Wave functions for quantized volume of the Universe

Ovidiu Racorean

Abstract

For a simple closed FRW minisuperspace model the Wheeler-DeWitt equation is arranged in the form of a dimensionless quantum harmonic oscillator like equation. Solving this equation is trivial and give equally spaced quantized levels for the volume of the universe and the set of wave functions for this bound states of the Universe. A series of philosophical implications of the above scenario, like quantum jumps, impossibility of an inflationary phase and the problem of “absence” of time for the quantum Universe are further briefly discussed.

decontatorul@hotmail.com

1. Introduction

Attempts in solving Wheeler-DeWitt equation in minisuperspace hypothesis are mainly concentrate around considering it as a Schrodinger equation for a particle with zero mass acting in a potential $U(a) = \left(\frac{3\pi}{2G}\right)^2 a^2 \left(1 - \frac{8\pi G}{3}\rho_v a^2\right)$.

Such interpretation was fruitful in the ’80 leading to the well known tunneling and Hawking-Hartle wave functions for the Universe. Although, different in the perspective of the universe creation, both converge to the conclusion that universe nucleates from “nothing” and experienced a phase of inflation to its present state.

The present paper presents an alternative way to solve the Wheelet-DeWitt equation in hypothesis of a constant volume for the universe. In this manner a discrete spectrum for the volume of quantum universe had been found. The wave functions associated with the volume levels are also deduced.

The ground state wave function is associated with the universe having the lowest volume level, in this case V_1 and that makes unimaginable a universe with zero volume. Although, the present paper is not referring at the creation of the universe it will question the possibility that universe could nucleates from noting.

Being constrained by the quantized volume the universe can evolve only by quantum jumps between volume levels. The initial state for the universe is the ground state volume. At this level the universe is absolute steady. That is way the “absence “of time for the quantum universe is not a problem anymore, but a normal fact.

What makes the universe to jump for the first time from the ground state? The first jump could be the result of vacuum fluctuations or some kind of other quantum effects.

The layout of the paper is as follows. Section 2 is devoted to write Wheeler-DeWitt in the form of Schrodinger equation for a harmonic oscillator having the volume of the universe as a counterpart for the oscillator energy. Solving this equation led to discreteness of universe volume. In sec.3 the volume ground state is deduced and a brief attempt to reject the creation of universe from nothing is discussed. Sec.4 is dedicated to the derivation of the complete set of eigen-wave functions associated with the volume levels of the quantum universe.

The paper is concluded with a brief discussion on the problem of absence of time at the quantum level of the universe and also the impossibility of an inflationary phase to exist in the light of the volume discreteness hypothesis.

1. Quantization of the Universe volume

Consider Wheeler-DeWitt equation for a closed FRW minisuperspace in the simplest form:

$$\left[\frac{d^2}{da^2} - \left(\frac{3\pi}{2G}\right)^2 a^2 \left(1 - \frac{8\pi G}{3}\rho_v a^2\right)\right]\psi(a) = 0\,, \tag{1}$$

with the well known notations. Taking the volume of the universe as being $V = \frac{\pi^2}{2}a^4$ and rearranging the terms equation (1) can be written as:

$$\frac{d^2}{da^2}\psi(a) + \left[\frac{3\pi}{2G} 8\rho_v V - \left(\frac{3\pi}{2G}\right)^2 a^2\right]\psi(a) = 0\,. \tag{2}$$

At this point, the form resemble with the familiar Schrodinger equation for the quantum harmonic oscillator become obvious.

Although, similar in form the two equations have physically different meaning. Both equations are representations of the conservation low but in the case of the Universe eigenvalues for energy are excluded. It will be seen latter in this chapter that in the case of universe the volume has discrete values.

The further discussion follows to a certain point a scheme that can be find in J.H. Kung []. It is convenient to define several parameters:

$$\alpha = \sqrt{\frac{3\pi}{2G}} \tag{3}$$

$$\varepsilon = 8\rho_v V \tag{4}$$

In terms of these variables the Wheeler-DeWitt equation is

$$\frac{d^2}{da^2}\psi(a) + \alpha^2(\varepsilon - \alpha^2 a^2)\psi(a) = 0\,. \tag{5}$$

Rescaling variable

$$y = \alpha a$$

one can write Wheeler-DeWitt equation in the form

$$\frac{d^2}{dy^2}\psi(y) + (\varepsilon - y^2) = 0 \tag{6}$$

that can be easily identified as dimensionless Schrodinger equation for the harmonic oscillator. Equation (6) can be solved only for particular choice of parameter ε namely:

$$\varepsilon = 2n + 1, \qquad n = 0,1,2,\ldots$$

that force the volume of the universe to take only one of the discrete values:

$$V_n = \left(n + \frac{1}{2}\right)\frac{1}{4\rho_v} \quad (7)$$

giving the spectrum of volumes of the quantum universe. As it will be stated further, only the odd values of n will be retaining for the volumes spectrum.

Volume levels are equally spaced by $\frac{1}{4\rho_v}$.

2. From zero volume "nothing" to ground state volume

It is easy to remark that the ground state of the universe is defined as being the state with the lowest volume. In this case the ground state volume is that of the first odd n

$$V_1 = \frac{3}{8\rho_v} \quad (8)$$

The quantum universe is restricted to a series of discrete volumes with a lowest level V_1 which could take the place of "nothing" in the Hawking-Hartle [] and Vilenkin[] representations. The Vilenkin view of "nothing" is that of a state with no notion of spatiality. Hawking-Hartle approach of ground state of the universe is that of a point situated at the poles of a 4-dimensional sphere. As one easily can remark, both recipe for the lowest level of the universe is that of a zero volume.

In the discreteness of universe volume scenario the "nothing" as a zero volume is rejected because the minimum volume can not be less than the ground state V_1.

A complete set of eigen-wave functions will be derived in the next section and will replace the consecrated tunneling and Hawking-Hartle wave functions.

3. Wave functions for quantized volume of the Universe

Having stated the quantized levels for the universe volume it is straightforward to look for the associated wave functions. For each discrete values V_n of the volume given by (7), there corresponds one, and only one, acceptable eigenfunction:

$$\psi_n(y) = N_n e^{-\frac{y^2}{2}} H_n(y) \quad (9)$$

where N_n is a normalization constant and H_n the Hermite polynomials. Returning to the original variables and taking $N_n = \left(\frac{\sqrt{3}}{\sqrt{G}2^{\left(n+\frac{1}{2}\right)}n!}\right)^{\frac{1}{2}}$ the eigenfunctions for the Universe are given by

$$\psi_n(a) = \left(\frac{\sqrt{3}}{\sqrt{G}2^{\left(n+\frac{1}{2}\right)}n!}\right)^{\frac{1}{2}} H(\alpha a)e^{-\frac{(\alpha a)^2}{2}} \quad (10)$$

The boundary condition $a > 0$, impose the wave function to vanish at $a = 0$ and that selects for the eigenfunctions of the universe only the odd states. In this case the wave function for the ground state of the universe is

$$\psi_1(a) = \left(\frac{\sqrt{3}}{\sqrt{G}2^{\left(\frac{3}{2}\right)}}\right)^{\frac{1}{2}} H(\alpha a)e^{-\frac{(\alpha a)^2}{2}} \tag{11}$$

This is the wave function of the lowest volume level of the universe.

4. Discreteness of volume and "absence" of time for quantum Universe

Existence of discrete spectrum for the volume of the Universe at the quantum level is the main concept of this paper. Discussions about discreteness of space volume and area appear before in the construction of loop quantum gravity [] and in the approaches of black holes quantization []. In [] Smolin L. argued that discrete volumes and areas of space may be the final test for the proving the correctness of LQG theory.

In this paper I emphasized the concept of a quantum universe that can only occupy one of the steady states predefined by the eigenfuctions latter deduced in chapter 4. The level specified by the ground state wave function is the initial state. The quantum universe in the ground state in not evolving, the space-time dimensions are freeze. To talk about time in an absolutely steady universe is meaningless. Time is not absent, is very present in the texture of 4-dimensional space-time. It just only makes sense to talk of time in classical, evolving Universe.

At the quantum level the evolution of the universe is manifest by quantum jumps between the bound states. That makes useless discussions about an inflationary phase in the very early universe, first because there is no " early " before the classical era and second because discreteness of volume will not allow other positions for the Universe except the bound states.

How universe came into the present state remain an open issue for future approaches.

5. Concluding remarks

Hypothesizing a constant volume for the universe the Wheeler-DeWitt equations for a close

simple FRW minisuperspace is rescaled in a dimensionless Schrodinger equation for harmonic oscillator form. Solving this particular equation, a discrete spectrum for the volume of the universe is found.

In this context, Hawkinh-Hartle and tunneling functions of the universe are rejected and replaced with a complete set of wave functions that reflect the volume levels.

The paper is concluded with brief remarks on the implications of discreteness of volume on time at the universe quantum level.

6. References

[1]. Hartle, J.B. and Hawking S.W. (1983) Phys. Rev. D28, 2960.

[2]. DeWitt, B.S. (1967) Phys. Rev. 160, 1113.

[3]. Vilenkin, A. (1984) Phys. Rev. D30 509.

[4]. Kung, J.H. (1993) Quantization of closed mini-superspace models as bound states, arXiv:hep-th/9302016v1

[5]. Bekenstein J.D. Lett Nuovo Cim 11 (1974) 467

[6]. Rovelli C. and Smlin L. Discretness of area and volume in Quantum Gravity, arXiv:gr-qc/9411005v1