Antimatter, CPT transformations and results for general relativity

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Abstract: Many of the profound ideas in nature manifest themselves as symmetries. Everything in physics that has been observed to date has a symmetrical opposite but equal property except for gravitational attraction. If antimatter is revealed to be gravitationally repulsive to matter through experimentation, then the equal and opposite of gravitational attraction or *anti-gravity* will be established. This paper reveals that if antimatter is in fact repulsive to matter as it should be according to a CPT transformation of the Einstein field equations for general relativity, then it has a pseudo-spherical volume mathematically. This paper first discusses the familiar Schwarzschild solution to emphasize how that leads to Newton's law of gravitational attraction. Then we discuss the CPT transformation of the Einstein field equations in the context of the Schwarzschild solution and how this leads to a repulsion between matter and anti-matter and how this repulsion is in sync with a pseudo-spherical geometry. Finally we discuss antimatter to antimatter attraction in general relativity and conclude with how this new derivation that is analogous to the Schwarzschild solution is important for physics.

Keywords: antimatter, symmetry in physics, anti-gravity, Schwarzschild solution, general relativity, CPT transformations, CPT invariance, Einstein vacuum field equations, pseudo-sphere, gravitational attraction, gravitational repulsion

1.) Introduction

To date, different scientists have theorized that antimatter is gravitationally attractive to matter or repulsive to matter. They have also theorized that antimatter is gravitationally attractive to itself or repulsive to itself. But in truth we really do not know how antimatter behaves gravitationally until an experiment is performed and validates what really happens. According to CPT transformations of the general relativity equations [8], it appears that antimatter is in fact repulsive to matter and attractive to itself. Starting from this premise and leveraging comparisons with the Schwarzschild solution, this paper reveals how antimatter mathematically has a pseudo-spherical volume and therefore compresses spacetime distances. This paper concludes with the consequences of this new derivation for physics and general relativity.

2.) A revisit of Newton's second law and the Schwarzschild solution

Most physicists and cosmologists are familiar with the Schwarzschild solution [1][2] in general relativity which to date is still the only vacuum solution of the Einstein field equations that reduces to Newton's law of gravitational attraction (and Kepler's third law) in the weak field approximation. That is the Schwarzschild solution for matter attracting matter to itself reduces to:

$$F = ma = \frac{-GMm}{r^2}, \tag{1}$$

as one would expect.

To arrive at Newton's second law through general relativity is not a simple task because the mathematics behind the general relativity equations is somewhat complex. But if one focuses solely on the radial and time components of the Schwarzschild solution in the weak field limit, then Newton's second law is readily derived. While the derivation here is definitely material that has been presented before and is probably familiar, it is important to revisit the solution and to show the steps in this derivation in order to draw analogies and elucidate the symmetries between the Schwarzschild solution presented here and the solution in the following section.

One of the Schwarzschild metrics typically used for the Schwarzschild derivation and that we use for our derivation here is:

$$ds^2 = -e^{M(r)}c^2dt^2 + e^{N(r)}dr^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2\,, \qquad (2)$$

So our metric tensor is:

$$g_{\alpha\beta} = \begin{pmatrix} -e^{M(r)} & 0 & 0 & 0 \\ 0 & e^{N(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}, \qquad (3)$$

Now we are just going to present the relevant equations that are pertinent to the derivation of Newton's second law using the Schwarzschild solution. The first important equation called the geodesic equation which leads to the equations of motion is:

$$\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0, \qquad (4)$$

for $\lambda, \mu, \nu \in \{0,1,2,3\}$. Considering that all three variables can cycle over four different values, the mathematical complexity of the equations starts to add up quickly. But in the Schwarzschild solution we can make the simplification that we only need to consider equations along the diagonal of the metric tensor for which μ is equivalent to ν. We can make the further simplification that since we are deriving Newton's law of gravitational attraction and since we are dealing with a static, isotropic spacetime that we only need consider the radial and time components in which λ is equivalent to one for the radial component and μ and ν are both equivalent to zero corresponding to the time component. With these simplifications, the only equation of motion we are really interested in solving for is:

$$\frac{d^2r}{d\tau^2} + \Gamma^1_{00}\left(\frac{cdt}{d\tau}\right)\left(\frac{cdt}{d\tau}\right) = 0, \qquad (5)$$

So now we will use the Einstein tensor values for a vacuum field solution to calculate $M(r)$ and $N(r)$. The only two Einstein tensor values we need are:

$$G_{00} = -\frac{e^{M(r)-N(r)}}{r}\left(N' - \frac{1}{r}\right) - \frac{e^{M(r)}}{r^2} = 0, \tag{6}$$

$$G_{11} = -\frac{M'}{r} - \frac{1}{r^2}\left(1 - e^{N(r)}\right) = 0, \tag{7}$$

where $M' = dM/dr$ and $N' = dN/dr$. These lead to our solutions for $M(r)$ and $N(r)$ considering that $N' = -M' = \frac{1}{r}(1 - e^{N(r)})$ as:

$$M(r) = -N(r), \tag{8}$$

$$e^{N(r)} = \frac{Cr}{1 + Cr}, \tag{9}$$

which lead us to the calculations for the previously unknown metric tensor values as:

$$g_{00} = -e^{M(r)} = -e^{-N(r)} = -\frac{1}{g_{11}} = -\left(1 + \frac{1}{Cr}\right), \tag{10}$$

Additionally with these values we can now calculate the value of the Christoffel coefficient Γ^1_{00} as:

$$\Gamma^1_{00} = -\frac{1}{2g_{11}}\frac{\partial g_{00}}{\partial r} = -\frac{1}{2Cr^2} - \frac{1}{2C^2r^3}, \tag{11}$$

which for the weak field approximation reduces to:

$$\Gamma^1_{00} \approx -\frac{1}{2Cr^2}, \tag{12}$$

Now we need to step outside of general relativity and by equating the weak field approximation of equation (5) to Newton's second law, we can calculate the constant C as:

$$C = -\frac{c^2}{2GM}, \tag{13}$$

which indicates gravitational attraction between two matter objects one that composes the gravitational field and the other that is present in this field.

3.) Repulsion of antimatter to matter in general relativity

While various vacuum solutions to the Einstein field equations exist [1][2][4][6], only the Schwarzschild solution has been shown to represent something that actually occurs in nature which is the gravitational attraction between two matter bodies or the motion of the planets orbiting our Sun. Now consider a new situation in general relativity which is that of a planet entirely composed of antimatter being orbited by a satellite entirely composed of matter. According to equation (5) above, if we CPT transform this equation for this new situation the only component that should change in sign is the Christoffel coefficient Γ^1_{00}. It is readily apparent that by changing the sign of the Christoffel coefficient Γ^1_{00} that by relating the weak field approximation to Newton's law of gravitational attraction as in the previous section that we now have a gravitational repulsion between antimatter and matter.

Now according to this change in sign, what sort of volume or geometry confluences with it? We know from the Schwarzschild derivation presented in the previous section that a spherical volume will not lead to this required change in sign. But we will now show the derivation of the Christoffel coefficient for a pseudo-spherical volume and by comparison to the previous derivation reveal how this new derivation leads to a change in sign for the Christoffel coefficient Γ^1_{00}. Note that like the Schwarzschild solution, the pseudo-spherical derivation presented here is static and isotropic and for the vacuum field external to a point mass centered at the origin of the coordinate system.

For the pseudo-spherical derivation, our metric equation is going to change to:

$$ds^2 = -e^{M(r)}c^2dt^2 + e^{N(r)}dr^2 + r^2\left(\frac{1}{\cosh^2(\phi)}\right)d\theta^2 + r^2\tanh^2(\phi)d\phi^2, \qquad (14)$$

and the metric tensor becomes:

$$g_{\alpha\beta} = \begin{pmatrix} -e^{M(r)} & 0 & 0 & 0 \\ 0 & e^{N(r)} & 0 & 0 \\ 0 & 0 & r^2\left(\dfrac{1}{\cosh^2\phi}\right) & 0 \\ 0 & 0 & 0 & r^2\tanh^2\phi \end{pmatrix}, \qquad (15)$$

Now for a vacuum solution to the Einstein field equations, our Einstein tensor values are going to change for this new geometry. As in the previous section in order to calculate $M(r)$ and $N(r)$, we only need the first two Einstein tensor values which are:

$$G_{00} = -\frac{e^{M(r)-N(r)}}{r}\left(N' - \frac{1}{r}\right) + \frac{e^{M(r)}}{r^2} = 0, \qquad (16)$$

$$G_{11} = -\frac{M'}{r} - \frac{1}{r^2}\left(1 + e^{N(r)}\right) = 0, \qquad (17)$$

where $M' = dM/dr$ and $N' = dN/dr$. Compare these two equations with equations (6) and (7) and notice the subtle differences. These two equations lead to two new solutions for $M(r)$ and $N(r)$ considering that $N' = -M' = \frac{1}{r}(1 + e^{N(r)})$ as:

$$M(r) = -N(r), \qquad (18)$$

$$e^{N(r)} = \frac{Cr}{1 - Cr}, \tag{19}$$

which lead us to the calculations for the previously unknown metric tensor values as:

$$g_{00} = -e^{M(r)} = -e^{-N(r)} = -\frac{1}{g_{11}} = \left(1 - \frac{1}{Cr}\right), \tag{20}$$

Additionally with these values we can now calculate the value of the Christoffel coefficient Γ^1_{00} for the pseudo-spherical volume as:

$$\Gamma^1_{00} = -\frac{1}{2g_{11}} \frac{\partial g_{00}}{\partial r} = \frac{1}{2Cr^2} - \frac{1}{2C^2 r^3}, \tag{21}$$

which for the weak field approximation reduces to:

$$\Gamma^1_{00} \approx \frac{1}{2Cr^2}, \tag{22}$$

which fulfills our requirement for an antimatter gravitational field being repulsive towards a matter satellite assuming that the constant C does not change in sign which it should not if it is a universal constant (matter is always going to be positive definite whether in a pseudo-spherical or spherical volume.) If one further considers the situation of a matter planet being orbited by an antimatter satellite, then in equation (5) the Christoffel coefficient no longer changes in sign since it now corresponds to a spherical volume but all other components in the equation do change in sign according to the CPT transformation of the satellite and the two bodies are still gravitationally repulsive towards each other.

3.) Attraction of antimatter to antimatter in general relativity

If we consider further the case of an antimatter satellite orbiting an antimatter planet, then not only does the Christoffel coefficient Γ^1_{00} change in sign corresponding to a pseudo-spherical volume but all the other components in equation (5) flip their signs as well according to a CPT transformation of the orbiting satellite and we have a mutual gravitational attraction between the two antimatter bodies.

4.) Conclusion

The importance of revealing that antimatter has a pseudo-spherical volume if it is repulsive to matter is that a pseudo-spherical volume is the geometry of a spacetime that is spacelike. In a spacelike spacetime, speeds that are faster than the speed of light can be achieved. Also spacetime distances in a spacelike spacetime are shortened and spacetime is effectively compressed whereas it is expanded in a timelike spacetime. So, *if* antimatter is gravitationally repulsive to matter and through experimentation it is revealed that antimatter also compresses spacetime, antimatter can eventually lead to conventional space travel in our lifetimes. Math does not lie or look at someone's credentials so please follow the mathematics and consider the real possibility of this theory and its ramifications for the future of space travel.

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