Formulas for generating primes involving emirps, Carmichael numbers and concatenation

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Abstract. Observations on generating primes or products of very few primes from reversible primes and Carmichael numbers using the method of concatenation.

I. *On the numbers obtained through concatenation from emirps and Carmichael numbers using only the digits of the number itself and the digits of its square*

Note: First we notice that, if p is a reversible prime and the number q is the number obtained through concatenation of the digits of p^2 with the digits of p, then the number q/p is often the product of very few primes (for a list of emirps see the sequence A006567 in OEIS).

Observation: If p is a reversible prime and the number q obtained through concatenation of the digits of p^2 with the digits of p has the sum of digits equal to 29, then the number q/p is often a prime or a semiprime.

16913/13 = 1301 is prime;
136937/37 = 3701 is prime;
624179/79 = 7901 is prime;
564001751/751 = 751001 is prime;
10180811009/1009 = 101*99901 is semiprime;
17450411321/1321 = 7*1887143 is semiprime.

Note that the first digits of the resulted primes are the same with the digits of p. The pairs of primes [13,1301], [37,3701], [79,7901], [751,751001] and so on deserve further study.

Conjecture: There is an infinity of reversible primes p with the property that the number obtained through concatenation of the digits of p with a number of n digits of 0, where n is equal to one less than the digits of p, and finally with the digit 1 is a prime.

Note: We also notice that, if C is a Carmichael number and the number s is the number obtained through concatenation of the digits of C^2 with the digits of C, then the number C/s is often the product of very few primes (for a list of Carmichael numbers see the sequence A002997 in OEIS):

Few examples:
314721561/561 = 7^2*107^2;
79405921/8911 = 59*1510339;
79580412821/2821 = 28210001;
435732016601/6601 = 2593*25457;
7115017141014721843505561/84350561 = 3*2811685366666667.

Note the interesting value of C/s for C = 2821.

II. *On the numbers obtained through concatenation from emirps and Carmichael numbers using the digits of the number itself, the digits of its square and the digits 0001*

Observation: If C is a Carmichael number then the number obtained through the concatenation of the digits of C with the digits 0001 is often a product of very few primes.

Few examples:
5610001 = 1129*4969;
17290001 = 1051*16451;
28210001 is prime;
66010001 = 2593*25457;
89110001 = 59*1510339;
105850001 = 911*116191;
158410001 is prime.

Note the values obtained for 2821 and 15841, both divisible with 31.

Observation: If C is a Carmichael number divisible by 31 then the number obtained through the concatenation of the digits of C with the digits 0001 is often a product of very few primes.

28210001 is prime;
158410001 is prime;
753610001 is prime;
1720810001 is prime;
21009010001 = 7*3001287143;
9912830875210001 is prime.

Observation: If C is a Carmichael number having 561 (a Carmichael number, also) as the last digits then the following numbers are often a product of very few primes:

: M, obtained through the concatenation of the digits of C with the digits 0001;
: N, obtained through the concatenation of the digits of C^2 with the digits 0001.

Few examples:

C = 340561; C^2 = 115981794721

M = 3405610001 is semiprime;
N = 1159817947210001 is semiprime;

C = 8134561; C^2 = 66171082662721

M = 81345610001 is prime;
N = 661710826627210001 is prime;

C = 10024561; C^2 = 100491823242721

M = 100245610001 is prime;
N = 1004918232427210001 is semiprime;

C = 10402561; C^2 = 108213275358721

M = 1104025610001 is semiprime;
N = 1082132753587210001 is semiprime;

C = 45318561; C^2 = 2053771971110721

M = 453185610001 is semiprime;
N = 20537719711107210001 is semiprime.

C = 84350561; C^2 = 7115017141014721

M = 843505610001 is semiprime;
N = 71150171410147210001 is semiprime.

Note: Probably the formulas could be extrapolated for Carmichael numbers having as the last digits not 561 but another Carmichael number but the results that we obtained, *exempli gratia*, with Carmichael numbers 1729 and 6601 were not encouraging.

III. *On the numbers obtained through concatenation from emirps using only the digits of the number itself*

Observation: We noticed that, through succesive concatenation of the digits of a reversible prime with the digits of its reversal, is obtained an interesting sequence of primes.

Primes obtained through concatenation of the digits of the numbers p, q, p, q and p, where p is an emirp and q is its reversal (this formula also conducts to products of very few primes):

1331133113, 9779977997, 769967769967769,
151111511151111511511

Observation: There is an infinity of primes formed this way.

IV. *On the extension of few of these observations from the set of emirps to set of all primes*

Note: We observed three interesting series of primes.

(1) Primes q of the form n/p, where p is prime and n is formed through concatenation this way: first digits of n are the digits of the square of p and last digits of n are the digits of p itself:

First few such primes:

31, 71, 1301, 1901, 3701, 6101, 6701, 7901, 103001, 109001, 181001 (...).

(the corresponding p: 3, 7, 13, 19, 37, 61, 67, 79, 103, 109, 181)

(2) Primes formed through succesive concatenation of the digits of the prime p with the digits of its reversal, not necessarily prime, q (this formula also conducts to products of very few primes).

First few such primes:

1331133113, 2992299229, 4334433443, 9779977997,
127721127721127 (...).

Note that, from the primes obtained this way, we can also obtain interesting primes from adding numbers of the form 18*10^k.

Few examples:

: 1331133113 + 18*10^8 = 3131133113 prime(which is the concatenation of q, q, p, q, p);
: 9779977997 + 18*10^7 = 9959977997 prime;
: 9779977997 + 18*10^9 = 27779977997 prime;
: 769967769967769 + 18*10^9 = 769985769967769 prime;
: 769967769967769 + 18*10^11 = 771767769967769 prime.

(3) Primes q formed through concatenation of the digits of the squares of the primes p with the digits 0001.

First few such primes:

90001, 490001, 2890001, 8410001, 18490001, 22090001

(the corresponding p: 3, 7, 17, 29, 43, 47).