Title: **Fermat's Last Theorem**

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Abstract: **Recall the theorem states that the equation $a^n + b^n = c^n$ cannot exist if all quantities are positive integers and n>2. Fermat maintained he had a short proof but it has never been found, nor has a short proof been supplied by anyone since. This attempt uses simple mathematics and methods reminiscent of those taught in English grammar schools in the 1950's.**

Fermat's Last Theorem
"Hanson Boys' G. S. Proof"

Statement of the Theorem

Fermat's Last Theorem **(FLT)** states that:
positive integers a, b, and c cannot be found satisfying the equation

$$a^n + b^n = c^n \qquad \textbf{(T)}$$

for any integer value of n greater than 2.

Proof

Assume that all common factors have been cancelled, noting that all or none of {a,b,c} have a common factor. **(A)**

Assume the theorem is false and n is an integer >2 such that positive integers {a,b,c} **do** exist satisfying the equation:

$$a^n + b^n = c^n$$

Clearly c>{a,b} and a ≠ b as this would require $c = (2a)^{1/n}$ and c must be irrational.

Assume a<b, thus a<b<c.

We will now examine the conclusions if n>2.

Let $a + h = b + i = c$ {h,i integers, h>i, h>1}

We can rewrite **(T)** in terms of a and b in the following 2 different ways:

(i) Using the Binomial Theorem

$$a^n + b^n = (a + h)^n = (b + i)^n = c^n$$
$$a^n = (b + i)^n - b^n = nb^{n-1}i + n(n-1)/(2!)b^{n-2}i^2 + \ldots\ldots + i^n$$
$$b^n = (a + h)^n - a^n = na^{n-1}h + n(n-1)/(2!)a^{n-2}h^2 + \ldots\ldots + h^n$$

(ii) By Factoring

$$a^n = c^n - b^n$$
$$= (c - b)(c^{n-1} + c^{n-2}b + \ldots. + b^{n-1})$$
$$= i(c^{n-1} + c^{n-2}b + \ldots. + b^{n-1})$$
$$b^n = c^n - a^n$$
$$= (c - a)(c^{n-1} + c^{n-2}a + \ldots. + a^{n-1})$$
$$= h(c^{n-1} + c^{n-2}a + \ldots. + a^{n-1})$$

let b = Fx {F,x integers>0, F = product of primes **not in** h, x = product of primes **in** h}
and a = Gy {G,y integers>0, G = product of primes **not in** i, y = product of primes **in** i}

∴ **x>y**

(i) can now be written

$$(Fx)^n = na^{n-1}h + n(n-1)/(2!)a^{n-2}h^2 + ... + h^n$$
$$(Gy)^n = nb^{n-1}i + n(n-1)/(2!)b^{n-2}i^2 + ... + i^n$$

∴ h divides x^n, h <= x^n ; i divides y^n, i <= y^n

Two cases must be considered:
(I) the primes of n are missing from x,y
(II) the primes of n are contained in x or y (not both ∵ of **(A)**)

Case (I)
In the equation containing $(Fx)^n$, **h = x^n** , otherwise, after cancelling h from each term on the RHS, with x's on the left, x will still occur in the h's in every term on the RHS except the first, and must therefore exist in the first term as a factor of a, violating **(A)**.

Similarly **i = y^n** .

(ii) can now be written

$$(Gy)^n = y^n(c^{n-1} + c^{n-2}b + ...\ b^{n-1})$$
$$G^n = (c^{n-1} + c^{n-2}b + ...\ b^{n-1})$$
$$(Fx)^n = x^n(c^{n-1} + c^{n-2}a + ...\ a^{n-1})$$
$$F^n = (c^{n-1} + c^{n-2}a + ...\ a^{n-1})$$

∴ **G>F but Fx>Gy {∵ Fx=b, Gy=a} (B)**

and since $a + h = b + i = c$
$Gy + x^n = Fx + y^n = c$
$Fx - Gy = x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y .... xy^{n-2} + y^{n-1})$
$= R(x-y)$ $\{R = (x^{n-1} + x^{n-2}y ....\ xy^{n-2} + y^{n-1})$ {x,y coprime ∵ of **(A)**}
i.e. $Fx - Gy = R(x-y)$ **(C)**
∴ $Fx – Gy = Fx – (F+ w)y = F(x – y) - wy = R(x-y)$ {w integer >0}
and **G>F>R**

let $F = (R + u), G = (R + v)$ {u,v integers, u>v>0}
then $(R + u)x - (R + v)y = R(x-y)$
∴ $ux = vy$

further, because {u,v} are supposedly positive integers and (x,y) are coprime this requires:

u=y, v=x.

This is an impossibility because u>v and y<x.

∴ this proves FLT for Case (I).

Case (II)
We will consider the factors of n to be contained in b but the logic is similar.

Fx can now be written $Fx = Fpn^t$ {p,t positive integers, t>0}.

Considerations of the first term on the RHS similar to those for Case **(I)** requires $h = p^n n^{t-1}$.

(T) can thus be written:

$$a^n + (Fpn^t)^n = (a+ p^n n^{t-1})^n$$

$$Fpn^t = a((1 + (p^n n^{t-1})/a)^n - 1)^{(1/n)}$$

This requires that b ($=Fpn^t$), is a non-integer because $(p^n n^{t-1})/a$ is a non-integer ∵ of **(A)**.

Thus Case (II) also proves FLT.