Two Conjectures on Primality

by princeps

March 15, 2013

Abstract : Generalizations of Wilson's theorem and Kilford's theorem are introduced .
Keywords : Primality test , Generalization
AMS Classification :11A51

1 Introduction

In number theory , both Wilson's [1] and Kilford's [2] theorem , represent general , uncoditional and deterministic primality tests .

2 Main result

Conjecture 1.(generalization of Wilson's theorem)
For $m \geq 2$ *, natural number* n *greater than one is prime iff :*

$$(n^m-1)! \equiv (n-1)^{\left\lceil \frac{(-1)^{m+1}}{2} \right\rceil} \cdot n^{\frac{n^m-mn+m-1}{n-1}} \pmod{n^{\frac{n^m-mn+m+n-2}{n-1}}}$$

Maxima implementation

```
m;n;
(f:1 , for i from 1 thru n^m-1 do(f:mod(f*i,n^((n^m-m*n+m+n-2)/(n-1)))))$
(if (f=((n-1)^ceiling(((-1)^(m+1))/2))*n^((n^m-m*n+m-1)/(n-1)))
then print("prime") else print("composite"));
```

Conjecture 2.(generalization of Kilford's theorem)
Natural number n *greater than two is prime iff :*

$$\prod_{k=1}^{n-1}\left(b^k-a\right) \equiv \frac{a^n-1}{a-1} \pmod{\frac{b^n-1}{b-1}}$$

where $b > a > 1$

Maxima implementation

```
a; b; n;
p:1;
(for k from 1 thru n-1 do (p : mod(p*(b^k-a),(b^n-1)/(b-1))))$
(if (p=(a^n-1)/(a-1)) then print("prime") else print("composite"));
```

References

[1] Hazewinkel, Michiel, ed. (2001), "Wilson theorem", Encyclopedia of Mathematics, Springer, ISBN 978-1-55608-010-4
[2] L. J. P. Kilford, A generalization of a congruence due to Vantieghem only holding for primes, 2004, arXiv:math/0402128