Alternative Classical Mechanics

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Abstract

This paper presents an alternative classical mechanics, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Universal Reference Frame

The universal reference frame is a reference frame fixed to the center of mass of the universe.

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to the universal reference frame $\mathring{\mathrm{S}}$, are given by:

$$\mathring{\mathbf{r}}_a = (\mathbf{r}_a)$$

$$\mathring{\mathbf{v}}_a = d(\mathbf{r}_a)/dt$$

$$\mathring{\mathbf{a}}_a = d^2(\mathbf{r}_a)/dt^2$$

where $\mathbf{r}_a$ is the position of particle A relative to the universal reference frame $\mathring{\mathrm{S}}$.

The dynamic position $\breve{\mathbf{r}}_a$, the dynamic velocity $\breve{\mathbf{v}}_a$, and the dynamic acceleration $\breve{\mathbf{a}}_a$ of a particle A of mass m_a, are given by:

$$\breve{\mathbf{r}}_a = \int\int (\mathbf{F}_a/m_a)\, dt\, dt$$

$$\breve{\mathbf{v}}_a = \int (\mathbf{F}_a/m_a)\, dt$$

$$\breve{\mathbf{a}}_a = (\mathbf{F}_a/m_a)$$

where $\mathbf{F}_a$ is the net force acting on particle A.

General Principle

The total position $\tilde{\mathbf{r}}_a$ of a particle A, is given by:

$$\tilde{\mathbf{r}}_a = \mathring{\mathbf{r}}_a - \breve{\mathbf{r}}_a$$

The general principle establishes that the total position $\tilde{\mathbf{r}}_a$ of a particle A is always in equilibrium.

$$\tilde{\mathbf{r}}_a = 0$$

Observations

Applying the general principle to a particle A, it follows:

$$\begin{tikzcd}
\boxed{m_a\mathring{\mathbf{r}}_a - m_a\breve{\mathbf{r}}_a = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{r}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{r}}_a^2 = 0} \arrow[d] \\
\boxed{m_a\mathring{\mathbf{v}}_a - m_a\breve{\mathbf{v}}_a = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{v}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = 0} \arrow[d] \\
\boxed{m_a\mathring{\mathbf{a}}_a - m_a\breve{\mathbf{a}}_a = 0} \arrow[r] \arrow[ur] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{a}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{a}}_a^2 = 0}
\end{tikzcd}$$

Substituting $\breve{\mathbf{r}}_a$, $\breve{\mathbf{v}}_a$ and $\breve{\mathbf{a}}_a$ from page [1] into the above equations, we obtain:

$$\begin{tikzcd}
\boxed{m_a\mathring{\mathbf{r}}_a - \int\int \mathbf{F}_a \, dt \, dt = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{r}}_a^2 - \tfrac{1}{2}\, m_a(\int\int (\mathbf{F}_a/m_a) \, dt \, dt)^2 = 0} \arrow[d] \\
\boxed{m_a\mathring{\mathbf{v}}_a - \int \mathbf{F}_a \, dt = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{v}}_a^2 - \int \mathbf{F}_a \, d\mathring{\mathbf{r}}_a = 0} \arrow[d] \\
\boxed{m_a\mathring{\mathbf{a}}_a - \mathbf{F}_a = 0} \arrow[r] \arrow[ur] & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{a}}_a^2 - \tfrac{1}{2}\, m_a(\mathbf{F}_a/m_a)^2 = 0}
\end{tikzcd}$$

Where $\tfrac{1}{2}\,\breve{\mathbf{v}}_a^2 = \int \breve{\mathbf{a}}_a \, d\breve{\mathbf{r}}_a \;\rightarrow\; \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = \int m_a \,\breve{\mathbf{a}}_a \, d\breve{\mathbf{r}}_a \;\rightarrow\; \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = \int \mathbf{F}_a \, d\breve{\mathbf{r}}_a \quad (\breve{\mathbf{r}}_a = \mathring{\mathbf{r}}_a)$

Transformations

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to a reference frame S, are given by:

$$\mathring{\mathbf{r}}_a = \mathbf{r}_a + \breve{\mathbf{r}}_S$$

$$\mathring{\mathbf{v}}_a = \mathbf{v}_a + \breve{\omega}_S \times \mathbf{r}_a + \breve{\mathbf{v}}_S$$

$$\mathring{\mathbf{a}}_a = \mathbf{a}_a + 2\,\breve{\omega}_S \times \mathbf{v}_a + \breve{\omega}_S \times (\breve{\omega}_S \times \mathbf{r}_a) + \breve{\alpha}_S \times \mathbf{r}_a + \breve{\mathbf{a}}_S$$

where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A relative to the reference frame S; $\breve{\mathbf{r}}_S$, $\breve{\mathbf{v}}_S$, $\breve{\mathbf{a}}_S$, $\breve{\omega}_S$, and $\breve{\alpha}_S$ are the dynamic position, the dynamic velocity, the dynamic acceleration, the dynamic angular velocity and the dynamic angular acceleration of the reference frame S.

The dynamic position $\breve{\mathbf{r}}_S$, the dynamic velocity $\breve{\mathbf{v}}_S$, the dynamic acceleration $\breve{\mathbf{a}}_S$, the dynamic angular velocity $\breve{\omega}_S$, and the dynamic angular acceleration $\breve{\alpha}_S$ of a reference frame S fixed to a particle S, are given by:

$$\breve{\mathbf{r}}_S = \int\int (\mathbf{F}_0/m_s)\; dt\; dt$$

$$\breve{\mathbf{v}}_S = \int (\mathbf{F}_0/m_s)\; dt$$

$$\breve{\mathbf{a}}_S = (\mathbf{F}_0/m_s)$$

$$\breve{\omega}_S = \pm \left| (\mathbf{F}_1/m_s - \mathbf{F}_0/m_s) \cdot (\mathbf{r}_1 - \mathbf{r}_0)/(\mathbf{r}_1 - \mathbf{r}_0)^2 \right|^{1/2}$$

$$\breve{\alpha}_S = d(\breve{\omega}_S)/dt$$

where $\mathbf{F}_0$ and $\mathbf{F}_1$ are the net forces acting on the reference frame S in the points 0 and 1, $\mathbf{r}_0$ and $\mathbf{r}_1$ are the positions of the points 0 and 1 relative to the reference frame S, and m_s is the mass of particle S (the point 0 is the origin of the reference frame S and the center of mass of particle S) (the point 0 belongs to the axis of dynamic rotation, and the segment 01 is perpendicular to the axis of dynamic rotation) (the vector $\breve{\omega}_S$ is along the axis of dynamic rotation)

The magnitudes $\breve{\mathbf{r}}$, $\breve{\mathbf{v}}$, $\breve{\mathbf{a}}$, $\breve{\omega}$, and $\breve{\alpha}$ are invariant under transformations between reference frames.

Inertial Reference Frame

A reference frame S is inertial if $\breve{\omega}_S = 0$ and $\breve{\mathbf{a}}_S = 0$, but it is non-inertial if $\breve{\omega}_S \neq 0$ or $\breve{\mathbf{a}}_S \neq 0$.

This paper considers that the universal reference frame $\mathring{\mathrm{S}}$ is always inertial.

Equation of Motion

From the general principle and the transformations it follows that the acceleration $\mathbf{a}_a$ of a particle A of mass m_a relative to a reference frame S fixed to a particle S of mass m_s, is given by:

$$\mathbf{a}_a = \mathbf{F}_a/m_a - 2\,\breve{\omega}_S \times \mathbf{v}_a - \breve{\omega}_S \times (\breve{\omega}_S \times \mathbf{r}_a) - \breve{\alpha}_S \times \mathbf{r}_a - \mathbf{F}_s/m_s$$

where $\mathbf{F}_a$ is the net force acting on particle A, and $\mathbf{F}_s$ is the net force acting on particle S.

In contradiction with Newton's first and second laws, from the last equation it follows that particle A can have non-zero acceleration even if there is no force acting on particle A, and also that particle A can have zero acceleration (state of rest or of uniform linear motion) even if there is an unbalanced force acting on particle A.

In order to apply Newton's first and second laws in a non-inertial reference frame, it is necessary to introduce fictitious forces.

However, this paper considers that Newton's first and second laws are false. Therefore, in this paper there is no need to introduce fictitious forces.

Kinetic Force

The kinetic force $\mathbf{K}_{a|b}$ exerted on a particle A of mass m_a by another particle B of mass m_b, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{K}_{a|b} = \frac{m_a m_b}{m_{cm}} (\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b)$$

where m_{cm} is the mass of the center of mass of the universe, $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the universal accelerations of particles A and B.

From the above equation it follows that the net kinetic force $\mathbf{K}_a$ acting on a particle A of mass m_a, is given by:

$$\mathbf{K}_a = m_a \mathring{\mathbf{a}}_a$$

where $\mathring{\mathbf{a}}_a$ is the universal acceleration of particle A.

Now from page [2], we have:

$$m_a \mathring{\mathbf{a}}_a - \mathbf{F}_a = 0$$

That is:

$$\mathbf{K}_a - \mathbf{F}_a = 0$$

Therefore, the total force $(\mathbf{K}_a - \mathbf{F}_a)$ acting on a particle A is always in equilibrium.