum effects cause the BH to emit radiation [1]. A further analysis, again by Hawking [2], has shown that the detailed form of the radiation emitted by a BH should be thermal and independent of the structure and composition of matter that collapsed to form the BH. Hence, the radiation state is considered a completely mixed one which cannot carry information about how the BH is formed.

After Hawking's original claim, enormous time and effort were and are currently devoted to solve the paradox. Notice that consequences of the BH information puzzle are not trivial. If information was lost in BHs, pure quantum states arising from collapsed matter would decay into mixed states arising from BH evaporation and quantum gravity wouldn't be unitary [3]! Various scientists worked and currently work on this issue. Some of them remained convinced that quantum information should be destroyed in BH evaporation. Other ones claimed that Hawking's original statement was wrong and information should be, instead, preserved. Susskind wrote a pretty and popular science book on details of the so called "Black Hole War" [4]. In fact, the paradox was introduced into physics folklore [4, 5]. Hawking made two famous bets, one, with Thorne like co-signer, with Preskill, another with Page, that BH does destroy information [4]. After almost 30 years, Hawking reversed his opinion and agreed that information would probably be recovered [3]. Historical notes on the paradox's controversy and on various attempts to solve it can be found in [3]-[7]. Recently, Hawking changed again his opinion by verbatim claiming that "there is effective information loss" [36].

A key point, not only in the framework of the BH information paradox, but in the whole BH quantum physics, is that analysing Hawking radiation as tunnelling, Parikh and Wilczek showed that the radiation spectrum cannot be strictly thermal [8, 9], differently from Hawking's original computations [1, 2]. The energy conservation enables the BH to contract during the process of radiation [8, 9]. Thus, the horizon recedes from its original radius to a new,

smaller radius [8, 9]. As a consequence, BHs cannot strictly emit thermally [8, 9]. This is consistent with unitarity [8] and has profound implications for the BH information paradox. In fact, Parikh [8] correctly stresses that arguments that information is lost during BH evaporation rely in part on the assumption of strict thermal behavior of the spectrum [2]. Assuming the non-thermal spectrum of Parikh and Wilczek, Zhang, Cai, Zhan and You recently found the existence of correlations among Hawking radiations which are elegantly described as hidden messengers in BH evaporation [10, 11, 12]. Thus, they claimed to have recovered the information loss in Hawking radiation and to have solved the paradox [10, 11, 12]. This issue generated a controversy with Mathur, who claimed that, instead, they failed to solve the paradox [13]. Mathur thinks that the foundation of the information problem is the growing entanglement entropy between the inside and outside of the BH [7, 13]. He also claims that only string theory, in the framework of the so called "fuzzball", can ultimately solve the paradox. The subsequent strong rebuttal by Zhang, Cai, Zhan and You [14] claims that, instead, Mathur's argument on the growing entanglement entropy is wrong and the correlations that they found are sufficient to resolve the paradox. This controversy has an important scientific value, as the contenders received the First Award [15] and Third Award [16] respectively in the important Gravity Research Foundation Essay Competition 2013 for their works on the information paradox. In this work, we discuss our proposal to solve the information puzzle. Following 't Hooft [6], we think that the foundation of the BH information problem is that BHs look to do not obey Schröedinger equations, which would allow pure states to evolve only into pure states. They look indeed to obey probabilistic equations of motion that are not purely quantum mechanical [6]. We will show that, instead, a time dependent Schröedinger equation allowing pure states to evolve only into pure states can be found in the correspondence between Hawking radiation and BH QNMs that we recently discussed in a series of papers [17, 18, 19], also together with collaborators [20, 38, 39]. In those papers, we naturally interpreted BH QNMs in terms of quantum levels discussing a model of excited BH somewhat similar to the historical semi-classical Bohr model of the structure of a hydrogen atom [31, 32]. In Section 4 of this paper, we show that the present approach permits also to solve the entanglement problem, discussed in [13], connected with the information puzzle.

We also stress that consistence between the analysis in the present paper and a recent approach to solve the BH information paradox [10, 11, 12, 14] has been recently highlighted in [40].

2 Quasi-normal modes in non-thermal approximation

BH QNMs are frequencies of radial spin-j perturbations which obey a time independent Schröedinger-like equation, see [17, 18, 19, 21]. They are the BH modes of energy dissipation which frequency is allowed to be complex [21]. The

intriguing idea to model the quantum BH in terms of BH QNMs arises from a remarkable paper by York [22]. For large values of the quantum "overtone" number n, where $n = 1, 2, ...,$ QNMs become independent of both the spin and the angular momentum quantum numbers [17, 18, 19, 21, 23, 24], in perfect agreement with *Bohr's Correspondence Principle* [33], which states that "transition frequencies at large quantum numbers should equal classical oscillation frequencies". In other words, Bohr's Correspondence Principle enables an accurate semi-classical analysis for large values of the principal quantum number n, i.e, for excited BHs. By using that principle, Hod has shown that QNMs release information about the area quantization as QNMs are associated to absorption of particles [23]. Hod's work was refined by Maggiore [24] who solved some important problems. On the other hand, as QNMs are *countable* frequencies, ideas on the *continuous* character of Hawking radiation did not agree with attempts to interpret QNMs in terms of emitted quanta, preventing to associate QNMs modes to Hawking radiation [21]. Recently, Zhang, Cai, Zhan and You [10, 11, 12, 14, 15] and ourselves and collaborators [17, 18, 19, 20] observed that the non-thermal spectrum of Parikh and Wilczek also implies the countable character of subsequent emissions of Hawking quanta. This issue enables a natural correspondence between QNMs and Hawking radiation, permitting to interpret QNMs also in terms of emitted energies [17, 18, 19, 20]. In fact, QNMs represent the BH reaction to small, discrete perturbations in terms of damped oscillations [17, 18, 19, 20, 21, 22, 23, 24]. The capture of a particle which causes an increase in the horizon area is a type of discrete perturbation [21, 22, 23, 24]. Then, it is very natural to assume that the emission of a particle which causes a decrease in the horizon area is also a perturbation which generates a reaction in terms of countable QNMs as it is a discrete instead of continuous process [17, 18, 19, 20]. On the other hand, the correspondence between emitted radiation and proper oscillation of the emitting body is a fundamental behavior of every radiation process in Science. Based on such a natural correspondence between Hawking radiation and BH QNMs, one can consider QNMs in terms of quantum levels also for emitted energies [17, 18, 19, 20]. This important point is in agreement with the general idea that BHs can be considered in terms of highly excited states in an underlying quantum gravity theory [17, 18, 19, 20]. Working with $G = c = k_B = \hbar = \frac{1}{4\pi\epsilon_0} = 1$ (Planck units), in strictly thermal approximation the probability of emission of Hawking quanta is [1, 8, 9]

$$\Gamma \sim \exp(-\frac{\omega}{T_H}), \tag{1}$$

where ω is the energy-frequency of the emitted particle and $T_H \equiv \frac{1}{8\pi M}$ is the Hawking temperature. By taking into account the energy conservation, i.e. the BH contraction which enables a varying geometry, one gets the fundamental correction of Parikh and Wilczek [8, 9]

$$\Gamma \sim \exp[-\frac{\omega}{T_H}(1 - \frac{\omega}{2M})], \tag{2}$$

where the additional term $\frac{\omega}{2M}$ is present. We have recently finalized the Parikh and Wilczek tunnelling picture showing that the probability of emission (2) is

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indeed associated to the two distributions [25]

$$< n >_{boson} = \frac{1}{\exp\left[4\pi\left(2M - \omega\right)\omega\right] - 1}, \quad < n >_{fermion} = \frac{1}{\exp\left[4\pi\left(2M - \omega\right)\omega\right] + 1}, \tag{3}$$

for bosons and fermions respectively, which are *non*-strictly thermal. It is important to stress that, as it is always $\omega \leq M$ because the BH cannot emit more energy than its total mass, we have neither negative number of particles nor divergences for finite values of $\omega > 0$ in eq. (3) [42].
By introducing the *effective temperature* [17, 18, 19, 20, 25]

$$T_E(\omega) \equiv \frac{2M}{2M - \omega}T_H = \frac{1}{4\pi(2M - \omega)}, \tag{4}$$

one rewrites eq. (4) in a Boltzmann-like form similar to eq. (1)

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp(-\frac{\omega}{T_E(\omega)}), \tag{5}$$

where $\exp[-\beta_E(\omega)\omega]$ is the *effective Boltzmann factor,* with $\beta_E(\omega) \equiv \frac{1}{T_E(\omega)}$. Thus, the effective temperature replaces the Hawking temperature in the equation of the probability of emission [17, 18, 19, 20, 25]. The effective temperature depends on the energy-frequency of the emitted radiation and the ratio $\frac{T_E(\omega)}{T_H} = \frac{2M}{2M - \omega}$ represents the deviation of the BH radiation spectrum from the strictly thermal feature [17, 18, 19, 20, 25]. It is better to clarify the definition of effective temperature [42] that we introduced in BH physics in [17, 18]. The probability of emission of Hawking quanta found by Parikh and Wilczek, i.e. eq. (2), shows that the BH does NOT emit like a perfect black body, i.e. it has not a strictly thermal behavior. On the other hand, the temperature in Bose-Einstein and Fermi-Dirac distributions is a perfect black body temperature. Thus, when we have deviations from the strictly thermal behavior, i.e. from the perfect black body, one expects also deviations from Bose-Einstein and Fermi-Dirac distributions. How can one attack this problem? By analogy with other various fields of Science, also beyond BHs, for example the case of planets and stars. One defines the effective temperature of a body such as a star or planet as the temperature of a black body that would emit the same total amount of electromagnetic radiation [41]. The importance of the effective temperature in a star is stressed by the issue that the effective temperature and the bolometric luminosity are the two fundamental physical parameters needed to place a star on the Hertzsprung–Russell diagram. Both effective temperature and bolometric luminosity actually depend on the chemical composition of a star, see again [41].

On the other hand, one recalls that the definition of temperature in Bose-Einstein or Fermi-Dirac distribution comes from the coefficient of ω in the exponential and that is itself "independent" of frequency [42]. But the key point here is that we have a deviation from the perfect thermal state and, in turn, we expect deviations from the exact Bose-Einstein and Fermi-Dirac distributions.

We also stress that the tunnelling is a *discrete* instead of *continuous* process as two different *countable* BH physical states have to be considered, the first before the emission of the particle and the latter after the emission of the particle. The emission of the particle is, in turn, interpreted like a *quantum transition* of frequency ω between the two different discrete states. In fact, the tunnelling mechanism works considering a trajectory in imaginary or complex time which joins two separated classical turning points[8, 9]. As a consequence the radiation spectrum is also discrete. This important issue needs to be clarified in a better way. At a well fixed Hawking temperature the statistical probability distribution (2) is a continuous function. But the Hawking temperature in (2) varies in time with a character which is *discrete* because the forbidden region traversed by the emitting particle has a *finite* size [8, 9]. If one considers a strictly thermal approximation, the turning points have zero separation and it is not clear what joining trajectory has to be considered because there is not barrier [8]. One solves the problem if argues that it is the forbidden finite region from $r_{initial} = 2M$ to $r_{final} = 2(M-\omega)$ that the tunnelling particle traverses which works like barrier [8]. In other words, the intriguing explanation is that it is the particle itself which generates a tunnel through the horizon [8]. In this way, one obtains the effective temperature also in the deviation from the exact Bose-Einstein or Fermi-Dirac distribution. In fact, by using the definition (4) one can easily rewrite eq. (3) as

$$< n >_{boson} = \frac{1}{\exp\left(\frac{\omega}{T_E(\omega)}\right) - 1}, \quad < n >_{fermion} = \frac{1}{\exp\left(\frac{\omega}{T_E(\omega)}\right) + 1}, \quad (6)$$

In other words, one takes an intermediate value between the two subsequent values of the Hawking temperature, i.e. the effective temperature. Thus, the introduction of the effective temperature does not degrade the importance of the Hawking temperature. In fact, as the Hawking temperature changes with a discrete behavior in time, in a certain sense the effective temperature represents the value of the Hawking temperature *during* the emission of the particle. Hence, the effective temperature takes into account the non-strictly thermal character of the radiation spectrum and the non-strictly continuous character of subsequent emissions of Hawking quanta.

The introduction of the effective temperature permits the introduction of other *effective quantities*. Considering the initial BH mass *before* the emission, M, and the final BH mass *after* the emission, $M - \omega$, one introduces the *BH effective mass* and the *BH effective horizon*

$$M_E \equiv M - \frac{\omega}{2}, \ r_E \equiv 2M_E \qquad (7)$$

during the BH contraction, i.e. *during* the emission of the particle [17, 18, 19, 20, 25]. Such effective quantities are average quantities [17, 18, 19, 20, 25]. In fact, r_E is the average of the initial and final horizons while M_E is the average of the initial and final masses [17, 18, 19, 20, 25]. The effective temperature T_E is the inverse of the average value of the inverses of the initial and final

Hawking temperatures (*before* the emission T_H initial $= \frac{1}{8\pi M}$, *after* the emission T_H final $= \frac{1}{8\pi(M-\omega)}$)[17, 18, 19, 20, 25]. We have also recently shown [25] that one can use Hawking's periodicity argument [28, 29, 30] to obtain the *effective Schwarzschild line element* [25]

$$ds_E^2 \equiv -(1 - \frac{2M_E}{r})dt^2 + \frac{dr^2}{1 - \frac{2M_E}{r}} + r^2(\sin^2\theta d\varphi^2 + d\theta^2), \tag{8}$$

which takes into account the BH *dynamical* geometry during the emission of the particle.

The introduction of $T_E(\omega)$ can be applied to the analysis of the spectrum of BH QNMs [17, 18, 19, 20]. In fact, another key point is that the equation of the BH QNMs frequencies in [21, 23, 24] is an approximation as it has been derived with the assumption that the BH radiation spectrum is strictly thermal. To take into due account the deviation from the thermal spectrum one has to replace the Hawking temperature T_H with the effective temperature T_E in the equation of the BH QNMs frequencies [17, 18, 19, 20]. For large values of the principal quantum number n, i.e, for excited BHs, and independently of the angular momentum quantum number, the expression for the quasi-normal frequencies of the Schwarzschild BH, which takes into account the non-strictly thermal behavior of the radiation spectrum is [17, 18, 19, 20]

$$\omega_n = a + ib + 2\pi i n \times T_E(|\omega_n|)$$
$$\simeq 2\pi i n \times T_E(|\omega_n|) = \frac{in}{4M - 2|\omega_n|}, \tag{9}$$

where a and b are real numbers with $a = (\ln 3) \times T_E(|\omega_n|)$, $b = \pi \times T_E(|\omega_n|)$ for $j = 0, 2$ (scalar and gravitational perturbations), $a = 0$, $b = 0$ for $j = 1$ (vector perturbations) and $a, b \ll |2\pi i n T_E(|\omega_n|)|$. In complete agreement with Bohr's correspondence principle, it is trivial to adapt the analysis in [21] in the sense of the Appendix of [19] and, in turn, to show that the behavior (9) holds if j is a half-integer too. A fundamental consequence of eq. (9) is that the quantum of area obtained from the asymptotics $|\omega_n|$ is an intrinsic property of Schwarzschild BHs because for large n the leading asymptotic behavior of $|\omega_n|$ is given by the leading term in the imaginary part of the complex frequencies, and it does not depend on the spin content of the perturbation [17, 18, 19, 20, 24]. An intuitive derivation of eq. (9) can be found in [17, 18]. We rigorously derived such an equation in the Appendix of [19]. Eq. (9) has the following elegant interpretation [17]. The quasi-normal frequencies determine the position of poles of a Green's function on the given background, and the Euclidean BH solution converges to a *non-strictly* thermal circle at infinity with the inverse temperature $\beta_E(\omega_n) = \frac{1}{T_E(|\omega_n|)}$ [17]. Thus, the spacing of the poles in eq. (9) coincides with the spacing $2\pi i T_E(|\omega_n|) = 2\pi i T_H(\frac{2M}{2M-|\omega_n|})$, expected for a *non-strictly* thermal Green's function [17].

3 Bohr-like model and time dependent Schrödinger equation

We found the physical solution for the absolute values of the frequencies (9) in [17, 18]. One gets

$$E_n \equiv |\omega_n| = M - \sqrt{M^2 - \frac{n}{2}}. \tag{10}$$

E_n is interpreted like the total energy emitted by the BH at that time, i.e. when the BH is excited at a level n [17, 18, 19, 20]. Considering an emission from the ground state to a state with large n and using eq. (10), the BH mass changes from M to

$$M_n \equiv M - E_n = \sqrt{M^2 - \frac{n}{2}}. \tag{11}$$

In the transition from the state with n to a state with $m > n$ the BH mass changes again from M_n to

$$\begin{aligned} M_m \equiv M_n - \Delta E_{n \to m} &= M - E_m \\ &= \sqrt{M^2 - \frac{m}{2}}, \end{aligned} \tag{12}$$

where $\Delta E_{n \to m} \equiv E_m - E_n = M_n - M_m$ is the jump between the two levels due to the emission of a particle having frequency $\omega_{n,m} = \Delta E_{n \to m}$. The BH model that we analysed in [17, 18, 19] is somewhat similar to the semi-classical Bohr model of the structure of a hydrogen atom [31, 32, 33]. In our BH model [17, 18, 19], during a quantum jump a discrete amount of energy is indeed radiated and, for large values of the principal quantum number n, the analysis becomes independent of the other quantum numbers. In a certain sense, QNMs represent the "electron" which jumps from a level to another one and the absolute values of the QNMs frequencies represent the energy "shells" [17, 18, 19]. In Bohr model [31, 32, 33] electrons can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation with an energy difference of the levels according to the Planck relation $E = hf$, where h is the Planck constant and f the transition frequency. In our BH model [17, 18, 19], QNMs can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation (emitted radiation is given by Hawking quanta) with an energy difference of the levels according to equations which are in full agreement with previous literature of BH thermodynamics, like references [24, 34, 35]. More, the BH model in [17, 18, 19] is also in agreement with the famous result of Bekenstein on the *area quantization* [37]. In fact, we found an area quantum arising from a jump among two neighbouring quantum levels $n - 1$ and n having a value $|\triangle A_n| = |\triangle A_{n-1}| \simeq 8\pi$, see eq. (37) in [19], which is totally consistent with Bekenstein's result [37]. The similarity is completed if one note that the interpretation of eq. (10) is of a particle, the "electron", quantized with anti-periodic boundary conditions on a circle of length [17]

$$L = \frac{1}{T_E(E_n)} = 4\pi \left(M + \sqrt{M^2 - \frac{n}{2}} \right), \tag{13}$$

which is the analogous of the electron travelling in circular orbits around the hydrogen nucleus, similar in structure to the solar system, of Bohr model [31, 32, 33]. Clearly, all these similarities with the Bohr semi-classical model of the hydrogen atom and all these consistences with well known results in the literature of BHs, starting by the universal Bekenstein's result, *cannot* be coincidences, but are confirmations of the correctness of the analysis in [17, 18, 19] instead.

On the other hand, Bohr model is an approximated model of the hydrogen atom with respect to the valence shell atom model of full quantum mechanics. In the same way, our BH model should be an approximated model of the quantum BH with respect to the definitive, but at the present time unknown, model of full quantum gravity theory.

Now, we show that the energy emitted in an arbitrary transition $n \to m$, with $m > n$, is proportional to the effective temperature $[T_E]_{n \to m}$ associated to the transition. Setting

$$\Delta E_{n \to m} \equiv E_m - E_n = M_n - M_m = K [T_E]_{n \to m}, \tag{14}$$

where M_n and M_m are given by eqs. (11) and (12), let us see if there are values of the constant K for which eq. (14) is satisfied. We recall that

$$[T_E]_{n \to m} = \frac{1}{4\pi (M_n + M_m)}, \tag{15}$$

because the effective temperature is the inverse of the average value of the inverses of the initial and final Hawking temperatures [17, 18, 19, 20, 25]. Thus, eq. (14) can be rewritten as

$$\Delta E_{n \to m} = M_n^2 - M_m^2 = \frac{K}{4\pi}. \tag{16}$$

By using eqs. (11) and (12), for large m and n eq. (16) becomes

$$\frac{1}{2}(m - n) = \frac{K}{4\pi}, \tag{17}$$

which implies that eq. (14) is satisfied for $K = 2\pi (m - n)$. Hence, one finds

$$\Delta E_{n \to m} = 2\pi (m - n) [T_E(\omega_{n,m})]_{n \to m}. \tag{18}$$

Using eq. (5), the probability of emission between the two levels n and m can be written in the intriguing form

$$\Gamma_{n \to m} = \alpha \exp - \left\{ \frac{\Delta E_{n \to m}}{[T_E(\omega)]_{n \to m}} \right\} = \alpha \exp \left[-2\pi (m - n) \right], \tag{19}$$

with $\alpha \sim 1$. Thus, the probability of emission between two arbitrary levels characterized by the two "overtone" quantum numbers n and m scales like $\exp \left[-2\pi (m - n) \right]$. In particular, for $m = n + 1$ the probability of emission has its maximum value $\sim \exp(-2\pi)$, i.e. the probability is maximum for two adjacent levels, as one intuitively expects.

From the quantum mechanical point of view, one physically interprets Hawking radiation like energies of quantum jumps among the unperturbed levels (10) [17, 18, 19, 20]. In quantum mechanics, time evolution of perturbations can be described by an operator [26]

$$U(t) = \begin{array}{ll} W(t) & for\ 0 \leq t \leq \tau \\ 0 & for\ t < 0\ and\ t > \tau. \end{array} \tag{20}$$

Then, the complete (time dependent) Hamiltonian is described by the operator [26]

$$H(x,t) \equiv V(x) + U(t), \tag{21}$$

where $V(x)$ is the *effective Regge-Wheeler* potential of the time independent Schröedinger-like equation which governs QNMs, see [17, 18, 19] for details. Thus, for a wave function $\psi(x,t)$, one can write the correspondent *time dependent Schroedinger equation* for the system [26]

$$i\frac{d|\psi(x,t)>}{dt} = [V(x) + U(t)]\,|\psi(x,t)> = H(x,t)|\psi(x,t)> . \tag{22}$$

The state which satisfies eq. (22) is [26]

$$|\psi(x,t)> = \sum_m a_m(t)\exp\left(-i\omega_m t\right)|\varphi_m(x)>, \tag{23}$$

where the $\varphi_m(x)$ are the eigenfunctions of the time independent Schröedinger-like equation in [[17, 18, 19], and the ω_m are the correspondent eigenvalues. One considers Dirac delta perturbations [17, 18, 19, 20, 24] which represent subsequent absorptions of particles having negative energies which are associated to emissions of Hawking quanta in the mechanism of particle pair creation. Thus, in the basis $|\varphi_m(x)>$, the matrix elements of $W(t)$ can be written as

$$W_{ij}(t) \equiv A_{ij}\delta(t), \tag{24}$$

where $W_{ij}(t) = <\varphi_i(x)|W(t)|\varphi_j(x)>$ [26] and the A_{ij} are real. In order to solve the complete quantum mechanical problem described by the operator (21) one needs to know the probability amplitudes $a_m(t)$ due to the application of the perturbation described by the time dependent operator (20) [26], which represents the perturbation associated to the emission of an Hawking quantum. For $t < 0$, i.e. before the perturbation operator (20) starts to work, the system is in a stationary state $|\varphi_n(t,x)>$, at the quantum level n, with energy $E_n = |\omega_n|$ given by eq. (10). Therefore, in eq. (23) only the term

$$|\psi_n(x,t)> = \exp\left(-i\omega_n t\right)|\varphi_n(x)>, \tag{25}$$

is not null for $t < 0$. This implies $a_m(t) = \delta_{mn}$ for $t < 0$. When the perturbation operator (20) stops to work, i.e. after the emission, for $t > \tau$ the probability amplitudes $a_m(t)$ return to be time independent, having the value $a_{n\to m}(\tau)$

[26]. In other words, for $t > \tau$ the system is described by the *wave function* $\psi_{final}(x, t)$ which corresponds to the state

$$|\psi_{final}(x, t) >= \sum_{m=n}^{m_{max}} a_{n \to m}(\tau) \exp(-i\omega_m t) |\varphi_m(x) > .$$ (26)

Thus, the probability to find the system in an eigenstate having energy $E_m = |\omega_m|$ is given by [26]

$$\Gamma_{n \to m}(\tau) = |a_{n \to m}(\tau)|^2.$$ (27)

By using a standard analysis, one obtains the following differential equation from eq. (26) [26]

$$i\frac{d}{dt}a_{n \to m}(t) = \sum_{l=m}^{m_{max}} W_{ml}a_{n \to l}(t) \exp[i(\Delta E_{l \to m})t].$$ (28)

To first order in $U(t)$, by using the Dyson series, one gets the solution [26]

$$a_{n \to m} = -i\int_0^t \{W_{mn}(t') \exp[i(\Delta E_{n \to m})t']\} dt'.$$ (29)

By inserting (24) in (29) one obtains

$$a_{n \to m} = iA_{mn}\int_0^t \{\delta(t') \exp[i(\Delta E_{n \to m})t']\} dt' = \frac{i}{2}A_{mn}$$ (30)

Combining this equation with eqs. (19) and (27) one gets

$$\alpha \exp[-2\pi(m-n)] = \tfrac{1}{4}A_{mn}^2$$

$$A_{mn} = 2\sqrt{\alpha} \exp[-\pi(m-n)]$$ (31)

$$a_{n \to m} = -i\sqrt{\alpha} \exp[-\pi(m-n)].$$

As $\sqrt{\alpha} \sim 1$, one gets $A_{mn} \sim 10^{-2}$ for $m = n + 1$, i.e. when the probability of emission has its maximum value. This implies that second order terms in $U(t)$ are $\sim 10^{-4}$, i.e. the approximation is very good. Clearly, for $m > n + 1$ the approximation is better because the A_{mn} are even smaller than 10^{-2}. Thus, one can write down the final form of the ket representing the state as

$$|\psi_{final}(x, t) >= \sum_{m=n}^{m_{max}} -i\sqrt{\alpha} \exp[-\pi(m-n) - i\omega_m t] |\varphi_m(x) > .$$ (32)

The state (32) represents a *pure final state instead of a mixed final state.* Hence, the states are written in terms of an *unitary* evolution matrix instead of a density matrix and this implies the fundamental conclusion that information is not loss in BH evaporation. The result agrees with the assumption by 't Hooft that

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Schrödinger equations can be used universally for all dynamics in the universe [6]. We also stress that, as the final state of eq. (32) is due to potential emissions of Hawking quanta having negative energies which perturb the BH and "trigger" the QNMs corresponding to potential arbitrary transitions $n \to m$, with $m > n$, the subsequent *collapse of the wave function* to a new a stationary state

$$|\psi_m(x,t)> = \exp\left(-i\omega_m t\right)|\varphi_m(x)>, \qquad (33)$$

at the quantum level m, implies that the wave function of the particle having negative energy $-\Delta E_{n \to m} = \omega_n - \omega_m$ has been transferred to the QNM and it is given by

$$|\psi_{-(m-n)}(x,t)> \equiv -\exp\left[i(\omega_m - \omega_n)t\right]\left[|\varphi_m(x)> - |\varphi_n(x)>\right]. \qquad (34)$$

This wave function results entangled with the wave function of the particle with positive energy which propagates towards infinity in the mechanism of particle creation by BHs. We will see in the following Section that this key point solves the entanglement problem connected with the information paradox.

The analysis in this work is strictly correct only for $n \gg 1$, i.e. only for excited BHs. This is the reason because we assumed an emission from the ground state to a state with large n in the discussion. On the other hand, a state with large n is always reached at late times, maybe not through a sole emission from the ground state, but, indeed, through various subsequent emissions of Hawking quanta.

For the sake of completeness, it is helpful to discuss on the exact nature of time as it is defined in the Schrodinger equation and the covariant properties of that equation [43]. Following [26], we recall that time must be considered as a parameter in quantum mechanics instead of a quantum mechanical operator. In particular, time is *not* a quantum observable [26]. In other words, in quantum mechanics it is *not* possible to discuss on a "time operator" in the same way that we do, for example for the "position operator" and for the "momentum operator" [26]. In fact, within the framework of quantum mechanics there is no room for a symmetric analysis for both time and position, even if, from an historical point of view, quantum mechanics has been developed by De Broglie and Schroedinger following the idea of a covariant analogy between time and energy on one hand and position and momentum on the other hand [26]. This discussion works from the quantum mechanical point of view of the analysis. But, in the current analysis, we have to discuss about time also from the point of view of general relativity. As we discussed Hawking radiation as tunnelling in the framework of the analysis in [8, 9] we recall that in such works the Painlev́ and Gullstrand coordinates for the Schwarzschild geometry have been used. On the other hand, the radial an time coordinates are the same in both the Painlev́ and Gullstrand and Schwarzschild line elements. Thus, we conclude that the time in the operator for the time evolution of eq. (20) and in the subsequent equations is the Schwarzschild time.

Concerning the covariant properties of the Schrodinger equation, we recall that, in general, quantum mechanics has to be applied to systems moving with

speeds that are not negligible with respect to the speed of light. Then, relativistic corrections could be in principle necessary in various cases. This important issue implies that the laws of quantum mechanics, and, in turn, the Schrodinger equation, must be re-formuled in covariant form through the Lorentz transformations. The conseguence will be the apparence of news important quantum properties like the spin and the spin-orbit interaction. This is not the case of the analysis in this paper because we use Bohr's Correspondence Principle which enables an accurate semi-classical analysis for large values of the principal quantum number n.

4 Solution to the entanglement problem

One could claim that, although the above analysis provides a natural model of Hawking radiation, it makes no reference to the BH spacetime, where information is conserved. In fact, some authors claim that the challenge in addressing information loss is to reconcile models of Hawking radiation with the spacetime structure, where the information falling into the singularity is causally separated from the outgoing Hawking radiation, see [13] for example. In any case, these criticisms do not work for the analysis in this work. In fact, in the above analysis there is a subtle connection between Hawking radiation and the BH spacetime, where information is conserved. The key point of this approach to the Hawking information problem concerns the entanglement structure of the wave function associated to the particle pair creation [13]. In other terms, in order to solve the paradox, one needs to know the part of the wave function in the interior of the horizon [13], i.e. the part of the wave function associated to the particle having negative energy (interior, infalling modes). This is exactly the part of the wave function which in the Hawking computation gets entangled with the part of the wave function outside, i.e. the part of the wave function associated to the particle having positive energy which escapes from the BH [13]. If one ignores this interior part of the wave function, one misses the entanglement completely, and thus fails to understand the paradox [13]. But in the correspondence between Hawking radiation and BH QNMs the particle having negative energy which falls into the singularity transfers its part of the wave function and, in turn, the information encoded in such a part of the wave function, to the QNM. Hence, the emitted radiation results to be entangled with BH QNMs, which are the oscillations of the BH horizon. This fundamental point is exactly the subtle connection between the emitted radiation and the interior BH spacetime that one needs to find. In other words, although we do not know what happens in the interior spacetime structure, we know that the response of such a structure to the absorption of an interior, infalling mode is to add the frequency of that interior, infalling mode (and, in turn, of its wave function) to the QNM corresponding to the energy level E_n, in order to permit it to jump to the energy level E_m. In that way, the interior part of the wave function is now "within" the QNM corresponding to the quantum level E_m, which is, in turn, entangled with all the particles emitted at that time. Let us see this issue in

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detail. Once again, we stress that the correspondence between emitted radiation and proper oscillation of the emitting body is a fundamental behavior of every radiation process in Nature, and this is the key point which permits to solve the entanglement problem. One describes the mechanism of particles creation by BHs [1], as tunnelling arising from vacuum fluctuations near the BH horizon [8, 9, 25]. If a virtual particle pair is created just inside the horizon, the virtual particle with positive energy can tunnel out. Then, it materializes outside the BH as a real particle. In the same way, if one considers a virtual particle pair created just outside the horizon, the particle with negative energy can tunnel inwards. In both of the situations, the particle with negative energy is absorbed by the BH. Again, let us assume a first emission from the BH ground state to a state with large n, say $n = n_1 \gg 1$. The absorbed particle having negative energy $-|\omega_{n_1}|$ generates a QNM corresponding to an energy-level of emitted energies $E_{n_1} = |\omega_{n_1}|$ and the BH mass changes from M to

$$M_{n_1} \equiv M - E_{n_1} = \sqrt{M^2 - \frac{n_1}{2}}. \tag{35}$$

In other words, the energy of the first absorbed particle having negative energy is transferred, together with its part of the wave function, to the QNM which is now entangled with the emitted particle having positive energy. By using eq. (34) and setting $n = 0$ and $m = n_1$ one finds that the part of the wave function in the interior of the horizon, i.e. the part of the wave function associated to the particle having negative energy (infalling mode) which has been transferred to the QNM is

$$|\psi_{-n_1}(x, t) > = - \exp\left(i\omega_{n_1} t\right) |\varphi_{n_1}(x) > . \tag{36}$$

Now, let us consider a second emission, which corresponds to the transition from the state with $n = n_1$ to a state with, say, $n = n_2 > n_1$. The BH mass changes from M_{n_1} to

$$\begin{aligned} M_{n_2} &\equiv M_{n_1} - \Delta E_{n_1 \to n_2} = M - E_{n_2} \\ &= \sqrt{M^2 - \frac{n_2}{2}}, \end{aligned} \tag{37}$$

where $\Delta E_{n_1 \to n_2} \equiv E_{n_2} - E_{n_1} = M_{n_1} - M_{n_2}$ is the jump between the two levels. The energy of the second absorbed particle having negative energy is transferred, together with its part of the wave function, again to the QNM, which now corresponds to an increased level of energy $E_{n_2} = |\omega_{n_{12}}|$ and is now entangled with both the two emitted particles having positive energy. By using again eq. (34) and setting $n = n_1$ and $m = n_2$, one finds that the part of the wave function of the second infalling mode which has been transferred to the QNM is

$$|\psi_{-(n_2 - n_1)}(x, t) > = - \exp\left[i(\omega_{n_2} - \omega_{n_1})t\right] \left[|\varphi_{n_2}(x) > - |\varphi_{n_1}(x) >\right]. \tag{38}$$

Let us consider a third emission, which corresponds to the transition from the state with $n = n_2$ to a state with, say, $n = n_3 > n_2$. The BH mass changes from M_{n_2} to

$$M_{n_3} \equiv M_{n_2} - \Delta E_{n_2 \to n_3} = M - E_{n_3}$$
$$= \sqrt{M^2 - \frac{n_3}{2}}, \tag{39}$$

where $\Delta E_{n_2 \to n_3} \equiv E_{n_3} - E_{n_2} = M_{n_2} - M_{n_3}$ is the jump between the two levels. Again, the energy of the third absorbed particle having negative energy is transferred, together with its part of the wave function, to the QNM corresponding now to a further increased energy level $E_{n_{13}} = |\omega_{n_3}|$ and being entangled with the three emitted particles which have positive energy. Now, eq. (34) with $n = n_2$ and $m = n_3$ gives the part of the wave function of the third infalling mode which has been transferred to the QNM as

$$|\psi_{-(n_3 - n_2)}(x, t) > = -\exp[i(\omega_{n_3} - \omega_{n_2})t] \left[|\varphi_{n_3}(x) > -|\varphi_{n_{12}}(x) > \right]. \tag{40}$$

The process will continue again, and again, and again... till the *Planck distance* and the *Planck mass* are approached by the evaporating BH. At that point, the Generalized Uncertainty Principle prevents the total BH evaporation in exactly the same way that the Uncertainty Principle prevents the hydrogen atom from total collapse [27] and one needs a full theory of quantum gravity for the further evolution.

In any case, we stress again that the energy E_n of the generic QNM having quantum "overtone" number n is interpreted like the total energy emitted by the BH at that time, i.e. when the BH is excited at a level n [17, 18, 19, 20]. This implies that such a QNM is entangled with all the Hawking quanta emitted at that time.

Thus, all the quantum physical information falling into the singularity is not causally separated from the outgoing Hawking radiation, but it is instead recovered and codified in eq. (32) which leads the time evolution of the correspondence between Hawking radiation and BH QNMs. In other words, in our approach the "smoothness of the horizon" is achieved by the issue that the horizon is oscillating through QNMs and all the emitted particles are entangled with such oscillations. As the solution to the information problem should be to find a physical effect that one might have have missed [13], here we have shown that the natural correspondence between Hawking radiation and BH QNMs *is exactly that missed physical effect.*

5   Conclusion remarks

Through an analysis of BH QNMs in terms of an unitary evolution, governed by a time dependent Schroedinger equation of a Bohr-like model for BHs as "hydrogen atoms, " we falsified Hawking's claim on the information loss in BH evaporation. We stress that it is an intuitive but general conviction that BHs result in highly excited states representing both the "hydrogen atom" and the "quasi-thermal emission" in quantum gravity. Here we have shown that such an intuitive picture is more than a picture, showing that a model of quantum BH somewhat similar to the historical semi-classical model of the structure of

a hydrogen atom introduced by Bohr in 1913 [31, 32, 33] has a time evolution obeying a time dependent Schrödinger equation, in perfect agreement with quantum mechanics. Clearly, this cannot be a coincidence. In the same way, they cannot be coincidences the consistences with various papers in the literature of BH thermodynamics, see for example [24, 34, 35] and the consistence with the famous result of Bekenstein on the area quantization [37].

In the final Section of this paper we have also show that the present approach permits to solve the entanglement problem connected with the information paradox.

We also recall that consistence between the time evolution of our Bohr-like model and a recent approach to solve the BH information paradox [10, 11, 12, 14] has been recently highlighted in [40].

6 Acknowledgements

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