Germs on a manifold

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1 Abstract

Let M be a smooth manifold. In this paper we review the definition of a germ and show that since it is an equivalence relation, the concept is only locally defined.

2 Preliminaries

We begin by introducing the technical definition of a germ.

Definition 2.1[Bj] A germ on a topological space X is the equivalence relation of maps $f : U_f \to X$, on open neighborhoods of points.

Recall that an equivalence relation is a relation R that is transitive, commutative, and reflexive.

In this paper, we will intuitively show that if we let two functions f, g are equal at different points on an open neighborhood on a topological space, then we obtain a definition of locality, such as the homgenous coordinates $(x_1, \dots, x_n)$ on a smooth manifold.

Definition 2.2 The locally constant sheaf $\mathcal{O}_c$ is the sheaf where for functions f, g, $f = g$.

We will see that a consequence of Definition 2.1 is that we can only locally define germs.

Definition 2.3. A derivation D is a $\mathbb{K}$- linear map that satisfies Leibniz's rule $D(ab) = D(b)a + D(a)b$.

Thus the partial derivatives of functions are derivations.

3 Germs are only locally defined

Suppose that $f = g$, $f, g \in M$, as by the definition of a germ. Then since we have a smooth manifold, we know that the functions are "the same" on that set of coordinates. However, this can only be locally defined, since unless it is the same smooth manifold, it will look much different globally, hence the locally constant sheaf only exists locally (Refer to Definiton 2.2).

4 A construction of the tangent bundle

Now supposing that the functions f, g are products (and hence differentiable), we can construct the tangent space T_pM by letting the functions be differentiated once. One can see that from calculus, this will look like the tangent line to a function. Differentiating again and again, taking greater approximations, finally gives us the sum of all these derivations, or equivalently, the tangent bundle TM. With that one can easily deduce that the map $\pi : TM \to M$ is a homomorphism.

5 Note

The author of this article is 15 years old; and is stuck in "school math" that doesn't show the capablities in mathematics young people can have, such as in this article.

6 References

[Bj] Dundas, Bjørn. 10 Jan 2013.*Differential Topology*, available at http://www.uib.no/People/nmabd/dt/0
Typset in $\mathcal{AMS}$-TEX.