Discrete structure of spacetime

Nicola D'Alfonso
Independent Scholar - Milan, Italy
nicola.dalfonso@hotmail.com

PACS 02.30.Lt: Sequences, series, and summability
PACS 03.30.+p: Special relativity

Abstract

In this paper, I introduce a particular discrete spacetime that should be seriously considered as part of physics because it allows to explain the characteristics of the motion properly, contrary to what happens with the continuous spacetime of the common conception.

1 Paradox of the dichotomy

We know from our observations of reality that the motions are possible. However, in a continuous spacetime the motion seems unable to happen, at least according to the paradox of the dichotomy presented here first in the spacial version and then in time version.

A person has to do a stretch of road along a straight line which extends in any direction, whose ends are identified by the letters A and B, and whose distance is AB. Clearly this person to get to B will have to arrive before at the halfway position C. Once he arrived in C, to get to B will have to arrive once again at the halway position D, and so on in an infinite progress. This means that in a continuous space, provided with an unlimited number of positions, the passage from A to B will require infinite operations, but no person is able to perform a truly infinite number of operations, and therefore no motion should be completable and in this way possible. We can observe with regard to this the following figure 1:

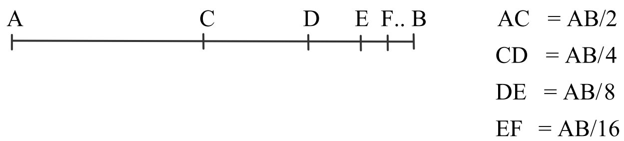

Figure 1: spatial representation of the paradox of the dichotomy

We can consider now the temporal version of the paradox. Let T_{AB} be the total time that the person should take to go from A to B. Clearly this person to reach B at time T_{AB} will have to move before through all the intermediate time $T_{AB}/2$. Once the time $T_{AB}/2$ is expired, to reach B at time T_{AB} he will have to move once again for all the intermediate time $T_{AB}/4$, and so on in an infinite progress. This means that in a continuous time, provided with an unlimited number of moments, the passage from A to B will require infinite operations, but no person is able to perform a truly infinite number of operations, and therefore no motion should be completable and in this way possible. We can observe with regard to this the following figure 2:

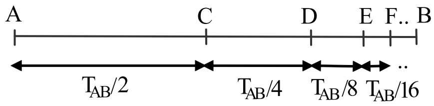

Figure 2: temporal representation of the paradox of the dichotomy

The supposed solution of the paradox comes from the use of mathematics. In fact the mathematics can assign a finite value also to the sum of infinite terms. In our case, it is sufficient to make reference to the formulas 1:

$$AB \cdot \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \left(\frac{AB}{2}\right) + \left(\frac{AB}{4}\right) + \left(\frac{AB}{8}\right) + \left(\frac{AB}{16}\right) + \ldots = AB \tag{1a}$$

$$T_{AB} \cdot \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \left(\frac{T_{AB}}{2}\right) + \left(\frac{T_{AB}}{4}\right) + \left(\frac{T_{AB}}{8}\right) + \left(\frac{T_{AB}}{16}\right) + \ldots = T_{AB} \tag{1b}$$

According to these formulas, the sums of the infinite spatial and temporal intervals considered can be associated with the whole distance AB and the length T_{AB}.

The problem is that this solution does not resolve the paradox on the physical plane. The reason is simple and depends on the fact that the previous formulas are based on the notion of limit, and the notion of limit shows the value at which a function can approach at pleasure, and not the value that actually takes. We can observe with regard to this what is stated in [1].

In practice, in the case of the addition of infinite terms what mathematics does is to verify if with the growth of their numbers, the sum is closer and closer to a certain value. If this happens, that value is taken as the "limit" to which the whole sum goes toward, but it does not represent really its result. So, what the above formulas allow us to say is that an interval after the other we will be able to approach at pleasure to point B and to the time T_{AB}, but not that these targets are actually achievable.

Since identifying a limit does not correspond in any way to do infinite operations nor it allows to determine what might be their result, the paradox of the dichotomy cannot be considered solved for a continuous spacetime.

2 Solution of the paradox in a discrete spacetime

Let us consider a discrete spacetime, in which each spatial interval is constituted by a finite number of positions arranged along any direction, and each time interval by a finite number of moments in the direction towards the future. Since the number of moments and positions are finite, will also be finite the operations required to do any movement, solving the paradox of the dichotomy. It is not necessary for the resolution of the above paradox that the individual positions that compose the space are fixed or homogeneous.

Let us suppose that the space AB considered is composed of eleven positions. We can observe with regard to this the following figure 3 (where the positions are drawn fixed, for simplicity):

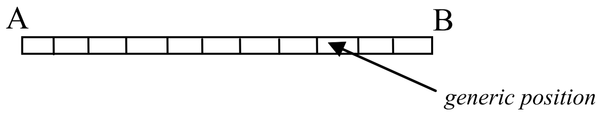

Figure 3: representation of the positions that compose the space AB

The person who starts from A does nothing but shifts of the 11 positions that separate it from B. The possibility to subdivide this distance will be limited by the necessity that each spatial interval is coverable by an integer number of positions. For example again referring to the figure above, we can attribute to the person the position of $AB \cdot (5/11)$ placed after the first five positions from A. No person could instead be in the position $AB/2$, to which would correspond 5.5 positions from A.

The fact that all positions of the distance AB, for example $AB/2$, seem to be accessible is due to the large number of positions whose are composed the observable spatial intervals. In substance the accessible positions fill the space so much that we are unable to identify those not reachable.

With regard to the temporal intervals, it is necessary to identify the moments as the opportunities available to the nature to show any change. Furthermore, the possibility to reconstruct the motion performed will be limited by the necessity for each temporal interval to be coverable by an integer number of moments. For example, considering the time T_{AB} composed of 11 moments, the person who starts from A advances of one position at any consecutive moment (we are assuming that the total positions are always 11). In this sense at the time $T_{AB}/11$ we can assign to the person the first position from A. Instead it does not make sense to wonder in which position the person will be placed at the time $T_{AB}/2$ because it would correspond to 5.5 moments.

The fact that all moments required to complete the distance AB, for example $T_{AB}/2$, seem to be observable is due to the contiguity of the motion in the space. In other words: since every moving object always passes through all the positions that compose his path, we will always observe a reality able to evolve through all possible intermediate stages.

3 Maximum speed

Due to the special theory of relativity, we know that there is a speed limit (the speed of light) beyond which the objects cannot move, as we can observe in [2]. Even this characteristic of the nature calls into question the concept of a continuous spacetime.

To understand how this is possible we must refer to the contiguity of the motion. The fact is that the need to consider the motion as contiguous in the space forces us to relate the speed of an object to the moments it takes to pass from a position to that contiguous. In this sense, in a discrete spacetime the nature will be intrinsically subject to a maximum speed: the one in correspondence of which an object reaches the position contiguous to each consecutive moment of time. The other speeds, slower than the one just considered, are possible whenever the moving object needs of more than one moment to pass from one position to another. For example an object that moves at half of the maximum speed will pass from one position to that contiguous at each pair of consecutive moments of time.

Vice versa in a continuous spacetime, the contiguous space movements should not be subject to any speed limit because given any speed, there will always be another greater. For this purpose it will be sufficient to think that the object in question uses a smaller time interval to pass through the same positions. And there is no limit to what a time interval can be thought small in a continuous spacetime.

In this sense, a truly continuous spacetime, in addition to the problems pointed out by the paradox of the dichotomy, would be subject to a maximum speed which is not justified by the structure of reality, at least in terms of space and time.

4 Conclusion

The possibility that spacetime has precisely the discrete structure presented here should be considered a very serious hypothesis in physics, because besides describing in a coherent manner what we observe about the motion, overcomes the problems due to the paradox of the dichotomy and gives a structural explanation of the existence of the maximum speed.

References

[1] Hardy G.H. A couse of pure Mathematics, Cambridge University Press, Cambridge 1908, pp. 117-123.

[2] Rindler W. Relativity. Special, general, and cosmological. Second Edition, Oxford University Press, New York 2006, pp. 54-75.