

# **New finding of number theory**

By Liu Ran

## **Contents**

1. Introduce
2. Prime density regularity
3. Odd composite number density regularity
4. The limitation of odd number is composite number
5. Natural number is limited
6. Prime is limited
7. Zeno's paradox
8. Conclusion

### **1. Introduce**

To prove a theorem, I have found some new phenomenon in number theory. To explain the phenomenon, I have given my explanation and deduction. Some deduction is surprised and revolutionary, which is unbelievable, but it can be verified by fact and logic.

### **2. Prime density regularity**

The occurrence of prime is irregular, but the density of prime is more regular.

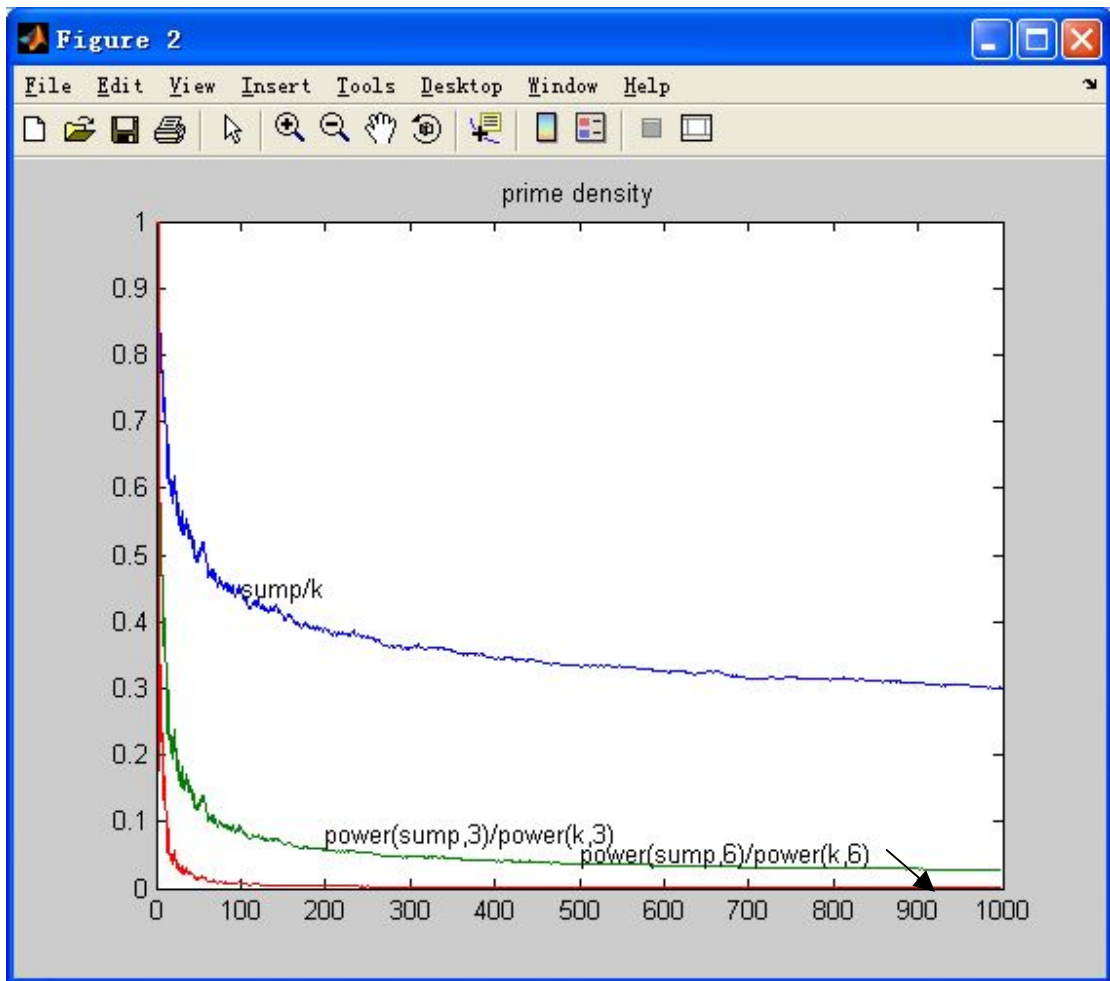
The density of prime((count of prime)/(count of odd number)) is

oscillating to trend to 0 when odd number is increasing.

Prime density data table like below, which displays the regularity very clearly.

$2K+1$	$sum(prime)/K$	$sum(p)^3 / K^3$	$sum(p)^6 / K^6$
00000003	1.0000	1.0000	1.0000
00000005	1.0000	1.0000	1.0000
00000007	1.0000	1.0000	1.0000
00000011	0.8000	0.5120	0.2621
00000013	0.8333	0.5787	0.3349
...	...	...	...
00000673	0.3601	0.0467	0.0022
00000677	0.3609	0.0470	0.0022
00000683	0.3607	0.0469	0.0022
00000691	0.3594	0.0464	0.0022
00000701	0.3571	0.0456	0.0021
...	...	...	...
00098807	0.1920	0.0071	0.0001
00098809	0.1920	0.0071	0.0001
00098837	0.1919	0.0071	0.0000
00098849	0.1919	0.0071	0.0000
...	...	...	...

The function plot like below.



### 3. Odd composite number density regularity

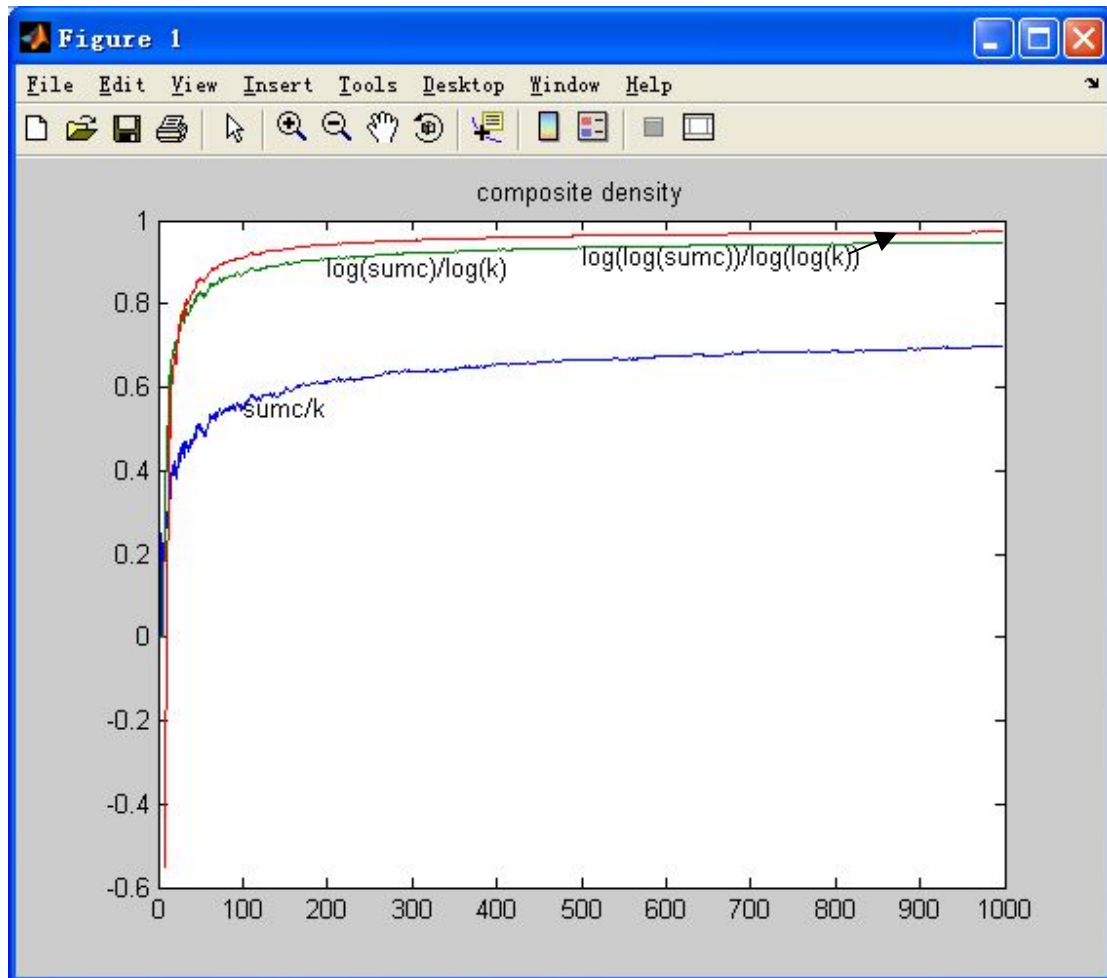
The density of odd composite number((count of odd composite number)/(count of odd number)) is oscillating to trend to 1 when odd number is increasing.

Odd composite number density data table like below,

$2K+1$	$sum(composite)/K$	$ln(sum(c))/lnK$	$lnln(sum(c))/lnlnK$
00000009	0.2500	0.0000	-INF
00000015	0.2857	0.3562	-0.5505
00000021	0.3000	0.4771	0.1128
00000025	0.3333	0.5579	0.3588
00000027	0.3846	0.6275	0.5052
...	...	...	...
00000671	0.6418	0.9237	0.9549
00000675	0.6409	0.9236	0.9549
00000679	0.6401	0.9234	0.9548
00000681	0.6412	0.9238	0.9550
...	...	...	...
00098805	0.8081	0.9803	0.9916
00098811	0.8080	0.9803	0.9916
00098813	0.8080	0.9803	0.9916
00098815	0.8080	0.9803	0.9916
...	...	...	...

12669203	0.8691	0.9910	0.9967
12669205	0.8691	0.9910	0.9967
12669207	0.8691	0.9910	0.9967
12669209	0.8691	0.9910	0.9967
...	...	...	...

The function plot like below.



#### 4. The limitation of odd number is composite number

Theorem (4.1):

When an odd number trends to infinity, this odd number must be an odd composite number.

First to define a prime set  $P = \{p | p=1 \times p; \{p/(p-k)\} \neq 0; [p/(p-k)] \neq 0; p>1, k \geq 1, k < p; p, k \in \mathbb{N}\}$ .  $x = [x] + \{x\}$  is Gaussian function.  $[x]$  expresses the maximum integer but not above  $x$ . Set  $[X] = \{[x] | [x] \leq x, [x] > x-1; x \in \mathbb{R}, [x] \in \mathbb{Z}\}$ ;  $\{x\}$  expresses the non-negative decimal fraction. Set  $\{X\} = \{\{x\} | \{x\} \geq 0, \{x\} < 1, \{x\} = x - [x]; x \in \mathbb{R}, [x] \in \mathbb{Z}\}$

(4.4.1) Suppose when an odd number trends to infinity, there is at least one odd number is prime.

i.e. Exist  $p_1$  is an odd number and  $\lim_{p_1 \rightarrow \infty} p_1 \in P$

$$\begin{aligned} \text{Because } p_1 \text{ is an odd number} &\Rightarrow \lim_{p_1 \rightarrow \infty} (p_1 / (p_1 - [p_1/2])) = \\ \lim_{p_1 \rightarrow \infty} (p_1 / (p_1 - (p_1-1)/2)) &= \lim_{p_1 \rightarrow \infty} (p_1 / (p_1/2 + 1/2)) = \lim_{p_1 \rightarrow \infty} (2p_1 / (p_1 + 1)) = \\ 2 \lim_{p_1 \rightarrow \infty} (1 - 1/(p_1 + 1)) &= 2 \Rightarrow \{2\} = \{ \lim_{p_1 \rightarrow \infty} (p_1 / (p_1 - [p_1/2])) \} = 0 \quad (4.1.1.1) \end{aligned}$$

Because  $p_1 \in P$  and  $p_1$  trends to infinity  $\Rightarrow \{p_1 / (p_1 - [p_1/2])\} \neq 0$  and  $p_1$  trends to infinity  $\Rightarrow \{ \lim_{p_1 \rightarrow \infty} (p_1 / (p_1 - [p_1/2])) \} \neq 0$ .

It's self-contradictory with (4.1.1.1). So  $p_1$  is not a prime, because we have found a divisor  $p_1 - [p_1/2]$  beside  $p_1$  and 1. According to prime definition ( $p = 1 \times p$ ),  $p_1$  does not belong to prime set.

Because  $\lim_{p_1 \rightarrow \infty} p_1 = \infty \Rightarrow \lim_{p_1 \rightarrow \infty} p_1 \neq 0$  and  $\lim_{p_1 \rightarrow \infty} p_1 \neq 1$ .

$\lim_{p_1 \rightarrow \infty} p_1 \neq 0$ ,  $\lim_{p_1 \rightarrow \infty} p_1 \neq 1$  and  $\lim_{p_1 \rightarrow \infty} p_1 \notin P \Rightarrow \lim_{p_1 \rightarrow \infty} p_1$  is an odd

composite number. Preliminary theorem (4.1) is true.

Theorem (4.1) is a very key theorem. To explain clearly, let me talk from a Series  $X_k = p_k / (p_k - 1)$ ,  $k \in \mathbb{N}$ ,  $p_k \in \mathbb{P}$ . i.e.  $X_k = 2/1, 3/2, 5/4, 7/6, 11/10, 13/12, 17/16, \dots$ . It's easy to calculate the limitation of  $X_k$ .  $\lim_{pk \rightarrow \infty} pk / (pk - 1) = 1$ . Similarly,  $\lim_{pk \rightarrow \infty} pk / (pk - [pk / 2]) = 2$ . It's strictly "equal to".

But we have found 2 divisors ( $p_k - 1$  and  $p_k - [p_k / 2]$ ) of  $p_k$ , according to prime definition ( $p = 1 \times p$ ),  $p_k$  does not belong to prime set. It has become a composite number.

Just like limitation of polygon becomes a circle, that is a qualitative change. The limitation of prime becomes a composite number that is also a qualitative change.

It's not easy to state clearly preliminary theorem (4.1). I have another statement for theorem (4.1)

Theorem (4.2):

The distribution of odd number serial can divide into 2 parts: a + b.

Serial a is a compound body of primes and odd composite numbers, density of prime become more and more lower;

Serial b is a pure body of odd composite numbers after density of prime being zero;

For example: 1,3,5,7,9,11,13,....,c1,c2,c3,c4,....

c1,c2,c3,c4,.... are very big odd composite numbers.



## 5. Natural number is limited

In ancient times, people can only calculate by hands. The natural number is from 1 to 10. Such as 11, 12, ..., 21, 22, ..., 100, ..., 1000, ... is recorded as one number  $10+$ .

In 32 bit computer, the biggest number is  $2^{32} = 4294967296$ . Any number is more than 4294967296 recorded as  $4294967296+$ . Similarly, In 64 bit computer, the biggest number is  $2^{64} = 18446744073709551616$ . Any number being bigger than  $2^{64}$  is record as one number  $2^{64}+$ . The natural number is from 1 to 18446744073709551616 in computer.

Since human invent algebra, any big number can be expressed as one character N. But it has occupied 1 character.  $N + 1$  occupied 1 character also. A very very ... big number  $N + \dots$  can exhaust all characters finally. It only changes the start number of natural number from 1 to N.

Maybe some people say that we can think a bigger number than limitation. But if the big number has occupied human's entire brain cell, human can't even think a bigger number than limitation.

Any number needs something to store, such as finger, paper, computer memory and brain cell. We can exhaust everything in the world to store a big number, including sun, earth, atom, and particle, everything in the world. Because the matter is limited, the big number is also limited. Any number being bigger than it can't be measured or calculated, because we can't store such a big number. This number should be the biggest number

in our world and it's the limitation of natural number.

Theorem (5.1):

Natural number is limited.

We can exhaust everything in the world to store a big number record as INF.

$INF + 1 \geq INF$  for algebra calculating rule;

But we have exhausted everything in the world to store INF,  $INF + 1 \leq INF$  for maximum store matter.

Because  $INF + 1 \geq INF$  and  $INF + 1 \leq INF \Rightarrow INF + 1 = INF$ .

Similarly,  $INF + k = INF$ ,  $k \geq 1$ ,  $k \leq INF$ .

Because  $INF + k = INF$ ,  $k \leq INF \Rightarrow INF + INF = INF \Rightarrow 2 \times INF = INF$ ;

Similarly,  $k \times INF = INF$ ,  $k \geq 1$ ,  $k \leq INF$ .

Because  $INF \times k = INF$ ,  $k \leq INF \Rightarrow INF \times INF = INF \Rightarrow INF^2 = INF$ ;

Similarly,  $INF^k = INF$ ,  $k \geq 1$ ,  $k \leq INF$ .

Because  $INF^k = INF$ ,  $k \leq INF \Rightarrow INF^{INF} = INF$ .

Summary below:

Theorem (5.2):

$INF + k = INF$ ;

$INF \times k = INF$ ;

$INF^k = INF$

$$k \geq 1, k \leq INF$$

Because  $INF + k = INF \Rightarrow$  natural number is limited.

We can prove strictly natural number being limited.

Suppose natural number is infinite.

Denote the distance from infinite as  $d(n) = \infty - n$ .

If the distance from infinite is constant, it means that  $d(n) = C = \infty - n = \infty - (n+1) \Rightarrow \infty = \infty - 1 \Rightarrow \infty = (\infty - 1) - 1 = \infty - 2, \dots \Rightarrow \infty = \infty - n = C \Rightarrow C = \infty$ ,

If the distance from infinite becomes more and more big, it means that  $d(n) = \infty - n < \infty - (n+1) \Rightarrow \infty + 1 < \infty, \Rightarrow 1 = \lim_{n \rightarrow \infty} (n+1)/n = (\infty + 1)/\infty < 1 \Rightarrow 1 < 1$ .

If the distance from infinite becomes more and more small, when  $d(n) = (\infty - n) < \varepsilon$ ,  $\varepsilon$  is smaller than any number, it means it can be smaller than 1. If  $\varepsilon < 1$ , then  $(\infty - n) < \varepsilon < 1, \Rightarrow n+1 > \infty, \Rightarrow$  natural number  $n+1$  has exceeded infinite.

So supposition is false and natural number is finite.

## 6. Prime is limited

It seems that it's contradictory with the famous Euclid's proof. Suppose prime is limited,  $P = \{p|2, 3, 5 \dots p_k\}$ . Constructing a number  $p_{k+1} = 2 \times 3 \times 5 \times \dots \times p_k + 1$ . Either  $p_{k+1}$  is a prime or  $p_{k+1}$  is a composite number that can resolve a prime being bigger than  $p_k$ .

It's correct in classical number theory.

If  $p_{k+1} = \text{INF}$ , because of theorem (5.2),  $\text{INF} = p_k \times \text{INF} \Rightarrow p_{k+1}$  is a composite number. But  $p_{k+1}$  can resolve a divisor of  $\text{INF}$ , and  $\text{INF}$  is a composite number. It's not sure to resolve a prime being bigger than  $p_k$ . So the famous proof is not correct when natural number is limited.

Actually, because natural number is limited, prime is natural number  $\Rightarrow$  prime is limited.

## **7. Zeno's paradox**

In the paradox of Achilles and the Tortoise, Achilles is in a footrace with the tortoise. Achilles allows the tortoise a head start of 100 meters, for example. If we suppose that each racer starts running at some constant speed (one very fast and one very slow), then after some finite time, Achilles will have run 100 meters, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, say, 10 meters. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Therefore, because there are an infinite number of points Achilles must reach where the tortoise has already been, he can never overtake the tortoise.

It's very interesting. Zeno's paradox becomes the evidence to verify that natural number is limited.

Infinitesimal can be regarded as  $1/\infty$ . Because infinity is limited,  $1/\infty$  is limited also.

In fact, Achilles has one moment to overtake the tortoise. At this moment, infinitesimal can't be divided again. If infinitesimal is really infinite small, that can't explain Zeno's paradox with satisfaction. If natural number can never reach infinity, infinitesimal can never reach zero. The contradiction is when Achilles had overtaken the tortoise? But if natural number is limited, it's so natural to explain Zeno's paradox.

So we can prove strictly natural number being limited.

Assume that Achilles allows the tortoise a head start of 100 meters, Achilles' speed is 10 meters per 1 second, and tortoise's speed is 1 meter per 1 second.

The distance serial is that 100, 10, 1,  $1/10$ ,  $1/100$ ,  $1/1000$ , ...,  $1/(10^n)$ , ...

(7.1) Suppose natural number is limitless.

The distance  $S > 0$  denote Achilles is behind of tortoise;  $S = 0$  denote Achilles overtakes tortoise;  $S < 0$  denote Achilles is ahead of tortoise.

Because the fact is Achilles had overtaken tortoise. There must be

a distance  $S = 0$ , assume  $S(n) = 1/(10^n) = 0 \Rightarrow S(n+1) = 1/(10^{(n+1)}) = S(n)/10 \leq S(n) = 0 \Rightarrow S(n+1) \leq 0$ .

Because  $S(n+1) = 1/(10^{(n+1)}) \geq 0$ , and  $S(n+1) \leq 0 \Rightarrow S(n+1) = 0 = S(n) \Rightarrow 1/(10^n) = 1/(10^{(n+1)}) \Rightarrow n = n+1$ . Similarly,  $\Rightarrow n = n+k, k \geq 1, k \in \mathbb{N}$ . It's contradictory with (7.1). So (7.1) is false.

Natural number is limited.

## 8. Conclusion

When odd number increases, the density of odd composite number trends to 1 with oscillation; the density of prime trends to 0 with oscillation.

Natural number is really the quantity of world matter.

Number need matter to store, it imply that number map really to the matter quantity.

Theorem (8.1)

The distribution of odd number serial is  $1, 3, 5, \dots, \text{INF}, \text{INF}, \dots$

The distribution of natural number serial is  $1, 2, 3, \dots, \text{INF}, \text{INF}, \dots$

Theorem (8.2)

Infinitesimal can be regarded as  $1/\text{infinity}$ . Because infinity is limited, infinitesimal is limited also.

## References

- [1] John Friedlander and Henryk Iwaniec, The polynomial  $X^2 + Y^4$  captures its primes, 148 (1998), 945-1040
- [2] John Friedlander and Henryk Iwaniec, Asymptotic sieve for primes, 148 (1998), 1041-1065
- [3] University of Tongji, Higher mathematics, 465(1991)
- [4] E. Bombieri, The asymptotic sieve, Mem. Acad. Naz. dei XL, 1/2 (1976), 243-269.
- [5] W. Duke, J.B. Friedlander, and H. Iwaniec, Equidistribution of roots of a quadratic congruence to prime moduli, Ann. of Math. 141 (1995), 423-441.
- [6] C.L. Stewart and J. Top, On ranks of twists of elliptic curves and power-free values of binary forms, J. Amer. Math. Soc. 8 (1995), 943-973.
- [7] E. Fouvry and H. Iwaniec, Gaussian primes, Acta Arith. 79 (1997), 249-287.
- [8] J. Friedlander and H. Iwaniec, Bombieri's sieve, in Analytic Number Theory, Proc. Halberstam Conf., Allerton Park Illinois, June 1995, ed. B. C. Berndt et al., pp. 411-430, Birkhäuser (Boston), 1996.
- [9] , The polynomial  $X^2 + Y^4$  captures its primes, Ann. of Math. 148 (1998), 945-1040.
- [10] G. Harman, On the distribution of  $\pi$  modulo one, J. London Math. Soc. 27 (1983),9-18.
- [11] H. Iwaniec, A new form of the error term in the linear sieve, Acta Arith. 37 (1980),307-320.
- [12] H. Iwaniec and M. Jutila, Primes in short intervals, Ark. Mat. 17 (1979), 167-176.
- [13] A. Selberg, On elementary methods in primenumber-theory and their limitations, in Proc. 11th Scand. Math. Cong. Trondheim (1949), Collected Works Vol. I, pp. 388-397, Springer (Berlin), 1989.
- [14] D. Wolke, A new proof of a theorem of van der Corput, J. London Math. Soc. 5 (1972), 609-612.