The new system that is concerned about Rindler theory

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ABSTRACT
In the general relativity theory, discover the new system that is concerned about Rindler coordinate theory. In this time, \( a_o = \frac{m_o c^3}{h} \). The new system uses the tetrad on the new method and it discovers the new inverse-coordinate transformation of the new system.

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    The Rindler’s theory
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    The tetrad

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I. Introduction

This theory is that it discovers new system that is concerned about Rindler theory.

Think the motion that use following the formula.

\[
x \approx \frac{1}{2} a_0 t^2 = \frac{1}{2} \left( \frac{m_0}{h} c^3 \right) t^2, \quad a_0 = \frac{m_0 c^3}{h}
\]  

(0)

It treats the motion by Eq(0).

Finding the new coordinate theory, use following the formula.

\[
x = \frac{c}{a_0} \left( \cosh \left( \frac{a_0 \tau}{c} \right) - 1 \right) = \frac{h}{m_0 c} \left( \cosh \left( \frac{m_0 c^2 \tau}{h} \right) - 1 \right),
\]

\[
t = \frac{c}{a_0} \sinh \left( \frac{a_0 \tau}{c} \right) = \frac{h}{m_0 c^2} \sinh \left( \frac{m_0 c^2 \tau}{h} \right)
\]

(1)

\(x\) and \(t\) is the coordinate and the time in the inertial system, \(\tau\) is invariable time, \(c\) is light speed in the inertial system in the free space-time, \(m_0\) is the particle’s stationary mass, \(h\) is the plank constant.

\[
dt = \cosh \left( \frac{m_0 c^2}{h} \tau \right) d\tau,
\]

\[
dx = c \sinh \left( \frac{m_0 c^2}{h} \tau \right) d\tau,
\]

\[
dy = dy' = 0, \quad dz = dz' = 0
\]

\[
V = \frac{dx}{dt} = c \tanh \left( \frac{m_0 c^2}{h} \tau \right)
\]

(2)

II. Additional chapter-I

The tetrad \(e^{a}_{\mu}\) is the unit vector that is each other orthographic and it use the following formula.

\[
e^{a}_{\mu} e^{\nu}_b g^{\mu \nu} = \eta_{ab}
\]

(3)

\(e^{a}_{\mu}\) is

\[
e^{a}_{\mu} = \eta^{ab} g^{\mu \nu} e^{\nu}_b
\]

(4)

and it is \(e^{a}_{\mu}\) ’s inverse-matrix. And it is

\[
e^{a}_{\mu} e^{\mu}_b = \delta^{a}_{b}, \quad e^{a}_{\mu} e^{\nu}_a = \delta^{\nu}_{\mu}
\]

\[
e^{a}_{\mu} e^{\mu}_b \eta_{ab} = g^{\mu \nu}
\]

(5)

The \(e^{\mu}_{\rho}(\tau)\) is the tetrad that if \(\xi^{1} = \xi^{2} = \xi^{3} = 0, d\xi^{1} = d\xi^{2} = d\xi^{3} = 0\). In this time, in Eq(5) it
does $g_{\mu \nu} = \eta_{\mu \nu}$.

Therefore, Eq(5) is

$$
\eta_{\alpha \beta} e^\alpha_0(\tau) e^\beta_0(\tau) = \eta_{00} = -1
$$

$$
d\tau^2 = \frac{1}{c^2} \eta_{\alpha \beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}
$$

$$
\rightarrow -1 = \eta_{\alpha \beta} \left( \frac{1}{c} \frac{dx^\alpha}{d\tau} \right) \left( \frac{1}{c} \frac{dx^\beta}{d\tau} \right) = \eta_{\alpha \beta} e^\alpha_0(\tau) e^\beta_0(\tau)
$$

(6)

According to Eq(2), Eq(6)

$$
e^\alpha_0(\tau) = \frac{1}{c} \frac{dx^\alpha}{d\tau} = \left( \cosh \left( \frac{m_0 c^2}{h} \tau \right), \sinh \left( \frac{m_0 c^2}{h} \tau \right), 0, 0 \right)
$$

(7)

About $y$ -axis's and $z$ -axis's orientation

$e^\alpha_2(\tau) = (0, 0, 1, 0), e^\alpha_3(\tau) = (0, 0, 0, 1)$

And the other unit vector $e^\alpha_1(\tau)$ has to satisfy the tetrad condition, Eq (5)

$$
e^\alpha_1(\tau) = \left( \sinh \left( \frac{m_0 c^2}{h} \tau \right), \cosh \left( \frac{m_0 c^2}{h} \tau \right), 0, 0 \right)
$$

(8)

III. Additional chapter-II

According to the tetrad $e^\alpha_\mu$ in the flat Minkowski space, the inertial coordinate system $S(t, x, y, z)$ transform the new system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$. Therefore,

$$
d\tau^2 = dt^2 - \frac{1}{c^2} \left[ dx^2 + dy^2 + dz^2 \right]
$$

$$
= -\frac{1}{c^2} \eta_{\alpha \beta} \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{\partial x^\beta}{\partial \xi^\nu} d\xi^\mu d\xi^\nu
$$

$$
= -\frac{1}{c^2} \eta_{\alpha \beta} e^\alpha_\mu e^\beta_\nu d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu \nu} d\xi^\mu d\xi^\nu
$$

$$
e^\alpha_\mu = \frac{\partial x^\alpha}{\partial \xi^\mu}
$$

(9)

Therefore, for saving the new system in the mathematical way, the $e^\alpha_\mu(\xi^0)$ is used by Eq (7), Eq(8) that used $\xi^0$ instead of $\tau$.

The unit vector $e^\alpha_1(\xi^0)$ is

$$
e^\alpha_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = \left( \sinh \left( \frac{m_0 c^2}{h} \xi^0 \right), \cosh \left( \frac{m_0 c^2}{h} \xi^0 \right), 0, 0 \right)
$$

(10)

$$
\frac{\partial e^\alpha_1(\xi^0)}{c \partial \xi^0} = \frac{\partial^2 x^\alpha}{\partial \xi^1 \partial \xi^0} = \frac{\partial e^\alpha_0(\xi^0)}{\partial \xi^1}
$$

(11)

Therefore, the vector $e^\alpha_0(\xi^0)$ is
\[ e^{\alpha_0}(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^0} \]
\[ = ((1 + \frac{m_0c}{h} \xi^1) \cosh(\frac{m_0c^2}{h} \xi^0), (1 + \frac{m_0c}{h} \xi^1) \sinh(\frac{m_0c^2}{h} \xi^0), 0, 0) \]  

(12)

About \( Y \)-axis's and \( z \)-axis's orientation, the unit vector \( e^{\alpha_2}(\xi^0) \) and \( e^{\alpha_3}(\xi^0) \) is

\[ e^{\alpha_2}(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^{\alpha_3}(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \]

The differential coordinate transformation is

\[ dx^\alpha = \frac{\partial x^\alpha}{\partial \xi^\mu} d\xi^\mu = \frac{\partial x^\alpha}{\partial \xi^0} c d\xi^0 + \frac{\partial x^\alpha}{\partial \xi^1} d\xi^1 + \frac{\partial x^\alpha}{\partial \xi^2} d\xi^2 + \frac{\partial x^\alpha}{\partial \xi^3} d\xi^3 \]
\[ = e^{\alpha_0}(\xi^0) c d\xi^0 + e^{\alpha_1}(\xi^0) d\xi^1 + e^{\alpha_2}(\xi^0) d\xi^2 + e^{\alpha_3}(\xi^0) d\xi^3 \]

\[ c dt = (1 + \frac{m_0c}{h} \xi^1) \cosh(\frac{m_0c^2}{h} \xi^0) d\xi^0 + \sinh(\frac{m_0c^2}{h} \xi^0) d\xi^1 \]
\[ dx = (1 + \frac{m_0c}{h} \xi^1) \sinh(\frac{m_0c^2}{h} \xi^0) c d\xi^0 + \cosh(\frac{m_0c^2}{h} \xi^0) d\xi^1 \]
\[ dy = d\xi^2, dz = d\xi^3 \]

(13)

(14)

(15)

Therefore if Eq(13), Eq(14) and Eq(15) integrate, finally the new system's coordinate transformation is found.

\[ ct = (\frac{h}{m_0 c} + \xi^1) \sinh(\frac{m_0 c^2 \xi^0}{h}) \]
\[ x = (\frac{h}{m_0 c} + \xi^1) \cosh(\frac{m_0 c^2 \xi^0}{h}) - \frac{h}{m_0 c} \]
\[ y = \xi^2, z = \xi^3 \]

(16)

(17)

Therefore, the inverse-coordinate transformation of the new system is

\[ \frac{ct}{(x + \frac{h}{m_0 c})} = \tanh(\frac{m_0 c^2 \xi^0}{h}) \]
\[ \xi^0 = \frac{h}{m_0 c^2} \tanh^{-1}\left(\frac{ct}{(x + \frac{h}{m_0 c})}\right) \]
\[ (x + \frac{h}{m_0 c})^2 - c^2 t^2 = (\frac{h}{m_0 c} + \xi^1)^2 \]
\[ \xi^1 = \sqrt{(x + \frac{h}{m_0 c})^2 - c^2 t^2} - \frac{h}{m_0 c} \]
\[ \xi^2 = y, \xi^3 = z \]

(18)

(19)
Therefore, the invariant time \( d\tau \) of the new system is by Eq(13), Eq(14), Eq(15)

\[
d\tau^2 = dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2]
\]

\[
=\left(1 + \frac{m_0c}{h} \xi^1\right)^2 (d\xi^0)^2 - \frac{1}{c^2} \left[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2\right]
\]

(20)

Hence, Riemann curvature tensor \( R_{\mu\nu\rho\lambda}^\delta(x) \), \( R_{\alpha\beta\gamma\delta}^\sigma(\xi) \) is

\[
g_{00} = -\left(1 + \frac{m_0c}{h} \xi^1\right)^2, \quad g_{11} = g_{22} = g_{33} = 1,
\]

\[
g^{00} = -1/\left(1 + \frac{m_0c}{h} \xi^1\right)^2, \quad g^{11} = g^{22} = g^{33} = 1,
\]

\[
\Gamma^1_{00} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial \xi^1}\right) = \frac{1}{2} \left[2\left(1 + \frac{m_0c}{h} \xi^1\right)^2 \cdot \frac{m_0c}{h} = \frac{1}{2} \frac{m_0c}{h} \right]
\]

\[
\Gamma^0_{10} = \Gamma^0_{01} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial \xi^1}\right) = \frac{1}{2} \left[-1/\left(1 + \frac{m_0c}{h} \xi^1\right)^2 \cdot \frac{m_0c}{h} = \frac{1}{2} \frac{m_0c}{h} \right]
\]

\[
R_{\alpha\beta\gamma\delta}^\sigma(\xi) = \frac{\partial \Gamma^\delta_{\alpha\beta}}{\partial \xi^\gamma} - \frac{\partial \Gamma^\delta_{\alpha\gamma}}{\partial \xi^\beta} + \Gamma^\gamma_{\alpha\rho} \Gamma^\rho_{\beta\gamma} - \Gamma^\gamma_{\beta\rho} \Gamma^\rho_{\alpha\gamma}
\]

\[
R_{\alpha\beta\gamma\delta}^\sigma(\xi) = -R_{\beta\alpha\gamma\delta}^\sigma(\xi) = \frac{\partial \Gamma_{\alpha\beta}^\gamma}{\partial \xi^\delta} - \Gamma_{\alpha\delta}^\gamma \Gamma_{\gamma\beta}^\delta - \Gamma_{\beta\delta}^\gamma \Gamma_{\gamma\alpha}^\delta = 0, \text{ otherwise } R_{\alpha\beta\gamma\delta}^\sigma(\xi) = 0
\]

(21)

Therefore, the new system is in the flat Minkowski space.

About \( x \)-axis’s light speed,

\[
dy = d\xi^2 = dz = d\xi^3 = 0, \quad y = \xi^2 = z = \xi^3 = 0
\]

\[
ct = dx, \quad ct = x
\]

\[
cd\xi^0 = \frac{d\xi^1}{1 + \frac{m_0c}{h} \xi^1}
\]

\[
c_\xi^0 = \frac{h}{m_0c} \ln \left| 1 + \frac{m_0c}{h} \xi^1 \right|
\]

\[
\rightarrow \left(1 + \frac{m_0c}{h} \xi^1\right) = e^{\frac{m_0c}{h} \xi^0} \rightarrow \left(\frac{h}{m_0c} + \xi^1\right) = h e^{\frac{m_0c}{h} \xi^0}
\]

(22)

In this time, if use the new system’s coordinate transformation, Eq(16), Eq(17)
\[ ct = \left( \frac{h}{m_0c} + \xi^1 \right) \sinh \left( \frac{m_0c^2 \xi^0}{h} \right) \]

\[ = \frac{h}{m_0c} e^{\frac{m_0c^2 \xi^0}{h}} \left( e^{\frac{m_0c^2 \xi^0}{h}} - e^{-\frac{m_0c^2 \xi^0}{h}} \right) \]

\[ = \frac{h}{m_0c} \left( e^{\frac{2m_0c^2 \xi^0}{h}} - 1 \right) \]

\[ x = \left( \frac{h}{m_0c} + \xi^1 \right) \cosh \left( \frac{m_0c^2 \xi^0}{h} \right) - \frac{h}{m_0c} \]

\[ = \frac{h}{m_0c} e^{\frac{m_0c^2 \xi^0}{h}} \left( e^{\frac{m_0c^2 \xi^0}{h}} + e^{-\frac{m_0c^2 \xi^0}{h}} \right) - \frac{h}{m_0c} \]

\[ = \frac{h}{m_0c} \left( e^{\frac{2m_0c^2 \xi^0}{h}} - 1 \right) \]

(23)

**IV. Conclusion**

It found the new system that used the tetrad on the new method.

**Reference**


