On the Velocity of the Photon

J. Shim

Abstract

In a previous paper, "On the Applicability of the Lorentz Transformations", the issue of the physical vs. apparent velocity of a photon, c vs. $(c\pm v)$, was examined in the cases of the emitter moving relative to the detector and vice versa. In the present paper, the case is examined of both the emitter and detector moving relative to the origin of a third coordinate system, the case used by Einstein for his derivation of the Lorentz transformations. It is shown that both Einstein's derivation and a more general derivation are inconsistent with the behavior of the photon, and that the physical (not apparent) velocity of the photon as measured in the stationary coordinate system in this case must be $(c\pm v)$. It is also noted that because the velocity of the photon is independent of its momentum and energy, the dispersion relations between conjugate Fourier variables of energy and time, $\nabla E\nabla t \leq \hbar/2$, and momentum and position, $\nabla p\nabla x \leq \hbar/2$, do not affect its velocity. As a result, unlike for non-zero rest mass quanta, the position of a photon as a function of time can in principle be determined with an arbitrary accuracy.

To illustrate the dependence of the velocity of a photon on the velocity of the emitter and detector relative to the origin of a third coordinate system, let there be an emitter, E, with a detector, D attached in front of it at a fixed distance, d. Let the coordinates in the coordinate system of E be ξ and τ, and the coordinates in the stationary system be x and t. At τ=t=0, the emitter E is located at ξ=x=0 and both E and detector D are moving at a velocity v along the x-axis of the stationary system.

At $\tau_0=t_0=0$, a photon is emitted from E towards D. If the clocks in both coordinate systems are synchronized, then the photon in the moving system will arrive at D at $\xi = \xi_1= d$, at the time $\tau = d/c = \tau_1= t_1$. Since the velocity of the detector D is v, and its location at τ=t=0 is ξ=x=d, then its position along the x-axis of the stationary system at any time $t > 0$ will be d+vt. At $\tau_1=t_1$, it will have traveled a distance vt_1, so that its location at ξ_1 in the moving system will be $x=d+vt_1$ in the stationary system. Therefore, in the stationary system, the velocity of the photon emitted at $\tau_0 =0$ and arriving at D at τ_1 must be $(d+vt_1)/t_1$, or $(c+v)$. If another photon at τ_1 is emitted from D to E, then it will arrive at the emitter E at $\xi = \xi_2 = 0$, and $\tau = \tau_2 = 2\,\tau_1= 2vt_1$. Since $\xi = 0$ at τ_2, this will correspond to $x = 2vt_1$, so the second photon will have traveled a distance of $d - vt_1$ in the stationary coordinate system. Therefore the velocity of the second photon in the stationary system must be $(c – v)$.

Einstein [1] attempts to derive a general set of coordinate transformations between ξ, τ and x, t by expressing the average time of travel of the photon in the moving coordinate system from ξ_0 = 0 to ξ_1 = d, and back to ξ_2 = 0, in terms of the position coordinates in the moving system, and time coordinates in the stationary system, basing these on the time of travel of the photons as measured in the stationary system. Using his original notation of x' for the coordinate ξ_1 = d of the detector D in the moving system in our example, the equation is

$$\tfrac{1}{2}\left[\tau_0(0,0,0,t) + \tau_2(0,0,0,t + x'/(c-v) + x'/(c+v))\right] = \tau_1(x',0,0,t + x'/(c-v)). \qquad (1)$$

But the determination of the time values in the arguments of τ in this equation is only possible by already knowing the coordinate transformations at $\xi_0 = 0$, $\tau = \tau_0$, $\xi_1 = d$, $\tau = \tau_1$, and $\xi_2 = 0$, $\tau = \tau_2$. That is, the time of travel of the photons in either direction as measured in the stationary system cannot be specified without already knowing the coordinate transformations at these points.

But let us put this issue aside for the moment, and look at the correctness of (1) based on the coordinate values in the stationary coordinate system, corresponding to the emission and detection of the photons at E and D in the moving system, in our example. We are basing our determination of these values on the same expressed or implied measurement assumptions as Einstein, namely, that clocks in both coordinate systems are synchronized, the photons travel at velocity $\pm c$ in the moving coordinate system between E and D, we can measure the velocity of E and D as v in the stationary system, and we can measure the time and location of arrival of the photons at both D and E on the x axis of the stationary coordinate system.

There are several problems with (1). First, obviously x' in the time coordinates cannot represent a position coordinate, but only the magnitude of a coordinate. Otherwise, if we put the initial detector at -x', then the time coordinates of the photons would be $[t - x'/(c+v)]$, and $[t - x'/(c+v) - x'/(c-v)]$. Second, both of the times of travel of the photons, $x'/(c-v)$ and $x'/(c+v)$, are incorrect according to our example. The time of travel of the photons measured in the stationary system is the same as in the moving system: $d/c = \tau_1 = t_1$. The difference between the measurements of the photons in the two coordinates systems is not in the time of travel, but in the distance traveled in the same time interval. In the stationary system, the photon moving in the +x directions travels a distance $d + vt_1$, and the in the $-x$ direction a distance of $d - vt_1$. Since the time of travel of both photons in the moving and stationary systems is $d/c = \tau_1 = t_1$, their velocity in the stationary system must be $(c + v)$ in the + x direction and $(c - v)$ in the -x direction. The correct expression of (1) would then be

$$\tfrac{1}{2}\left[\tau_0(0,0,0,t) + \tau_2(0,0,0,t + |x'|/c + |x'|/c\right] = \tau_1(x',0,0,t + |x'|/c) \qquad (2)$$

If we convert (2) into a differential equation in the manner of Einstein, treating τ, x' and t as arbitrary coordinates, then we get

$$\tfrac{1}{2}(1/c + 1/c)\, \partial\tau/\partial t = \partial\tau/\partial|x'| + 1/c\ \partial\tau/\partial t\,, \text{ or}$$

$\partial\tau/\partial|x'| = 0$, rather than

$$\tfrac{1}{2}\,(1/(c-v) + 1/(c+v))\ \partial\tau/\partial t = \partial\tau/\partial x' + 1/(c+v)\ \partial\tau/\partial t \text{ , or}$$

$$\partial\tau/\partial x' = -\,c^2/(c^2-v^2)\ \partial\tau/\partial t \qquad (3)$$

But both (1) and (2) do not represent a relationship among arbitrary coordinates, τ, x', and t, as Einstein treats them in (3), and assumes in his subsequent derivation. That is, t does not represent the time coordinate in the stationary system corresponding to an arbitrary set of coordinates x' and τ in the moving system. Rather, it represents the arbitrary initial time of emission of the first photon. t is an initial condition, and merely defines an arbitrary term on both sides of (1) and (2), which we can extract as follows.

$$\tfrac{1}{2}\,[\tau_0(0,0,0,0) + \tau_2(0,0,0,x'/(c-v) + x'/(c+v))] + \tau'(t) = \tau_1(x',0,0,x'/(c-v)) + \tau'(t). \quad (1')$$

If clocks in both systems are synchronized, then τ' = t. So, except for an arbitrary choice of the initial time value, both sides of (1) are independent of t, and we have

$$\tfrac{1}{2}\,[\tau_0(0,0,0,0) + \tau_2(0,0,0,x'/(c-v) + x'/(c+v))] = \tau_1(x',0,0,x'/(c-v)), \qquad (1'')$$

By treating the initial condition, t, as the time variable itself, Einstein creates a relationship between the moving and stationary coordinates that does not exist.

More general derivations of the Lorentz transformations such as [2] seem to me to be little more than mathematical exercises, because they do not refer to specific physical measurement situations, so that it is impossible to judge the validity of the assumptions used in the derivation. The coordinate transformations we are seeking here are empirically derived relationships between the coordinates measured in a moving coordinate system, and the same coordinates determined using measurement devices based in a stationary coordinate system. In effect, they are measurement corrections, allowing a mathematical method of adjusting the measurements of a moving event by stationary devices, so that they become equivalent to the same event occurring in the stationary system. In order to be well defined, the transformations must be based upon the specification of a measurement process that completely defines the relationship between the set of coordinates measured in the moving system, and the measurement results in the stationary system. Without such a specification, the derivation of the transformations is based on inherent assumptions about the measurement process, which may or may not be correct. Therefore we will examine some general assumptions in [1] and [2] that are not appropriate for the measurement processes in this example, and also in the moving clock example in [3].

The first assumption is that the relationship between the coordinates in the moving system, ξ, τ, and the stationary system must be expressed as a function of only the coordinates in the stationary system, x, t, and the relative velocity, v, or the relative velocity plus c. That is, $\xi,\tau = f_{\xi,\tau}(x,t,v)$, or $f_{\xi,\tau}(x,t,v,c)$. This is not generally true, as we can see in the example used in [3] of measuring the time of travel of a photon-emitting clock moving at velocity v along the x-axis between stationary detectors at x=0 and x=x. In this case, t, the time of travel of the clock from x=0 to x=x measured in the stationary system is $t= \tau$ if measured by the front detector, and $t = \tau (1 – v/(c+v))$ if measured by the rear detector, so τ is not only a function of x,t,c,v, where x represents the position of the clock, but also the location of the two detectors relative to the position of the clock. And if they are located off-axis, then the measured time of travel of the clock will depend upon not only the x coordinate of the detector relative to the clock, but also the distance from the x-axis, $r = \sqrt{y^2+z^2}$.

So clearly the idea is unrealistic that transformations must generally be of the form

$$\xi = Ax + Bt, \quad \text{and}$$

$$\tau = Gx + Ht,$$

with a single set of coefficients, or with coefficients dependent solely on v, or on v and c. When the form is applicable, then the coefficients will be determined by the measurement assumptions. In our case, since clocks in both systems are synchronized at $\tau = t = 0$, and we can communicate a time τ from the moving to the stationary system in an arbitrarily short time interval, independent of the value of x, then we must have G = 0 and H = 1, so that $\tau = t$. Similarly, since we can determine the location of emitter E and detector D on the x-axis at any time, then at $\tau = t = 0$, for D we must have $\xi = x = d$, so that A = 1. For t > 0, since D moves with velocity v along the x-axis, we must have $\xi = x – vt$, so B = -v.

The assumption that the velocity of the moving coordinate system, v, can be determined consistently with the measurement system is a reasonable, but not a universally valid one. For example, for the moving clock in (3), if we define the velocity of the coordinate system of the clock in terms of the number of photons, N, it emits between x=0 and x=x, then its velocity will be x/N. If we base our measurement of the velocity on the number of photons counted by the front detector, then that is the velocity we will get. But if we base it on the number of photons counted by the rear detector, then we will measure a velocity of $v = x/N\ [c/(c+x/N)]$.

We also cannot treat the relationship between the moving and stationary coordinate systems as generally reversible—Einstein's idea of relativity. Changing the position of the observer, the inertial reference frame, usually changes what is being measured, or how it is being measured, which may or may not be reflected in the coordinate transformations themselves. In the case here, for example, moving the observer to the reference frame of the emitter E and detector D, treating it as stationary, and the other coordinate system as moving at a velocity of $v' = –v$, says nothing about the velocity of the photon when emitter and detector are moving relative to the observer. In this reversed case, the observer is merely measuring the movement of an empty coordinate system, whose origin travels a distance of $–vt_1$ in the time of travel of the photon from E to D. The coordinate transformations are the same, $\tau = t$, and $\xi = x – v't =$

$x - (-v)t$ as when the observer is moving relative to E and D, but the physical arrangement and significance of the experiment are not.

In the moving clock example, (3), changing the position of the observer changes the dynamics of the measurements. Now the clock is stationary at the origin of its own coordinate system, and the detectors are moving at a velocity of $-v$ relative to it. The detector at $x = 0$ is moving away from the clock, and the detector at $x = x$ is moving towards it. Therefore the velocity of the photons with respect to the detectors will be $(c - v)$ for the detector at $x = 0$, and $(c + v)$ for the one at $x = x$. This leaves the time interval of travel of the clock the same for the $x = x$ detector, but changes the measurement by the $x = 0$ detector to $t = \tau (1 - v/c)$ from $t = \tau (1 - (v/(c + v))$.

Based on these issues, it seems to me that the significance of the Lorentz transformations is highly uncertain, and their use in the determination of the velocities of the photons here inappropriate. So, with apparently realistic assumptions, we are put in the somewhat perplexing position of having the velocity of the photon in the stationary system be $(c\pm v)$, while at the same time traveling at a velocity c between the emitter and detector. This seems inconsistent with the view of the photon travelling freely in space, independent of the emitter and detector.

A possible explanation, one that appears generally consistent with the quantum field, is that the emitter and detector define boundaries of the field, or the portion of the field, that determines the photon's velocity, c, between the emitter and detector (although of course the quantum field, as presently understood, does not define the actual value of the velocity.) As long as the velocity of the photon is measured by its time of travel within the boundaries, in either the coordinate system of the emitter or detector, then the physical velocity of the photon is $\pm c$. But when the velocity is measured in a coordinate system in which the entire field between the emitter and detector is moving at velocity $\pm v$, then this velocity must be added to or subtracted from the velocity within the field in order to obtain the correct speed of the photon.

It is also interesting to note that, since the velocity of the photon between an emitter and detector stationary relative to each other is always c, an arbitrarily massive emitter located at $x=0$, emitting a photon in an arbitrarily short time interval at $t=0$ along the x-axis, will define the photon's x coordinate at $t=t$ with an arbitrary precision. The dispersion relations between the conjugate Fourier variables of the time interval of emission and energy of emission, $\nabla E \nabla t \leq \hbar/2$, and position interval and momentum interval, $\nabla p \nabla x \leq \hbar/2$, do not affect the velocity of the photon, so the photon in principle can be emitted in an arbitrarily short time interval, and small position interval, without affecting the velocity, thereby defining the position on the x-axis at any time with an arbitrary precision. This also implies that any spatial extent of the photon must be increasingly limited at arbitrarily large energies, since the emission will occur in a decreasing time interval, which will increasingly limit its possible dimensions at a constant velocity. That is, if ∇t is the time interval of emission, then the maximum spatial extent, ∇x, during emission must decrease as ∇t, since $\nabla x/\nabla t=c$.

[1] A. Einstein, *Annalen der Physik*, 17:891, 1905. 1923 English translation
[2] V. Yakovenko, Lecture Notes for Course Phys171H, U. of Maryland, 15 Nov. 2004

[3] J. Shim, 2013, viXra:1301.0028