# A Perdurable Defence to Weyl's Unified Theory

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**Abstract**

Einstein dealt a lethal blow to Weyl's unified theory by arguing that Weyl's theory was at the very least, it was beautiful and at best, un-physical, because its concept of variation of the length of a vector from one point of space to the other meant that certain absolute quantities, such as the "fixed" spacing of atomic spectral lines and the Compton wavelength of an Electron for example, would change arbitrarily as they would have to depend on their prehistories. This venomous criticism of Einstein to Weyl's theory remains much alive today as it was on the first day Einstein pronounced it. We demonstrate herein that one can overcome Einstein's criticism by recasting Weyl's theory into a new Weyl-kind of theory were the length of vectors are preserved as is the case in Riemann geometry. In this new Weyl theory, the Weyl gauge transformation of the Riemann metric $g_{\mu\nu}$ and the electromagnetic field $A_\mu$ are preserved.

*"Symmetry, as wide or narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection."*
***Hermann Klaus Hugo Weyl*** $(1885 - 1955)$

## 1 Introduction

It was shortly after Albert Einstein $(1897 - 1955)$ announced his GTR (on November $25$, $1915$), that Herman Weyl $(1885 - 1955)$ began an intensive study of the theory's mathematics and began publishing related scientific papers dealing with its physical applications. In $1918$, Weyl published his book *Raum-Zeit-Materie* (Space-Time-Matter), which

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provided the first fully comprehensive analysis of the geometric aspects of Einstein GTR and its relationship with spacetime and physics.

One of the topics covered in the book was Weyl's idea that gravity and electromagnetism might both be derivable from a generalization of Riemann geometry, the mathematical basis from which Einstein had developed his relativity theory. Weyl's idea was based on a new mathematical symmetry that he called gauge invariance. This theory gave birth to the modern concept of the gauge principle; a principle without which any of the modern effort to finding a unified theory could not be. It underlies all of the *Yang-Mills* theories and is a cornerstone in string theory and its more recent variant, M-theory.

Weyl's theory was given a still birth by its most able midwife – Einstein. Einstein greatly admired the theory but with equal passion, he would not make it take its "first gulp of air" nor have just "one photon fall on the retina" of the newly born theory. Einstein's razor sharp and agile intellect saw the "tiny devil in the detail of the beauty of Weyl's theory" and immediately thereafter, he delivered his all-enduring and lethal blow to it, a blow that made sure it would never raise. *Lo and behold!* What resurrected from Weyl's theory is the "ghost" of *gauge invariance*, this is about all that remained of its beauty. Today, this ghost (of gauge invariance) pervades all of physics that makes the endeavour on which Weyl was failed by Einstein *i.e.*, it pervades and permeates all unification efforts. For example in virtually all modern quantum gauge theories, Weyl's gauge concept is used to justify the "mathematical chicanery and shenanigan" called renormalization! Renormalization is when one subtracts an artificial infinity from a calculated infinity to get a usable or useful, finite answer.

In this reading, we stand right outside the tomb of Weyl's theory and call forth Weyl's theory back to life. We demonstrate that Einstein's criticism is easily overcome without much difficultly. We recast Weyl's theory by resetting its foundation stone in which process we completely overcome Einstein's lethal and venomous criticism. Having overcome Einstein's criticism – a fresh new life is thereby breathed into the nostrils of Weyl's theory; we see nothing hindering its further developed from thereon.

## 2 Length Invariance in Riemann Geometry

One will recall from elementary geometry that the square of the magnitude or length $(l)$ of a vector $\boldsymbol{v}$ is defined by the dot product $l^2 = \boldsymbol{v}\cdot\boldsymbol{v} = |\boldsymbol{v}|^2$. In tensor notation and in particular in Riemann geometry, for an arbitrary vector $V^\mu$, we write this as:

$$l^2 = g_{\mu\nu}V^\mu V^\nu, \tag{1}$$

where as usual $g_{\mu\nu} = g_{\nu\mu}$ is the symmetric metric tensor of spacetime. Now, taking the total derivative of this expression, we will have:

$$2ldl = g_{\mu\nu,\alpha}V^\mu V^\nu dx^\alpha + g_{\mu\nu}dV^\mu V^\nu + g_{\mu\nu}V^\mu dV^\nu, \tag{2}$$

and using the identity or fact that $dV^\mu = \Gamma^\mu_{\alpha\nu}V^\nu dx^\alpha$ where $\Gamma^\mu_{\alpha\nu}$ is the usual three Christophel symbol of Riemann geometry, the above reduces to:

$$2ldl = g_{\mu\nu,\alpha}V^\mu V^\nu dx^\alpha + g_{\mu\lambda}\Gamma^\lambda_{\nu\alpha}V^\mu V^\nu dx^\alpha + g_{\lambda\nu}\Gamma^\lambda_{\mu\alpha}V^\mu V^\nu dx^\alpha = g_{\mu\nu;\alpha}V^\mu V^\nu dx^\alpha, \tag{3}$$

that is:

$$g_{\mu\nu;\alpha}V^{\mu}V^{\nu}dx^{\alpha}=2ldl, \tag{4}$$

where:

$$g_{\mu\nu;\alpha}=g_{\mu\nu,\alpha}+g_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\nu\alpha}+g_{\lambda\nu}\hat{\Gamma}^{\lambda}_{\mu\alpha}, \tag{5}$$

is the usual covariant derivative of the metric tensor in Riemann geometry. In Riemann geometry $g_{\mu\nu;\alpha}\equiv 0$, the meaning of which is that $dl\equiv 0$, simple stated, the length of a vector is invariant under translation of the frame of reference or the coordinate system. Clearly, the covariant derivative $(g_{\mu\nu;\alpha})$ determines whether or not the length of a vector is invariant or not. The condition $g_{\mu\nu;\alpha}=0$ is the foundation stone of Riemann geometry as this condition defines the structure and nature of Riemann geometry.

At this point, Weyl wondered if Riemann geometry could be altered in such a way that $dl$ does not vanish as it does in ordinary Riemann geometry. While thinking about this, a mental visitation in the form of a thought must have occurred to Weyl, a thought to the effect that, if the metric is *re-gauged* or transformed from $g_{\mu\nu}$ to $\mathrm{e}^{\phi}g_{\mu\nu}$, *i.e.*:

$$g_{\mu\nu}\longmapsto \mathrm{e}^{\phi}g_{\mu\nu}, \tag{6}$$

where $\phi$ is a scaler; he would achieve his desire of a none vanishing $dl$ because if $g_{\mu\nu;\alpha}\neq 0$, it follows from (4) that $dl\neq 0$. If he wanted to do away with Riemann geometry, he must have not compromised and tried to remain within its confines and domains. As to what might have motivated Weyl into think of varying the length of the vectors may very well be the fact the in Riemann geometry while lengths of vectors are preserved, their directions in space are not, so Weyl might have wondered: if the directions are not preserved, why should the lengths be preserved?! With this noble thought, Weyl must then have embarked onto his goal of a non-Riemann geometry with non-preserved lengths of vectors.

# 3 Length Variation in Weyl Geometry

As a starting point, in laying down the foundation stone of his theory, Weyl having glimpsed into the structure and nature of Riemann geometry as lay-down above, he had two things in mind, first of which was the idea of a none vanishing covariant derivative *i.e.* $g_{\mu\nu;\alpha}\neq 0$ and second, the idea of a none vanishing length of vectors *i.e.* $dl\neq 0$. To achieve the former, Weyl inspected the identity or fact that $dV^{\mu}=\Gamma^{\mu}_{\alpha\nu}V^{\nu}dx^{\alpha}$. This identity gives the change of length of a vector $V^{\mu}$ under any given transformation or translation. Providentially, Weyl assumed that a change in the vector magnitude $dl$, must obey a similar expression. By the adroitness of the mind, Wely proceeded to make his analogue, which is:

$$dl=lA_{\alpha}dx^{\alpha}, \tag{7}$$

where $A_{\alpha}=A_{\alpha}(x)$ is a hitherto mysterious four vector quantity of unknown origins, its duty, sole purpose and ultimate mission is simple to keep $dl$ from vanishing – for as long as $A_{\alpha}\neq 0$, we will have $dl\neq 0$. The relationship (7), Weyl was to embellish by holding it sacrosanct as the *foundation stone* of this new theory; regrettably, it is a relationship which he would defend right up to his untimely and unexpected passing-over in 1955.

Further, having set the battle-lines through (7), Weyl proceed by the *sleight of hand* to insert this relationship into (4) thus obtaining:

$$2l^2 A_\alpha dx^\alpha = g_{\mu\nu;\alpha} V^\mu V^\nu dx^\alpha, \tag{8}$$

which reduces to:

$$g_{\mu\nu;\alpha} = 2A_\alpha g_{\mu\nu}. \tag{9}$$

The relationship (9) justified (7). Now, we have to calculate the resulting affine connections from the new covariant derivative (9). For this, we will have to write down the three expressions that result from a cyclic permutation of the indices $\mu, \nu$ and $\alpha$ in $g_{\mu\nu,\alpha}$ and $A_\alpha g_{\mu\nu}$, that is:

$$\begin{array}{rclr} g_{\mu\nu,\alpha} + g_{\mu\lambda}\hat{\Gamma}^\lambda_{\nu\alpha} + g_{\lambda\nu}\hat{\Gamma}^\lambda_{\mu\alpha} & = & 2A_\alpha g_{\mu\nu}, & \textbf{(a)} \\ g_{\alpha\mu,\nu} + g_{\alpha\lambda}\hat{\Gamma}^\lambda_{\mu\nu} + g_{\lambda\nu}\hat{\Gamma}^\lambda_{\alpha\mu} & = & 2A_\nu g_{\alpha\mu}, & \textbf{(b)} \\ g_{\nu\alpha,\mu} + g_{\nu\lambda}\hat{\Gamma}^\lambda_{\alpha\mu} + g_{\lambda\mu}\hat{\Gamma}^\lambda_{\nu\alpha} & = & 2A_\mu g_{\nu\mu}, & \textbf{(c)} \end{array} \tag{10}$$

where the hat on $\hat{\Gamma}^\lambda_{\mu\nu}$ has been put so that it is made clear that this affine is no longer the same Christoffel symbol as the one used in Riemann geometry.

Now, subtracting from equation (10) (a) the sum of (10) (b) and (c), one obtains:

$$\hat{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} + W^\alpha_{\mu\nu}, \tag{11}$$

where:

$$W^\alpha_{\mu\nu} = -\left(\delta^\alpha_\mu A_\nu + \delta^\alpha_\nu A_\mu - g_{\mu\nu} g^{\alpha\lambda} A_\lambda\right), \tag{12}$$

is the new object Weyl tensor and $\Gamma^\alpha_{\mu\nu}$ is the usual Christoffel three symbol of Riemann geometry. The geometry that we have just described with the affine connection $\hat{\Gamma}^\alpha_{\mu\nu}$ is what is called Weyl's geometry. This geometry tends to Riemann geometry as $A_\mu \longrightarrow 0$. What deeply intrigued Weyl and many others that came to admire the new theory is the fact that the Weyl connection $\hat{\Gamma}^\alpha_{\mu\nu}$, is invariant under the following transformations:

$$\begin{array}{rclr} g_{\mu\nu} & \longmapsto & e^{2\chi} g_{\mu\nu}, & \textbf{(a)} \\ A_\mu & \longmapsto & A_\mu + \partial_\mu \chi, & \textbf{(b)} \end{array} \tag{13}$$

where $\chi = \chi(x)$ is an arbitrary scaler function. If $\hat{\Gamma}^\alpha_{\mu\nu}$ is invariant under the transformation (13) *i.e.* $\delta\hat{\Gamma}^\alpha_{\mu\nu} \equiv 0$, the curvature tensor $\hat{R}^\alpha_{\mu\lambda\nu}$ is also invariant *i.e.* $\delta\hat{R}^\alpha_{\mu\lambda\nu} \equiv 0$. Given that Weyl knew very well that Maxwellian electrodynamics is described by a four vector such that the entire Maxwellian electrodynamics is invariant under the transformation (13 b), without wasting much time, Weyl seized the golden moment and identified $A_\mu$ with the Maxwellian four vector potential of electrodynamics. At this point, one can not help but endlessly and deeply admire Weyl's brilliantly convinced theory, and on the other hand, one can only be irretrievably and deeply sad to know that this theory, despite its esoteric grandeur and exquisite beauty, it does not correspond with experience.

## 4 Einstein's Criticism

Weyl's proposed theory lead to a rescaling of the fundamental metric tensor $g_{\mu\nu} \longmapsto e^{\chi} g_{\mu\nu}$. Weyl held that, this rescaling should have no effect on physics. As aforestated, Einstein initially loved the idea – alas, the devil was in the detail; he noted that the line element $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$ would also be rescaled according to $ds'^2 \longmapsto e^{\chi} ds^2$. Since $ds$ can be made to serve as a measuring rod or clock, the agile Einstein was quick note that this would mean that certain absolute quantities, such as the spacing of atomic spectral lines and the Compton wavelength of an Electron for example, would change arbitrarily and thus have to depend on their prehistory. With this, Einstein delivered the lethal and venomous blow to Weyl's theory and concluded that it must therefore be non-physical – despite its grandeur and beauty, it had no correspondence nor bearing with reality.

Thus, from a 'safe distance', the great Einstein was the first to publicly exhibit his passionate *albeit* backhanded admiration of Weyl's theory, he said of it:

> *"... apart from the agreement with reality,*
> *it is at any rate a grandiose achievement of the mind ...*
> *a first class [stroke of] genius."*
>
> (Abraham Pais 2005, *Subtle is the Lord*, p. 341)

With equal passion, he made the one all-enduring 'aerial bombardment' to it, a bombardment from which Weyl's theory would never recover to this day.

Stated in a different manner, the agile Einstein was quick and to point out that in Weyl's geometry, the frequency of the spectral lines of atomic clocks from different portions of the distant heavenly spaces would depend on the location and pre-histories of the atoms. This is in fragment disagreement with experience. The spectral lines are well-defined and sharp; they appear to be independent of an atom's pre-history. Atomic clocks define units of time, and experience shows they are integrally transported from one portion of the heavens to the other. So, with this criticism alone, Einstein gave Weyl's theory a stillbirth with his backhanded compliant.

Weyl's brilliant and beautiful theory was hopelessly thwarted and, to no avail, he made last ditch effort to save his theory in latters year (Weyl 1927*a*,7). Einstein's criticism lay deep in the nimbus of the foundation stone of Weyl's theory, which is that the length of a vector varied from one point of spacetime to another. In wrapping-up his criticism, he [Einstein] said:

> *"... I do not believe that his theory will hold its ground in relation to reality."*
>
> (Einstein 1952, *Sidelights of Relativity*, p. 23)

Much for the great Hermann Weyl and his *all-brilliant, beautiful but 'failed' theory*. Can it [the theory] be saved? Weyl himself made attempts (Weyl 1927*a*,7) together with such notable figures as Sir Arthur S. Eddington and even the great Paul A. M. Dirac. All attempts so far at trying to save they have not gain much attention or universal recognition simple because these attempt are stuck in Weyl's quagmire and conundrum by holding sacrosanct the relation (7). Shortly, we will demonstrate that indeed Weyl's theory can be saved first by drop the condition (7) and reverting back to the humble Riemann condition $dl \equiv 0$. In our recasting of Weyl's theory, we are to hold sacrosanct the Weyl gauge transformation (13) and not the Weyl hypothesis (7).

## 5 Recasting of Weyl's Theory

In recasting Weyl's theory so that it overcomes once and for altime Einstein's criticism, we will not take the traditional route that was taken by Weyl because in so doing, we will fall into the same trap which the great Weyl fell victim to. At our point of departure, we wave goodbye to Riemann geometry and efferently prepare to embrace a totally new geometry, a hybrid Riemann geometry which has the same feature as Weyl, less off cause the change of length of vectors under transformations or translations. The route that we are about to take is equivalent and the reason for changing the sails is that the present route allows us to demonstrate latter how Weyl would have overcome Einstein's criticism that gave the theory a still birth. Actually, this route allows us to pin-down exactly where Weyl's theory makes an un-physical assumption.

We begin with the usual covariant derivative $g_{\mu\nu;\alpha} = 0$ of Riemann geometry. As fore-alluded, the condition $g_{\mu\nu;\alpha} = 0$, is the sacrosanct foundation stone of Riemann geometry. We will uphold this covariant derivative condition under the Weyl conformal transformation $g_{\mu\nu} \longmapsto \hat{g}_{\mu\nu} = \mathrm{e}^{\phi} g_{\mu\nu}$ of the metric, *i.e.* $\hat{g}_{\mu\nu;\alpha} = 0$. Likewise, the condition $\hat{g}_{\mu\nu;\alpha} = 0$, is to be taken as the sacrosanct foundation stone of the new Weyl's geometry. Written in full, the equation $\hat{g}_{\mu\nu;\alpha} = 0$ is given by:

$$\hat{g}_{\mu\nu;\alpha} = e^{\phi}\left(g_{\mu\nu,\alpha} + g_{\mu\nu}\phi_{,\alpha} + g_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\nu\alpha} + g_{\lambda\nu}\hat{\Gamma}^{\lambda}_{\mu\alpha}\right) = 0, \tag{14}$$

and taking $\phi = A_\alpha x^\alpha$, this equation can be rewritten as:

$$g_{\mu\nu,\alpha} + g_{\mu\lambda}\bar{\Gamma}^{\lambda}_{\nu\alpha} + g_{\lambda\nu}\bar{\Gamma}^{\lambda}_{\mu\alpha} = -A_\alpha g_{\mu\nu}. \tag{15}$$

As is the case with Weyl's original geometry, the covariant derivative $g_{\mu\nu;\alpha}$ does not vanish since $g_{\mu\nu;\alpha} = -A_\alpha g_{\mu\nu}$. With this, Weyl had achieved the non-Riemann geometry he desired.

Now, we have to calculate the resulting affine connections and for this, we have to write down the three expressions that result from a cyclic permutation of the indices $\mu, \nu$ and $\alpha$ in $g_{\mu\nu,\alpha}$ and $A_\alpha g_{\mu\nu}$, that is:

$$\begin{array}{rcll} g_{\mu\nu,\alpha} + g_{\mu\lambda}\bar{\Gamma}^{\lambda}_{\nu\alpha} + g_{\lambda\nu}\bar{\Gamma}^{\lambda}_{\mu\alpha} & = & -A_\alpha g_{\mu\nu}, & \quad \textbf{(a)} \\ g_{\alpha\mu,\nu} + g_{\alpha\lambda}\bar{\Gamma}^{\lambda}_{\mu\nu} + g_{\lambda\nu}\bar{\Gamma}^{\lambda}_{\alpha\mu} & = & -A_\nu g_{\alpha\mu}, & \quad \textbf{(b)} \\ g_{\nu\alpha,\mu} + g_{\nu\lambda}\bar{\Gamma}^{\lambda}_{\alpha\mu} + g_{\lambda\mu}\bar{\Gamma}^{\lambda}_{\nu\alpha} & = & -A_\mu g_{\nu\mu}, & \quad \textbf{(c)} \end{array} \tag{16}$$

where the "bar" on $\bar{\Gamma}^{\lambda}_{\mu\nu}$ has been put so that it is made clear that this affine is neither Christoffel symbol nor the usual Weyl connection but the is the new Weyl connection of the new Weyl geometry.

Now, as before, subtracting from equation (16) (a) the sum of (16) (b) and (c), one obtains:

$$\bar{\Gamma}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + \mathcal{W}^{\alpha}_{\mu\nu}, \tag{17}$$

where:

$$\mathcal{W}^{\alpha}_{\mu\nu} = -\frac{1}{2}\left(\delta^{\alpha}_{\mu}A_\nu + \delta^{\alpha}_{\nu}A_\mu - g_{\mu\nu}g^{\alpha\lambda}A_\lambda\right), \tag{18}$$

is the new Weyl tensor and $\Gamma^{\alpha}_{\mu\nu}$ is the usual Christoffel three symbol of Riemann geometry. The geometry that we have just described is what we shall call the *New Weyl Geometry*. As is the case with Weyl's original geometry, this geometry tends to Riemann geometry as $A_{\mu} \longrightarrow 0$. Further, like in the case of the original Weyl geometry, this geometry is invariant under the following transformations:

$$\begin{array}{rcll} g_{\mu\nu} & \longmapsto & e^{\chi}g_{\mu\nu}, & \textbf{(a)} \\ A_{\mu} & \longmapsto & A_{\mu}+\partial_{\mu}\chi, & \textbf{(b)} \end{array} \tag{19}$$

where $\chi=\chi(x)$ is an arbitrary scaler function. If $\bar{\Gamma}^{\alpha}_{\mu\nu}$ is invariant under the transformation (19) *i.e.* $\delta\bar{\Gamma}^{\alpha}_{\mu\nu}\equiv 0$, the curvature tensor $\hat{R}^{\alpha}_{\mu\lambda\nu}$ is also invariant *i.e.* $\delta\bar{R}^{\alpha}_{\mu\lambda\nu}\equiv 0$. At this point we have successfully recast Weyl's theory into a new theory where we have up to now not worried about the change of length of vectors.

# 6 A Lasting Defence to Weyl's Geometry

Now, under the new Weyl geometry, the change of length of a vector has the same form and structure as happens in Riemann geometry *i.e.* (4) *albeit* with the conform Weyl transformation (6) effected to the metric $g_{\mu\nu}$ in (4), *i.e.*:

$$2ldl=\hat{g}_{\mu\nu;\alpha}V^{\mu}V^{\nu}dx^{\alpha}. \tag{20}$$

Now, given that we have set the foundation stone of the new Weyl geometry to be $\hat{g}_{\mu\nu;\alpha} = 0$, it follows that $dl=0$. Einstein's valid criticism is reduced to naught, it falls apart into nothing but shreds – it comes out flying through the window. At a stroke, in this way, the great Weyl could have thwarted the great Einstein. The problem is that Weyl set the condition (7) as the foundation stone of his theory, it is this that he tried to defend. With hindsight, we now see that what Weyl should have defended is not (7) but the invariance of the connection $\hat{\Gamma}^{\lambda}_{\mu\nu}$ under the transformation (13) because it is these transformations that guarantee the inclusion of the electromagnetic into the framework of gravitation and not the Weyl condition (7).

# 7 Discussion and Conclusion

## 7.1 General Discussion

Without any doubt, Weyl produced the first natural unified field theory for which the Maxwellian electromagnetic field $A_{\mu}$ and the gravitational field $g_{\mu\nu}$, appear side-by-side as geometrical properties of the fabric space-time and regrettably, for simple reasons of incredulity cast by Einstein's agile intellect, the theory largely lies forgotten. To say Weyl's theory lies forgotten is not true. Perhaps the correct statement to say is that Einstein's criticism continues – today as it did on the first day of its pronouncement; to hover above Weyl's theory so much that no physically meaningful thing can be derived from it as one must first overcome Einstein's spell over it.

Despite Einstein's criticism, 95 years on, research into Weyl's theory – while not very active, it is much alive (see *e.g.* Cattani et al. 2013, Scholz 2011). By active, we mean it is not embraced universally by the majority but is only take-up for academic reasons largely to do with curiosity than to do with seeking solid answers to the nature of physical and natural reality. Interesting to note is that Weyl's theory was not, and never has been, disproved, but only sideline because of Einstein's venomous criticism.

Since what we have done herein is to show that Einstein's criticism can be thrown out of the window, we are left wondering what becomes of Weyl's theory? In it acceptable because its greatest setback has been overcome? To ourself, we feel this opens the door for exploration of this theory. However, one fact that is likely to continue to haunt the theory is that it is not a true unification of gravitation and electricity as these fields non-mutually and non-intimately sit side-by-side in disjoint manner with no visible connection to one another. As in the case electricity and magnetism, there is no reciprocal action between the field $A_\mu$ and $g_{\mu\nu}$. This is a great setback for those seeking true unification.

### 7.2 Conclusion

As demonstrated herein, a Weyl geometry completely free from Einstein's criticism is possible. This geometry preserves the lengths of vectors and maintains a four vector field as in Weyl's original theory. This four vector field together with the gravitational field submit to Weyl's original simultaneous transformation of these two fields thus allowing us to identify the four vector field with the electromagnetic field as Weyl did in his original presentation.

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