Dark Energy: Order-Disorder In The Cosmos
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We are such things as dreams are made of (William Shakespeare)

Abstract; An Application of Statistical Mechanics to Dark Energy

A reexamination of a past application of order-disorder theory to a new two dimensional application produces a toy model of "dark energy." This model of the universe contains negative pressure. The model depicts sparse basic geometric figures having vertices covered with celestial objects. Most of the basic figures are empty or considered as vacuum covered. There are interactions between the occupied adjacent vertices of the basic figures. The basic figures change from a point to a bond to a triangle and, as larger basic figures are considered, the results become more accurate as the mathematics become more difficult. There is no need in this application to resort to phenomena as negative mass or anti matter, etc. in order to explain dark energy as has been done in some other publications. However, new publications that refer to entangled growth and coevolution of correlated multiplex networks seem appropriate.

Introduction: The big bang caused the universe to expand very fast and equally in all directions. Such a phenomenon happened 13.7 billion years ago. Recently astronomers can surmise that distant galaxies are moving away from us at an accelerated velocity, much above what is expected. This is said to be caused by "dark energy." It is believed that the entropy of the universe is always increasing. A previous application of order-disorder theory is utilized here to examine a scenario where the interaction between stars and galaxies cause events to be now construed as dark energy. It should be understood that what was called order-disorder theory some years ago is now called chaos theory. It is also sometimes referred to as a theory of cooperative phenomena. Such cooperative phenomena can be seen to cause repulsion as well as attraction.

Three papers by Hijmans and De Boer (1) developed a treatment of order-disorder theory that has been applied to a study of localized physical adsorption. This led to the Langmuir isotherm, when using a point as a basic figure, and the Fowler-Guggenheim isotherm, when using the bond as the basic figure. They will be called the zero- and first- order cases of a more generalized approach. The same procedure will be produced here and will be used to model the phenomenon known as dark energy in present cosmological descriptions.

Our universe is considered as a two dimensional plane containing basic figures which are the smallest localities that act as centers for the appearance of stars or galaxies. These sites may be occupied or empty and a sample of these sites may form geometric figures that rise in complexity. One can assign to each site an energy of attachment and a probability of occurrence. From these quantities, the energy and entropy of the assembly of sites can be computed. This yields an expression for the free energy and after minimization, the chemical potential of the attached phase is equated with the chemical potential of the free phase. Normalizing and consistency equations can also be derived in addition to the equilibrium relations that are given for the basic figure and its subfigures. The exact final resulting expressions can then lead to the isotherm equations that pertain to the largest basic figure in each approximation.

It is realized that, in the application of a gas adsorbed onto a surface, we were dealing with basic figures that were part of a lattice. This was indicated by a coordination number (Z), for the case of either a square or hexagonal lattice. There was also an understanding that the procedure could be used for three dimensional lattices. The two dimensional case could be extended to a three dimensional case by holography.

A method for deriving the chemical potential (μ) of particles adsorbed on a two dimensional surface has previously been derived (1, 2,) for lateral and next nearest neighbor interactions of the particles. In order to do so a parameter called K was used and $K = ((\exp((\mu-\epsilon/kT))(\Theta/1-\Theta)) =(P/Po)(\Theta/1-\Theta))$. In the last equation, ϵ is the adsorption energy, k is the Boltzman constant, T is the temperature in degrees Kelvin and P is the pressure. It was found from a series of normalizing, consistency, and equilibrium relations shown in papers by Hijmans and DeBoer (1) and used by Bumble and Honig (2) in a paper on the adsorption of a gas on a solid. In the above, μ is the chemical potential and ϵ is the adsorption energy. The numerical values of K were derived from computers for various lattices with different values of the interaction parameters for nearest neighbors (c) and next nearest neighbors (c′), where $c = \exp(-w/kT)$ and w is the interaction energy, and the "order" of such lattices were plotted as the values of $\exp(\mu-\epsilon/kT)$ or $P/Po=\exp((\mu-\epsilon/kT))$ versus Θ (the degree of occupancy of the lattice). A method for approximating the lattice was accomplished mathematically by selecting basic figures such as the point o, the bond o—o, the triangle Δ, or the rhombus ◇. The basic figure, the bond, has the point as a subfigure. If the point represents a planet, or a star or a galaxy, then the fully occupied bond can be thought of as two celestial objects with a connection between them. The occupation can be represented as a nucleation and the nucleating factor can be symbolized as K′. The two connected sites in a bond are connected by an energy denoted as c. We can have four bond situations: •-•(c(K′)^2), •-o(K′), o-•(K′), or o-o(1), in which case,

$\Theta =(K'+(K')^{2}c)/(1+2K'+(K')^{2}c)$,

Where the Greek letter theta stands for the fraction of sites that are occupied, K′ is the nucleating variable and $c = \exp(-w/kT)$ is the parameter for the interaction energy.

The value of K′ can be found using the methods of Hijmans and de Boer(1) to set up the normalizing, consistency and equilibrium relations in the original papers for a lattice of such sites. The mathematics in the original papers are reformulated in a simpler form without reducing the rigor of the original methods or results. This is presented here. It can be derived that $K'=K(\Theta/(1-\Theta)) =Ka$, where $a = \Theta/(1-\Theta)$, and thus the above equation becomes

$$\Theta = Ka(1+cKa)/(1+2Ka+c(Ka)^{2})$$

Solving for Θ, we find the quadratic equation

$$c (K)^{(2)}a+K(1-a) -1 = 0$$

and $K =(P/Po)(1-\Theta/\Theta))$, where Θ is the extent (varying from 0 to unity) of coverage of the adsorbent, μ is the chemical potential and ϵ is the energy of adsorption on the site. Setting the value of $(\Theta/1-\Theta)$ equal to a, then $P/Po = Ka$.

If $\beta = (1+4c\Theta(1-\Theta)(c-1))^{(1/2)}$,

then, $P/Po = (\Theta/(1-\Theta)(\beta-1+2\Theta)/(2\Theta c)$

This equation represents the isotherm for the adsorption for a particular value of c. At a certain value for c the isotherm may become flat, i. e., the pressure is constant for a number of values of Θ. This is the critical isotherm that occurs when the adsorbed gas turns into a liquid and many values of Θ share the same value of P/Po. The process has been carried out for the cases when the basic figure of the lattice is a bond, a triangle and a rhombus. They yield in succession more complex calculations and better approximations to the exact answers. Such isotherms are shown below in figure 1 where P/Po is plotted against Θ. Critical isotherms were obtained for the bond when c was approximately 1.75 (see figure 1); for the triangle when c was approximately 2.78 and for the rhombus when c was approximately 3. (See figure 1). We notice in the reference that a value for the coordination number (Z) was used as 6 which is not used in the cosmic application. This was because in the application for adsorption the basic figure was part of a lattice. Here the basic figure is a free agent.
Applying this format to the model of the sparse universe, we use the coordination number Z as unity so that we have the following equation as shown above:

$$P/Po =(\Theta/(1-\Theta)(\beta-1+2\Theta)/(2\Theta c)$$

A graph of the above equation with the interaction parameter, c, and the occupancy, Θ, varying is shown in figure 1.

It is emphasized that in the solution of the quadratic equation above, β was assumed to be positive. In the quadratic solution used in the cosmic bond solution, β can also be negative. Then the equation for P/Po is given below.

$$P/Po = (\Theta/(1-\Theta)(-\beta-1+2\Theta)/(2\Theta c)$$

and when P/Po is plotted against Θ, we obtain the isotherms shown below in figures 2 to 4 for various values of c. Notice in these graphs all values of P/Po are negative and that the value -1 is the limiting value. The graph can be regarded as holograms of a three dimensional field; the cosmos. This would mean that we have a toy model of what is taking place during the appearance of dark energy in 3 dimensions. Also note that the value of $P/Po/\rho$ is $< -1/3$, (where ρ is the density), as expected.

Discussion: Figure 1 shows isotherms for P/Po (pressure ratio of a gas) versus Θ (coverage of several basic figures formed on a triangular lattice of an adsorbent). The values of $c = (-w/kT)$, where w is the interaction energy between adjacent occupied sites of the lattice, is given for the bond, and is shown for several cases.

Considering the isolated basic figures, one can calculate critical values of c when the adsorbed matter changes its phase. The derivation for P/Po as a function of Θ is shown in the literature and a reprint of the article to show this will be sent to those who indicate their interest by email.

In the equation above, P/Po is shown as a $f(\Theta)$. In its derivation, the values of β can be either positive or negative. If we use the negative value of β, P/Po will always be negative. When β is positive, then P/Po will always be positive. Figures 2 to 4 show plots of when P/Po can be either. Where $c>1$, then the interaction is attractive, and P/Po is positive. When $c<1$, or the interaction energy is repulsive, then P/Po is negative. These latter cases correspond to dark energy. It is also noted that the slopes will also correspond to positive values when P/Po is positive and negative values when P/Po is negative, as seen in figure 5. This figure has P/Po varying continuously for c = 0.25, .5, .75, 1.5, 2, 2.5 and 3. We can also have a mixed plot of two values of c, one with positive energy (attraction) and the other with negative energy (repulsion) as long as the mole fraction sum is unity and this is shown in figure 6.

The temperature of the cosmos is 2.7 degrees Kelvin. Recently there has been the possibility of negative absolute temperatures (6). This could allow a negative value for β and then the phenomenon of negative pressure can also occur.
When the value of Θ is very small, then the value of P/Po becomes very negative. This occurs at the Big Bang and there the expansion is, of course, very large and very fast. If this continues for a long time we can have the big rip. The values of Θ can correspond to the time since the big Bang. However, most of the application of this toy model corresponds to values of Θ that are very low indicating that the occupancy is scant.

Further work done here used the triangle as the basic figure. This leads to a quartic equation in K, which was solved for its four roots of K. The real roots can lead to a singular-like solution where P/Po is driven to extremely negative values for a particular value of Θ. This again corresponds to the expansion of the universe in an extremely short time and indicates that it happened shortly after the "big bang". This can also be simulated in a close analogy using special values of c in the bond model. It is to be expected that the expansion is from a vacuum and the instantaneous expansion was accompanied with very high negative pressure.

Conclusions: As we uncover more about the universe around us, including exoplanets and “exostarsystems,” it may be necessary for more equipment such as better telescopes, the LHC, and other astrophysical instruments to refine or disprove the theoretical work presented here. Other recent publications that utilize entanglement or multiplex networks may be combined with the present work to improve the mode and results. There have been many graphs constructed similar to the figures, so this work has just begun. A universe consisting of phenomena associated with negative pressure can help us understand more clearly our place and future in the multiverse. Theory must be subject to experimental and/or observational tests and we will need to wait for further developments.

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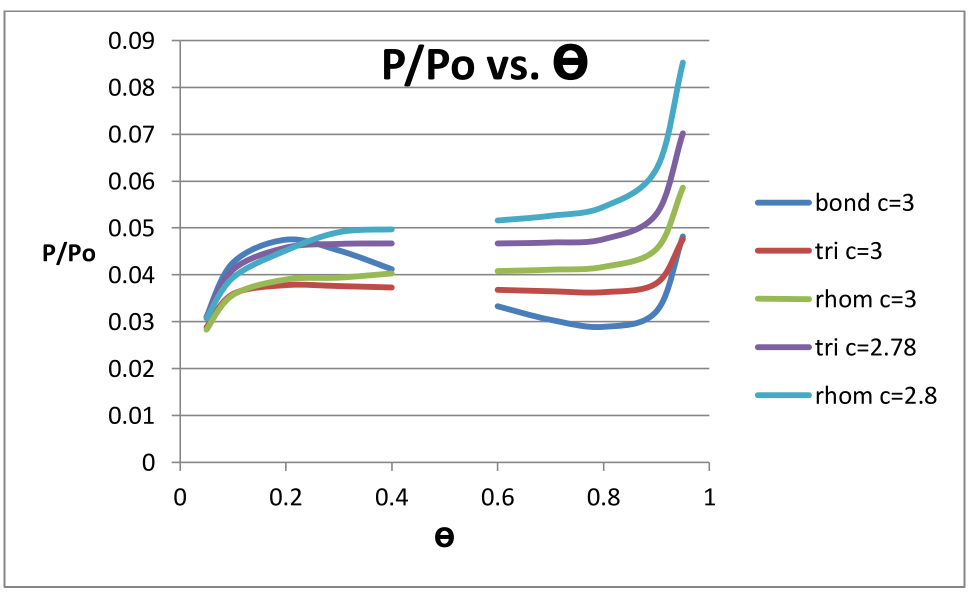

Figure 1

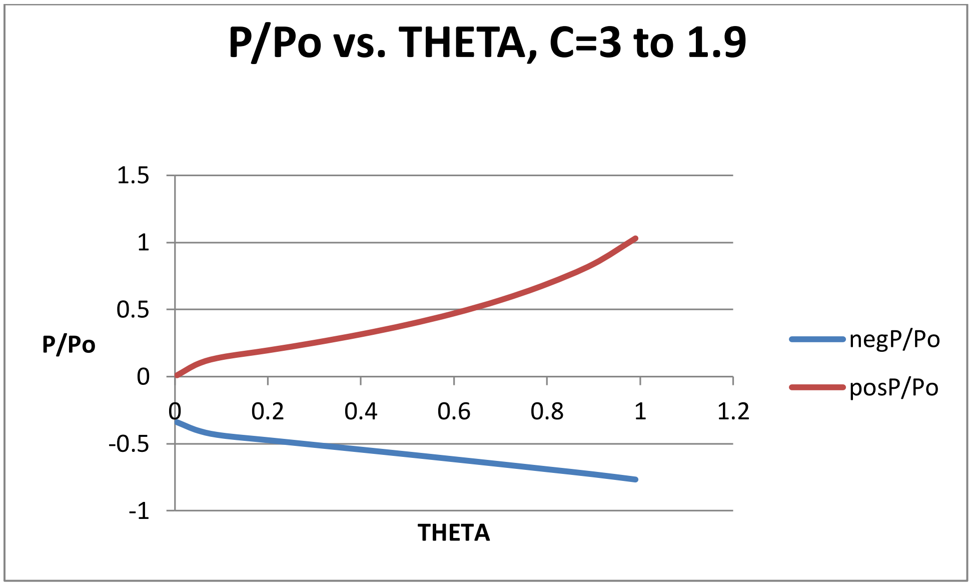

Figure 2

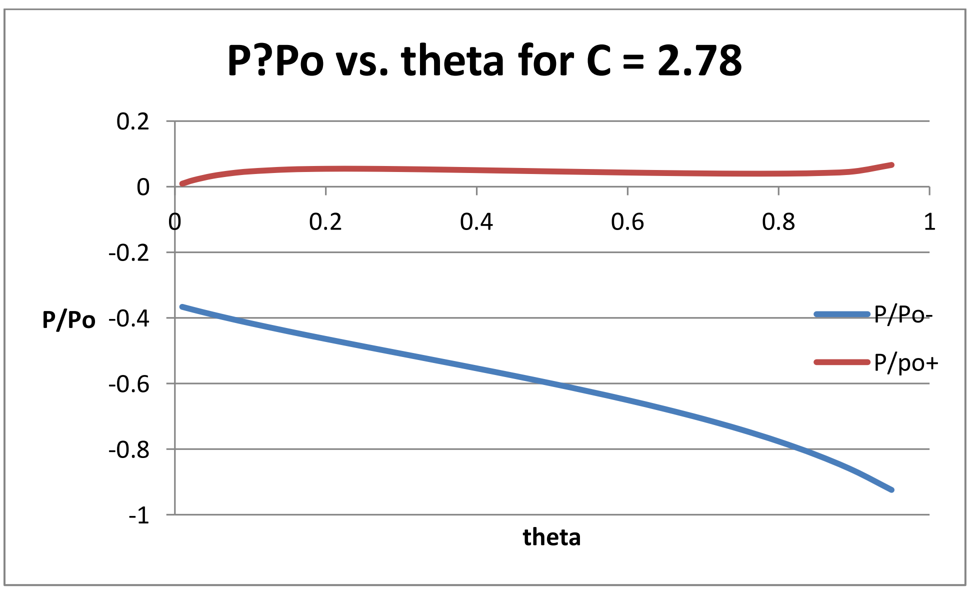

Figure 3

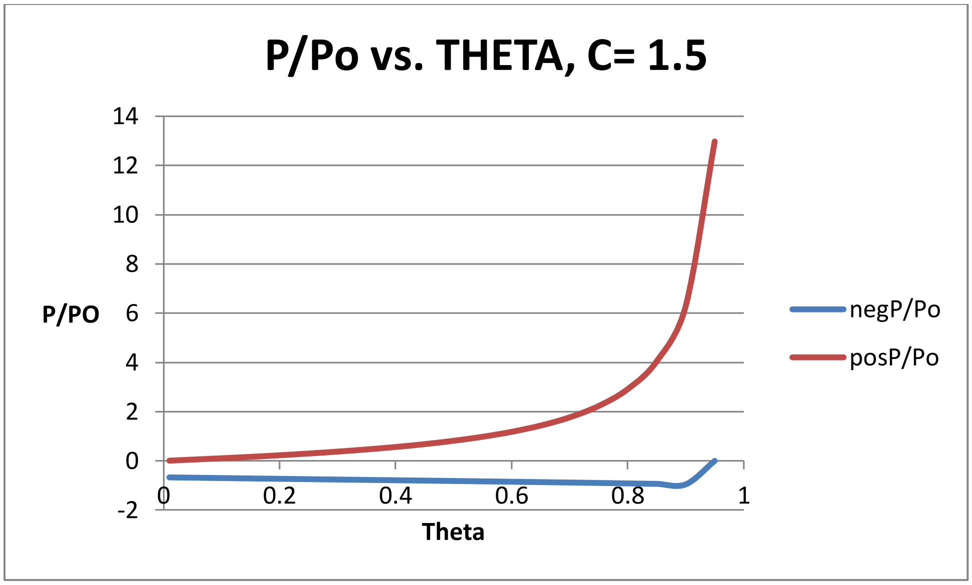

Figure 4

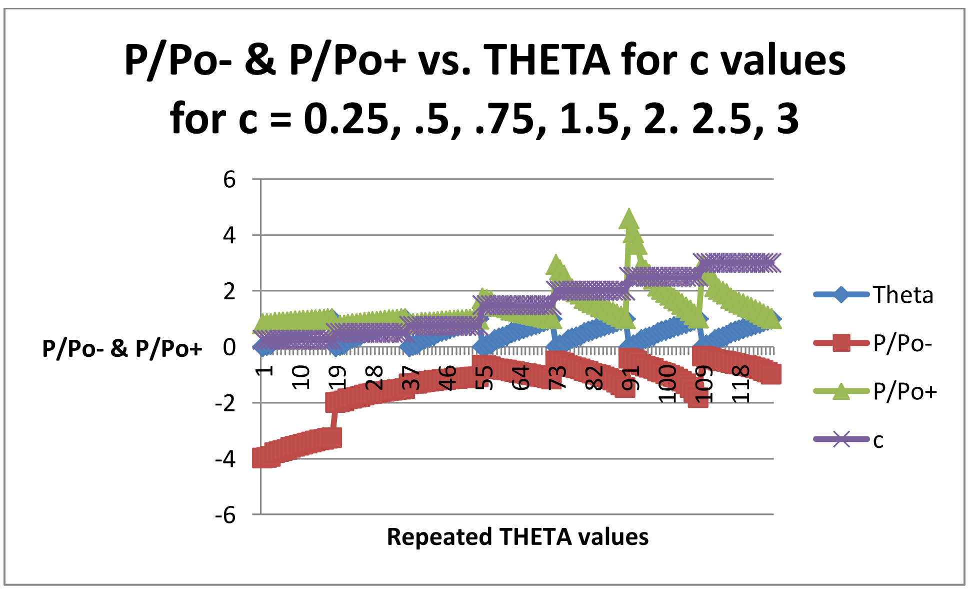

Figure 5

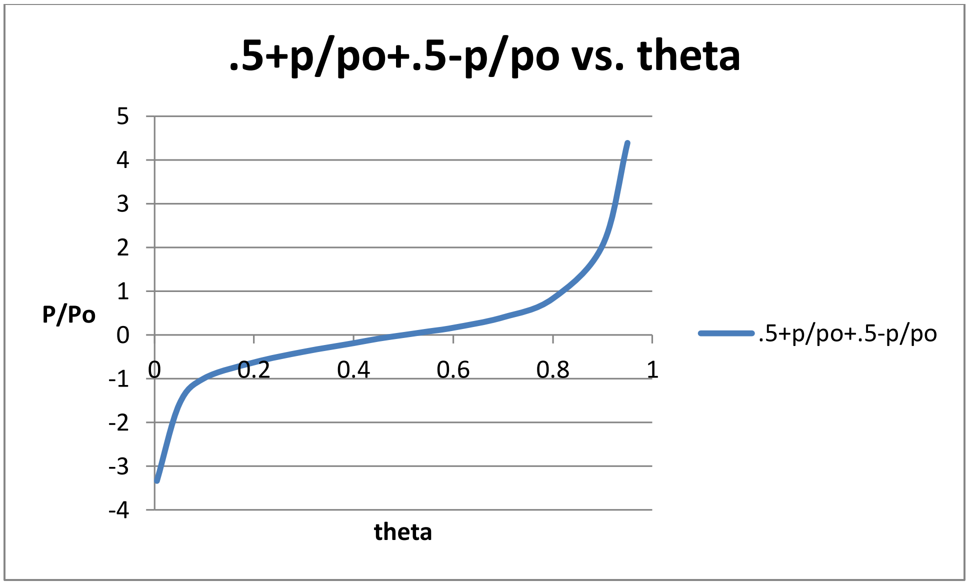
.5+p/po+.5-p/po vs. theta
5
4
3
2
1
0
-1
-2
-3
-4
P/Po
0
0.2
0.4
0.6
0.8
1
theta
.5+p/po+.5-p/po

Figure 6