Generalization of Lucas-Lehmer-Riesel Test

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Abstract: Generalization of Lucas-Lehmer-Riesel primality test is introduced.

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1 Introduction

In number theory the Lucas-Lehmer-Riesel test [2], is the fastest deterministic primality test for numbers of the form $k \cdot 2^n - 1$ with $k$ odd and $k < 2^n$. The test was developed by Hans Riesel and it is based on Lucas-Lehmer test [1].

2 Main result

Conjecture. Let $N = k \cdot b^n - 1$ with $b \equiv 0 \pmod{2}$, $b \not\equiv 0 \pmod{3}$, $b \not\equiv 0 \pmod{5}$, $k \equiv 1, 5 \pmod{6}$, $k < b^n$, $n > 2$, and

\[ S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(P_{b/2}(4)), \text{ where} \]

\[ P_m(x) = 2^{-m} \cdot \left( x - \sqrt{x^2 - 4} \right)^m + \left( x + \sqrt{x^2 - 4} \right)^m \]

then:

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Maxima implementation

```maxima
b;k;n;
(s:2*chebyshev_t(b*k/2,chebyshev_t(b/2,2)),
for i from 1 thru n-2 do(s:mod(2*chebyshev_t(b,s/2),k*b^-n-1)))$
(if(s=0) then print("prime") else print("composite"));
```
References