On the Applicability of the Lorentz Transformations

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Abstract

The Lorentz transformations are not true coordinate transformations as Einstein derives them. That is, they do not represent a one-to-one mapping of a single set of coordinate values in one coordinate system onto another set in an identical coordinate system moving at a constant velocity relative to the first. Rather, they represent a mapping of an average of two sets of coordinate values from the first coordinate system onto a single set of values in the second. If the measurement system used in an experiment is inconsistent with Einstein's averaging method, then the Lorentz transformations will give incorrect results. A simple example is given of a photon-emitting clock moving at a constant velocity, v, in a straight line between two photon detectors. The time of travel of the clock between detectors measured by the front detector would be $t = \tau$, and by the rear detector would be $t = \tau\,(1-v/(c+v))$. The Lorentz transformation gives a value of $t = \tau / \sqrt{(1-v^2/c^2)}$ for both detectors. It is also noted that the Lorentz transformations give results inconsistent with the coordinates of photons in a light pulse of the form $c^2t^2-x^2-y^2-z^2=0$, when measured in an inertial reference frame different from that of the source.

PACS code 03.30.+p

Einstein [1] gave no reason for his unusual method of determining coordinate values, and he seems to have regarded it as universally applicable. He simply starts his derivation of the Lorentz transformations by the definition of a time interval, τ_1, in a moving coordinate system, as the average of two values, $\frac{1}{2}(\tau_0 + \tau_2)$, where τ_0 represents the time of emission of a pulse of light along the x axis at the origin of the moving coordinate system, and τ_2 the time of return of the pulse to the origin after having been reflected from an arbitrary point on the x axis. He then expresses τ_2, τ_1, and τ_0 as functions of c, the coordinates of the stationary coordinate system, and the relative velocity of the two systems. Using rather general arguments, he then derives a set of coordinate transformations that leave invariant the form of a spherical pulse of light, $c^2t^2-x^2-y^2-z^2=0$, in a coordinate system moving relative to the emitting system. He previously uses a similar method of measuring the time of travel of a light pulse over the same distance in opposite directions in his definition of clock synchronization, and also in his definition of the velocity of light using the average of the two times. Both of these definitions seem to me largely irrelevant. Such a method of clock synchronization is not necessary, nor even used, as far as I know. And the measurement of the velocity of a photon certainly does not require an average of its time of travel over the same distance in opposite directions.

Personally, I do not believe Einstein derived the transformations this way, but rather in the manner he hints at in a footnote. It seems to me highly unlikely that he could recognize that this coordinate averaging scheme would result in transformations that would leave the form of a spherical light pulse invariant under a change in velocity of the coordinate system. He more likely started from the assumption that the spherical pulse was unchanged, found the transformations that left $c^2t^2-x^2-y^2-z^2=0$ formally invariant, and then figured out a method of coordinate determination which would give him the transformations. But this of course, is speculation. Regardless of which way he derived them, they seem to me inconsistent with the behavior of photons representing a spherical light pulse, and Einstein's example illustrates an important limitation of the Lorentz transformations. We can see this as follows:

A determination of the velocity of light requires the measurement of the time of travel of a photon between an emitter and a detector at a known distance. The photon has the peculiar property that its velocity is independent of the velocity of the emitter. But it is not independent of the velocity of the detector. That is, any velocity of the detector relative to the photon must be accounted for in order to obtain a correct result. This means that the measured velocity of the photon is constant in both the emitting and detecting coordinate systems if the emitter is moving relative to the detector, but it is not constant in the coordinate system of the detector if the detector is moving relative to the emitter. We can see this in a simple example:

If the emitter is located at x=0, and emits a photon at t=0 in the positive x direction towards a stationary detector located at x=x, then the photon will arrive at the detector at t=t, and the measured velocity of the photon will be x/t. If, however, the detector is moving towards the emitter with a velocity, v, then the photon will arrive at the detector at some t=t' (t'<t) at x=d (d<x), where d= x-vt'. The measured velocity will then be d/t', which will equal x/t as long as the physical speed of the photon is the same in both cases. This is true in the coordinate system of the emitter. But it is not so in the coordinate system of the detector.

If the detector is stationary, the coordinate system of the detector will give the same result as the coordinate system of the emitter. Similarly, the result will be the same if the emitter is moving relative to the detector. But, if the detector is moving relative to the emitter it will not. The coordinate system of a moving detector cannot account for the motion of the detector with respect to the photon. In the coordinate system of the moving detector, the detector is stationary, and the photon is emitted towards it at x= -x, and t=0. Since the detector remains at x=0, the photon must travel the entire distance, x, even though it arrives at the detector at t=t'. So the velocity of the photon as measured in the coordinate system of the detector will be x/t' rather than x/t. In order to obtain the correct result in the coordinate system of the detector, the measured velocity of the photon must be changed to c+v.

Einstein certainly recognized this necessity to some extent, as he uses this modified value of the speed of light, (c±v) for the determination of the average time interval, $\tau_1 = \frac{1}{2}(\tau_0 + \tau_2)$, in his derivation of the Lorentz transformations. But he seems not to have understood its significance. The emitter defines a unique inertial reference frame. All other coordinate systems become detector coordinate systems with respect to it, and must be treated as such. That is, the velocity of the detecting coordinate system relative to the emitter determines the velocity of the detector relative to the photon, and therefore the photon's velocity as measured in the detecting coordinate system. The issue of the consistency of Einstein's use of the modified value of the speed of light with his principle of relativity has been to some degree debated elsewhere [2,3], but without, in my view, any real illumination of the problem.

Because of the difference between the emitting and detecting coordinate systems, an examination of Einstein's example of the shape of a spherical pulse of light must be divided into two cases: the detecting coordinate system moving relative to the emitter, and the emitter moving relative to the detecting system. Since the coordinates equidistant in either direction from the origin along the x-axis represent radii of a sphere centered at the origin, we only need to look at what happens with two photons emitted at $t=0$ in opposite directions along the x-axis from the origin. In the coordinate system of the emitter, they will always represent the radii of a sphere. That is, detectors located along the x-axis will always, at $t=t$, measure the same distance of each photon from the origin.

But in a coordinate system moving at velocity v along the x-axis, whose origin is coincident with that of the emitting system at $t=0$, this will not be so. In this system, a detecting system, the measured velocity of the photon in one direction will be $c+v$, and in the other $c-v$. Detectors located along its x-axis will never see identical values for the length of travel of the two photons over the same time interval. The two x coordinates of the photons on the x-axis will be $x=t(c-v)$ and $x= -t(c+v)$, so the light pulse can never be represented by an expression of the form $c^2t^2-x^2-y^2-z^2=0$. By basing his derivation of the Lorentz transformations on a method of coordinate measure that averages values from light signals traveling in opposite directions, Einstein makes them independent of this effect. They define a modified set of coordinates in which the dependence on v of the velocities of the photons has been eliminated.

In the alternative case, where the emitter is moving at a constant velocity, v, along the x-axis of a stationary detecting coordinate system, the Lorentz transformations do not give correct results either, even though they retain the spherical shape of the light pulse. Let the coordinates in the stationary system be x,t , and the moving system X,T. The photons are emitted in either direction when the origins are coincident at $t=T=0$. Since the clocks in both systems measuring the time of travel of the photons are synchronized, it is always possible to define a time interval measured in either system such that $t=T$. Because the velocity of the photons is the same in both systems, the length of travel of the photons measured at time $t=T$ in both systems must be the same. That is, the length of travel measured in the stationary system will be $x= \pm ct$, and in the moving system $X= \pm cT= \pm ct = x$. The Lorentz transformation gives $X=(x-vt)/\sqrt{1-v^2/c^2}$ for the relationship between the coordinates in the two systems. Or, if we treat the case as equivalent to the detecting coordinate system moving at a velocity of $-v$ relative to the emitter, then we have $x= (X+vT)/\sqrt{1-v^2/c^2}$.

But let us look at a more practical example of the type of photon behavior that the Lorentz transformations would not represent correctly, an experiment that seems to me within the realm of possibility: measuring the time of travel of a photon-emitting clock in a straight line between two fixed detectors, located at $x=0$ and $x=x$. In this case, the emitter is moving relative to the detector, so the velocity of the photon is independent of the motion of the emitting clock relative to the detectors. In my view, such independence does not imply, as Einstein claimed, the existence of a universally applicable set of coordinate transformations, independent of the experimental arrangement under consideration.

If the clock, traveling at velocity v along the x-axis, emits photons at a fixed rate, starting at t=0 and x=0, then when it arrives at x=x at t=t, it will have emitted N photons in an arbitrary direction. In its own coordinate system, then, the value of t can be represented by N. To determine what the laboratory detectors will see, that is, the number of photons measured in the laboratory, we have to examine what happens to the photons emitted by the clock during the time of travel.

Depending on where the detectors are located, the results will differ. If the photons are emitted in the forward direction, then a detector stationed at the end of the path, at x=x, will have detected all N photons in the time of travel, t, because each photon emitted during the period of travel will move towards the detector more rapidly than the clock, and will have already arrived at the detector prior to, or along with the clock's arrival at x.

For photons emitted in the backward direction, however, this is not the case. There will be a threshold point, at some $x-\Delta x$, where the remaining time of travel of the clock will be less than the time of travel of emitted photons to the rear detector. That is, $\Delta x/v < (x-\Delta x)/c$. Equating the two, and solving for Δx, gives $\Delta x = vx/(c+v)$. During such a time period, the number of photons emitted will be $\Delta N=\Delta t\, dn/dt$, where Δt is the time of travel of the clock over the distance Δx, and dn/dt is the rate of photon emission of the clock in an arbitrary direction. These will not arrive at the detector during the time interval of travel. Therefore the number of photons detected at the rear detector in time t will be $N-\Delta N$. Since the rate of photon emission is constant over the length of travel, the ratio of $\Delta N/N$ will be the same as $\Delta x/x$, and we can then determine ΔN:

$$\Delta N/N = v/(c+v),\ \ \Delta N = Nv/(c+v), \text{ so that}$$

$$N-\Delta N = N-Nv/(c+v), \text{ or } N(1-v/(c+v)).$$

Using t and τ instead of $N-\Delta N$ and N for the time interval expressed in the two coordinate systems, for the rear detector we then have

$$t = \tau\,(1-v/(c+v)).$$

If we assume a clock velocity of .9c, the rear transformation gives a value for t of approximately $.526\tau$, which indicates that a significant number of photons would not be observed at high velocities. If $v = c$, then $t = .5\tau$, and of course half the photons would be excluded, as the velocities would be the same. Thus the time measured by the moving clock during time τ as seen by the stationary observer would be the same as an equivalent stationary clock if measured by the front detector, but reduced by this factor when measured by the rear detector. The Lorentz transformation, $\tau = t\sqrt{(1-v^2/c^2)}$, with $v = .9c$, gives $t = 2.294\tau$ measured by either detector.

[1] A. Einstein, *Annalen der Physik*, 17:891, 1905. 1923 English translation
[2] M. S. Khan, 2012, viXra:1201.0022
[3] J. Dunning-Davies, 2012, viXra:1202.0054