

## Fine Structure Constant And Relations Between Dimensionless Constants

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**Abstract.** The aim of this article is to determine dimensionless physical constants through mathematical constants and other dimensionless physical constants.

## 1. Introduction

If physical constants really are constant, then the relationship between them is also constant. Relation that is proposed for such a relationship has to be proven by mathematical derivation and confirmed by experimental evidence. In practice, however, we have cases where:

- Formulae and even entire theories are widely accepted, many times experimentally verified, and there is no irrefutable proof of their theoretical validity;
- Theories are widely accepted and only in some cases do not work;
- There is a large number of published articles with formulae that provide approximate values for several orders of magnitude beyond the experimentally determined values, but the authors are convinced that "there is something".

This article presents original relations, which have not been sufficiently theoretically confirmed and experimentally verified, but the values of dimensionless physical quantities calculated with them are in  $1\sigma$  confidence interval, always with the same number of significant digits as in [1]. For those who want to check these relations, all calculations are given in App 1-3. in the order of appearance.

## 2. Relations

In article [4] the following relation is presented:

$$N = \gamma^{1+\alpha'^2 \ln(\mu)/\ln(2)} \quad (1)$$

Where, The CODATA recommended constants published in 2010 [1] are:

Inverse fine structure constant:	$\alpha=137.035\ 999\ 074\ (44)$
Proton-electron mass ratio:	$\mu=1836.152\ 672\ 45\ (75)$
Neutron-proton mass ratio:	$\gamma=1.001\ 378\ 419\ 17\ (45)$
Mathematical constants:	$e=2.71828..$ and $\pi'=6.283185..$

Relation (1) gives the following value:  $N=6.387\ 08E+121$

From the relation (1) we see that it is possible to determine the fine structure constant over the three dimensionless values of physical constants,  $\alpha = f(\mu, \gamma, N)$

Relation (1) can also have the following form:

$$\alpha'^2 = \ln(2) * [\ln(N) / \ln(\gamma) - 1] / \ln(\mu) \quad (2)$$

If we define a new dimensionless physical constant:

$$\alpha'_2 = \ln(\gamma) / \ln(N / \gamma) = 4.91136733 \text{E} - 06 \quad (3)$$

Relation (2) can be written as:

$$\ln(\mu) / \ln(2) = 1 / (\alpha'^2 \alpha'_2) = 10.8424703 \quad (4)$$

That is:

$$\mu = 2^{1 / (\alpha'^2 \alpha'_2)} = 1836.15267 \quad (5)$$

$$\alpha' = \sqrt{\ln(2) / (\ln(\mu) \alpha'_2)} = 137.035999074 \quad (6)$$

From the previous relations we can see that it is possible to determine  $\mu$  and  $\alpha'$  only through two dimensionless constants,  $\mu = f(\alpha', \alpha'_2)$  or  $\alpha' = f(\mu, \alpha'_2)$ , if we know how constant  $\alpha'_2$  is physically manifested. It is likely that this constant is important in nuclear physics. Calculations related to this chapter are given in App 2.

### 3. Relations Between Three Dimensionless Physical Constants

The next goal is to determine the fine structure constant through only two dimensionless constants or to find a relationship between three dimensionless constants, one of which is the fine structure constant.

One obvious solution is the relation:

$$\mu = \pi' \alpha' \beta \quad (7)$$

Where  $\beta$  is the ratio of the classical radius of the electron and proton Compton wavelength:

$$\beta = r_e / \lambda_c \quad (8)$$

Where the  $r_e = 2.8179403267(27) \text{E}-15$ ,  $\lambda_c = 1.32140985623(94) \text{E}-15$  [1]  
Or using (8):  $\beta = 2.1325255850$

Then from (7) we get:

$$\alpha' = \mu / (\pi' \beta) \quad (9)$$

$$\alpha' = 1836.15267245 / (6.28318531 * 2.1325255850) = 137.035999074$$

We see from (9) that  $\alpha'$  is determined via mathematical constant  $\pi'$  and two dimensionless physical constants  $\alpha' = f(\mu, \beta)$ . Here we are sticking to the definition of the constant  $\beta$  according to equation (8), although this constant can be determined in several other ways.

In [5] the following relation was presented:

$$N = 2^q \quad (10)$$

Where:

$$q = [3 \exp(\pi')/2 - (\pi' \beta + 1)/(\pi' \beta + 2) - 1 + 3 \ln \pi' / \ln 2] / 2 = 404.628455366 ; \quad z_1 = (\pi' \beta + 1) / (\pi' \beta + 2) = 0.93506094352$$

And if from (3) we determine  $\ln \gamma$  we get:

$$\ln \gamma = \ln N / (1 + 1 / \alpha_2) = q \ln 2 / (1 + 1 / \alpha_2)$$

i.e., in a developed form:

$$\ln \gamma = \ln 2 [3 \exp(\pi') / 2 - (\pi' \beta + 1) / (\pi' \beta + 2) - 1 + 3 \ln \pi' / \ln 2] / [2(1 / \alpha_2 + 1)] \quad (11)$$

or:

$$\gamma = 2^{[3 \exp(\pi') / 2 - (\pi' \beta + 1) / (\pi' \beta + 2) - 1 + 3 \ln \pi' / \ln 2] / [2(1 / \alpha_2 + 1)]} = 1.001378492 \quad (12)$$

From (12),  $\gamma$  is obtained through mathematical constants:  $\pi'$ ,  $e$ ,  $\ln 2$  and two dimensionless physical constants  $\beta$  and  $\alpha_2$ . Mathematical constants  $\pi'$  and  $e$  are very common in physics, and their role is quite well-known, while constant  $\ln 2$  is rarely used, for example in [3]. The role of this constant is crucial for getting the results in this article and to a great extent is obtained thanks to my professor of meteorology and mathematician Marijan Čadež (1912-2009) [2]. Integers 1 and 2 and fraction  $3/2$  frequently appear in physics.

Let us clarify some of the articles in the formula (11). Part in (11) which has the value of:

$$p = \exp(\pi') / 2 - (\pi' \beta + 1) / (\pi' \beta + 2) - 1 = 265.8107668 \quad (13)$$

for the mass of the proton:

$$m_p = 1.672\,621\,777\,(74) \text{ e-27 kg [1]}$$

gives:

$$M = m_p * 2^p = 1.7394 \text{ e+53 kg} \quad (14)$$

which is the mass of the universe. I have to note that the values of physical quantities here are dependant on the system of measurement units and shown only to give a clue about the physical quantities mentioned. In fact, since the whole article compares only dimensionless relations, system of units is not important and a natural system is used in which the mass, radius and time of the universe are  $M = 1$ ,  $R = 1$ ,  $T = 1$ . The derived quantities are then parts of the unit.

Dimensionless constant  $p$  is in fact by definition  $p = \ln (M / m_p) / \ln 2$ , and hence (14). Relation (13) is proposed and is yet to be thoroughly explained. More relationships related to  $p$  are given in the App 2.

Part in (11) and (12) which has the following value:

$$q = \exp(\pi')/2 + p/2 + 3\ln(\pi')/[2\ln(2)] = 404.6284554 \quad (15)$$

For the Planck length:

$$r_{pl} = 1616 \ 199 \ E-35 \ (97) \ m \ [1]$$

if the following is applied:

$$R = r_{pl} * 2^{q/2} = 1.291653 \ e + 26 \ m \quad (16)$$

we obtain the value of the radius of the universe.

If we apply this radius and the known value of the speed of light  $c = 299 \ 792 \ 458 \ m/sec$  to the well-known formula:  $c = R / T$ , where  $T$  is the duration of the universe (the cycle of the universe), we get:

$$T = c/R = 4.30849 \ e + 17 \ sec = 13.6528 \ e + 9 \ years \quad (17)$$

Approximate values obtained here for the mass, radius and time of the universe are the values that have been mentioned in literature. The previous relationships are not coincidences. They were obtained from the same starting point, as well as the dimensionless relations in the next chapter. The same goes for many dimensional constants which will not be presented in this article.

Calculations for this chapter are given in the App 2.

#### 4. Relations Between Coulomb's And Newton's Forces Of Gravity

These relations are usually defined for pairs of proton-electron and electron-electron. The intensity of the Coulomb force for both pairs:

$$F_C = ke^2/r_e^2$$

Newtonian gravity is:

for the proton-electron  
for the electron-electron

$$F_{Npe} = Gm_p m_e / r_e^2$$

$$F_{Nee} = Gm_e^2 / r_e^2$$

So if we take that in [1]:

$$\begin{aligned} m_p &= 1.672621777E-27 \ kg \\ m_e &= 9.109382910E-31 \ kg \\ e &= 1.602 \ 176 \ 565 \ e-19 \ C \\ k &= 8.9875517874E+09 \ Nm^2/C^2 \\ G &= 6.67384E-11 \ m^3 \ kg^{-1} \ s^{-2} \end{aligned}$$

we get:

$$N_p = F_c / F_{Npe} = k * e^2 / Gm_p m_e = 2.26882E+39 \quad (18)$$

While using only dimensionless quantities:

$$N_p = F_c/F_{N_{pe}} = \beta \sqrt{2^{3p-\exp(\pi')} / \pi'} = 2.2688193641E+39 \quad (19)$$

where the value of  $\beta$  is from (8) and  $p$  from (13). For the electron-electron pair there is the following relation:

$$F_c/F_{N_{ee}} = \alpha' \beta^2 \sqrt{\pi' 2^{3p-\exp(\pi')}} = 4.16589873 \text{ } 87E + 42 \quad (20)$$

As in (19),  $p$  is a function only of  $\beta$  then the value of  $N_p$  dimensionless relation is also a function only of  $\beta$ . Then, in a developed form:

$$N_p = F_c/F_{N_{pe}} = \beta \sqrt{2^{\exp(\pi')/2-3(\pi'\beta+1)/(\pi'\beta+2)-3} / \pi'} \quad (21)$$

Gravitational coupling constant is defined as:

$$\alpha'_G = (m_e/m_{pl})^2 = \pi' G m_e^2 / hc = 1.751687E-45 \quad (22)$$

Similar to the above-presented relations:

$$\alpha'_G = 1/[\alpha'^2 \sqrt{\pi'} \beta^2 \sqrt{2^{3p-\exp(\pi')}}] = 1.751687458E-45 \quad (23)$$

Factor that occurs in all relationships in this chapter,

$$x = \sqrt{2^{3p-\exp(\pi')}} = 2.666831670E+39 \quad (24)$$

connects the four fundamental physical constants,  $x=f(h,c,G \text{ i } m_p)$ :

$$x = hc\sqrt{\pi'}/(Gm_p^2) = 2.66683E+39 \quad (25)$$

Calculations for this chapter are given in App 3.

## 5. Conclusion

There is one key difference between relations (18) and (21). In the former relation it is assumed that Gravity and Coulomb's force are different, while in the latter each force is only due to the immanent relations that exist in the universe between the whole and its parts. Thus, these two forces have the same cause. During the calculation, the starting point was not the fact that mass has inherent properties, especially not the feature to attract, rather that all the properties are a consequence of the relationship between the whole and the parts. The goal of many, to express the fine structure constant only through mathematical constants, has been replaced in this article with a more modest goal: to express that constant or some other constant with as few other dimensionless constants as possible. For the fine structure constant, we see that such a relation with the other two dimensionless constants is already known (9). To determine the constant of  $N_p$ , it takes only one dimensionless physical constant  $\beta$ , as shown. In this article I tried not to burden the reader with my views and opinions, rather, I used mathematics to express my views.

## References:

1. CODATA *internationally recommended values of the Fundamental Physical Constants, (2010) values of the constants.*
2. Marijan Čadež, *unauthorized lecture on the importance of numbers 2<sup>n</sup>*
3. Lloyd, S., *Computational capacity of the universe*, Phys. Rev. Lett. 88 (2002) 237901
4. Branko Zivlak, *Relation between dimensionless physical constants*, viXra: 1210.0076
5. Branko Zivlak, *Neutron, proton and electron mass ratios*, viXra: 1211.0090

# App 1.

(No)	Calculating: Relations	
	<i>Constant</i> or Relation	Value
	$\pi' =$	<b>6.2831853071796</b>
	$\acute{\alpha} =$	<b>137.035999074</b>
	$\mu =$	<b>1836.1526724501</b>
	$\gamma =$	<b>1.00137841917</b>
	$\acute{\alpha}^2 =$	<b>18778.8650422</b>
	$\ln \mu / \ln 2 =$	<b>10.842470305630</b>
(1)	$N = \gamma^{\ln \mu / \ln 2} [1 + \acute{\alpha}^2 \ln \mu / \ln 2] =$	6.3870326400E+121
	$\ln(N) =$	280.46706603722
	$\ln(\gamma) =$	<b>0.0013774700224</b>
(2)	$\acute{\alpha}^2 = \ln 2 (\ln N / \ln \gamma - 1) / \ln \mu =$	1.8778865042E+04
	$\ln(N/\gamma) =$	2.8046568857E+02
(3)	$\acute{\alpha}_2 = \ln(\gamma) / \ln(F/\gamma) =$	4.9113673385E-06
(4)	$\ln \mu / \ln 2 = 1 / (\acute{\alpha}^2 \acute{\alpha}_2) =$	10.842470305630
(5)	$\mu = 2^{\ln \mu / \ln 2} [1 / (\acute{\alpha}^2 \acute{\alpha}_2)] =$	1836.1526724501
(6)	$\sqrt{[\ln 2 / (\ln \mu * \acute{\alpha}_2)]} =$	137.03599907400

## App 2.

Calculating: Relations Between Three Dimensionless Physical Constants		
	Constant or Relation	Value
	<b>Classical electron radius</b> $2.817\ 940\ 3267\ e-15\ r_e=$	<b>2.8179403267E-15</b>
	<b>proton Compton wavelength</b> $1.321\ 409\ 856\ 23\ e-15\ \lambda_c=$	<b>1.32140985623E-15</b>
(8)	$\beta=r_e/\lambda_c=$	2.132525585014
	$\beta=\mu/(\pi'\acute{\alpha})=$	2.132525585004
(9)	$\acute{\alpha}=\mu/(\pi'\beta)=$	137.035999074
(10)	$N=2^q=$	6.3870771837E+121
	$z1=(\pi'\ \beta +1)/(\pi'\ \beta +2)=$	0.935060943518
	$q=[3\exp(\pi')/2-z1-1+3\ln\ \pi'/\ln2]/2=$	404.62845536602
(11)	$\ln(\gamma)=\ln2[3\exp(\pi')/2-z1-1+3\ln\ \pi'/\ln2]/[2*(1/\acute{\alpha}_2+1)]$	0.0013774700567
(12)	$\gamma=2^{[3\exp(\pi')/2-z1-1+3\ln\ \pi'/\ln2]/[2*(1/\acute{\alpha}_2+1)]}$	1.000955245443
(13)	$p=\exp(\pi')-(\pi'\ \beta +1)/(\pi'\ \beta +2)-1$	265.810766818864
	<b>proton mass</b> $1.672\ 621\ 777\ e-27\ m_p=$	<b>1.672621777E-27</b>
(14)	$M=m_p*2^p=$	1.7394491196E+53
	<b>universe radius</b> $R=$	<b>1.29165299E+26</b>
	<b>Planck mass</b> $2.17651\ e-8\ m_{pl}=$	<b>2.17651E-08</b>
	<b>Planck constant</b> $h=6.626\ 069\ 57\ e-34\ (kgm^2s^{-1})=$	<b>6.62606957E-34</b>
	<b>c</b> $=299792458\ (ms^{-1})=$	<b>2.997924580E+08</b>
	$p=4\ln(M/m_{pl})/\ln2-\exp(\pi')-3\ln\pi'/\ln2=$	265.81076656288
	$p=\exp(\pi')-2\ln(R/(r_e/\beta))/\ln2+\ln\pi'/\ln2$	265.81076681886
	$p=\ln[M/(\beta h/cr_e)]/\ln2$	265.81076681955
(15)	$q=ci/2+p/2+3\ln\pi'/(2\ln2)=$	404.62845536602
	<b>Planck length</b> $1.616\ 199\ e-35\ r_{pl}=$	<b>1.616199E-35</b>
(16)	$R=r_{pl}*2^{q/2}=$	1.2916531752E+26
(17)	$T=R/c$	4.3084912270E+17
(17)	$T/(365.25*24*3600*10^{\wedge}9)=(10^{\wedge}9\ \text{years})=$	13.652784834719



### App 3.

Calculating: Relations Between Coulomb's And Newton's Forces Of Gravity	
(No)	Constant or Relation
	Value
	<i>electron mass</i> $m_e=9.109382910E-31$ kg
	<b>9.109382910E-31</b>
	$e=1.602\ 176\ 565\ e-19$ C
	<b>1.602176565E-19</b>
	$k=8.9875517874E+09$ Nm <sup>2</sup> /C <sup>2</sup>
	<b>8.9875517874E+09</b>
	$G= 6.67384E-11$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>
	<b>6.67384E-11</b>
(18)	$F_C/F_{Npe}=ke^2/Gm_p m_e=$
	2.26882E+39
(19)	$Np=F_C/F_{Npe}=\beta\sqrt{[2^{3p-\exp(\pi')}/\pi']}=$
	2.268819364E+39
(20)	$F_C/F_{Nee}=\alpha\beta^2\sqrt{[\pi'2^{3p-\exp(\pi')}]}=$
	4.165898739E+42
(21)	$Np=F_C/F_{Npe}=\beta\sqrt{[2^{\exp(\pi')/2-3(\pi'\beta+1)/(\pi'\beta+2)-3}/\pi']}=$
	2.268819364E+39
(22)	$\dot{\alpha}_G=(m_e/m_p)^2=\pi'Gm_e^2/hc=$
	1.751687E-45
(23)	$\dot{\alpha}_G=1/\{\alpha^2\sqrt{\pi'}\beta^2\sqrt{[2^{3p-\exp(\pi')}]}\}=$
	1.751687458E-45
(24)	$x=\sqrt{[2^{3p-\exp(\pi')}]}=$
	2.666831670E+39
(25)	$x=hc\sqrt{\pi'}/(Gm_p^2)=$
	2.66683E+39