A Note on the Classical Vector Fields

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Using the density of the lines of force, we have obtained the laws of Coulomb, Biot-Savart and Newton.

Key words: density of the lines of force.

In geometry, we have that angle = arc/radius or $\theta = \frac{l}{r}$, and therefore solid angle = surface/radius$^2$ or $\Omega = \frac{S}{r^2}$. Now, we define the (classical) electric vector field produced by the source electric charge, $q_1$, as proportional, $k_e$, to the number of lines of force per unit area, $n = \frac{N}{S}$, per solid angle; then

$$\vec{E}_i = k_e \frac{N}{S} \frac{S}{r_i^2} \ddot{u}_r = k_e \frac{N}{r_i^2} \ddot{u}_r = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_i^2} \ddot{u}_r,$$

(1)

$\varepsilon_0$ being the electric permittivity of the vacuum, with

$$k_e N = \frac{1}{4\pi \varepsilon_0} q_1,$$

(2)

The electric force on a test electric charge, $q_2$, would be

$$\vec{F}_{e12} = q_2 \vec{E}_i = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \ddot{u}_r,$$

(3)

For the magnetic vector field, it would be the same:

$$\vec{B}_i = -k_m \frac{N}{S} \frac{S}{r_i^2} \ddot{u}_r = -k_m \frac{N}{r_i^2} \ddot{u}_r = -\frac{\mu_0 i_l}{4\pi} \frac{1}{r_i^2} \ddot{u}_r,$$

(4)

$\mu_0$ being the magnetic susceptibility of the vacuum, $i_l$ the source electric current and $l_l$ the conductor length, with

$$k_m N = \frac{\mu_0 i_l}{4\pi}$$

(5)

The magnetic force on a test electric current, $i_2$, of conductor length, $l_2$, would be

$$\vec{F}_{m12} = i_2 \frac{1}{4\pi} \frac{\mu_0 i_l l_2}{r_{12}^2} \ddot{u}_r,$$

(6)
And also for the gravitational vector field:

\[ \Gamma_1 = -k_g \frac{N}{S} \frac{m_1}{r_1^2} \hat{u}_r = -k_g \frac{N}{r_1^2} \hat{u}_r = -G \frac{m_1}{r_1^2} \hat{u}_r \]  

\( G \) being the Newton’s gravitational constant and \( m_1 \) the source mass, with

\[ k_g N = G m_1 \]

The gravitational force on a test mass, \( m_2 \), would be

\[ \vec{F}_{g12} = m_2 \vec{\Gamma}_1 = -G \frac{m_1 m_2}{r_{12}^2} \hat{u}_r \]

By convention, the electric lines of force are directed outward for positive electric charges and inward for negative electric charges. Then, two electric charges of the same sign repel each other because their lines of force go in opposite directions, and two electric charges of different sign attract each other because their lines of force go in the same direction. In two parallel conductors, the conductors attract if the currents go in the same direction and repel otherwise.

In the gravitational field, however, the gravitational lines of force would not drag the bodies in the same form as the electric lines of force, because so whether we consider those outgoing or incoming, the force would always be repulsive instead of attractive. We assume that the gravitational lines of force produce the gravitational attraction bending the space (as stipulates the general relativity of Einstein) and that this curvature decreases with the distance to the source field.

In summary, using the density of the lines of force, we have obtained the laws of Coulomb, Biot-Savart and Newton.