# What Causes Tides? 

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#### Abstract

It has been known for thousands of years that the waters of coastal seas ebb and flow on a daily basis. This oscillation is widely thought to be from the gravitational pulls of the Moon and Sun. Here we propose that, the Earth's curved motions about the centre of mass of the Earth-Moon system and about the Sun may generate centrifugal effect to stretch the Earth's body along the direction of equator; the majority of the Earth's surface is covered with water, this makes the Earth look like being submerged in the water; the deformed Earth, together with the Earth's daily rotation, makes the Earth's each part regularly rise from and fall into the water. The rise and fall of related part result in the fall and rise of water level.


## 1 Introduction

### 1.1 A brief introduction of tidal ideas

From antiquity it has been familiar that coastal seas always perform daily regular movements of water rise and fall. Since these movements are closely related to the frequently coastal activities, explaining them has undoubtedly tested human wisdom for millennia. Aristotle ( $384-322 \mathrm{BC}$ ) was highly perplexed and vaguely attributed it to the rocky nature of the coast. Galileo asserted that the Earth in the curved orbit is alternately accelerated and retarded to induce the sea to move and form tide. Early Chinese considered tides as the beating of the Earth's pulse and alternatively, it was believed to be caused by the Earth's breathing. Some people thought tides were caused by the different depths of ocean water. However, the majority certainly linked tidal action to the influence of the Moon and of the Sun. Seleucus (lived in the 2nd century BC) was the first to consider this connection. He concluded the height of tide was correlated with the Moon's position relative to the Sun. However, the exact determination of how the Moon and Sun cause tides is unknown. A few Arabic explanations proposed that the Moon uses its rays to heat the water and then expand it. Descartes argued that space was full of ethereal substance and the resulting stresses between the ether and the Earth's surface gave birth to tides when the Moon travelled round the Earth. In contrast, Kepler and Newton represent those who expressly define this influence as the attraction of the Moon and Sun on the water. Newton formulated an equilibrium mechanism to account for the tide, within it the Earth is supposed to be an ocean planet with no land, and the influence of inertia and currents are fully ignored. The gravity gradient of the Moon produces a pair of bulges of water on Earth (that is in the line of the Earth-Moon system) to form tides. As Newton and the well known 1740's Essayists (such as Daniel Bernoulli, Leonhard Euler, Colin Maclaurin, and Antoine Cavalleri, for instance) had assumed the oceans' response to the tidal driving force to be quasi-static, considering the complexity of actual oceans and currents, Laplace developed a set of hydrodynamic equations of continuity and momentum for a fluid on a rotating earth. Together with the following endeavors (including William Thomson, Baron Kelvin, Henri Poincaré, Arthur Thomas Doodson, etc.,), the idea of gravitation as the cause of tide (we name it the attractive mechanism in the following sections) was increasingly strengthened and became the cornerstone of modern tidal theory. A fuller review of the tidal history may be found in these works [1,2].

### 1.2 Problems of the attractive mechanism <br> Logic

The attractive mechanism may be approximately expressed with such a paradigm [1,3]: the Earth orbits about the centre of mass of the Earth-Moon system, this makes all particles of the Earth travel around in circles which have the same radius. The force necessary to give each particle of the Earth the acceleration to perform this revolution is the same as for the particle
at the centre of the Earth, for such a particle the Moon's gravitational force provides this necessary acceleration. For those particles nearer the Moon than the centre particle, the Moon's gravitational attraction is greater than that necessary to maintain the orbit. For those further away the forces are weaker. The differences between the forces necessary for orbit and the forces actually experienced generate the tides on the surface of the Earth. In detail, as demonstrated in Figure 1, a particle of mass $m$ located at $\mathrm{P}_{1}$ of the Earth's surface, the Moon's gravitational attraction on this particle is $G m m_{1} /(R-a)^{2}$, the force necessary for the revolution of this particle is the same as for a particle at $O$, which is equal to $G m m_{1} / R^{2}$ (the Moon's gravitational attraction on the particle at the centre of the Earth), the difference between the two is $G m m_{1}\left[1 /(1-a / R)^{2}-1\right] /(R)^{2}$, which is considered the tide producing force at $\mathrm{P}_{1}$. Due to $(a / R)^{2} \ll 1$ and expanding $\left[1 /(1-a)^{2}\right] \approx 1+2 a$ for small $a$, this finally results in a net tidal force (towards the Moon) of $2 G m m_{1} a / R^{3}$ at $\mathrm{P}_{1}$, where $m_{1}, a, R, G$ are respectively the Moon's mass, the Earth's radius, the distance between the Earth and Moon, and gravitational constant. Similar work for a particle located at $\mathrm{P}_{2}$ results in a net tidal force (away from the Moon) of $2 G m m_{1} a / R^{3}$. The net force at $\mathrm{P}_{3}$, directed towards the Earth's center, is given as $G m m_{1} a / R^{3}$ under a consideration that approximately treats $\sin \left(\mathrm{OMP}_{3}\right)=a / R$ and the force along $\mathrm{P}_{3} \mathrm{M}$ is given as $G m m_{1} / R^{2}$. The net effect is for particles at both $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ to be displaced always from the centre of the Earth, whereas particles at $\mathrm{P}_{3}$ are displaced towards the centre. This leads to an equilibrium shape (two bulges) for a fluid Earth which is slightly elongated along the axis between the centers of the Moon and the Earth.


Figure 1. The dynamics frame of the attractive mechanism. $R$ and $a$ represent the distance between the Earth and Moon and the Earth's radius, $O$ is the centre of the Earth. Dashed lines are the resulting bulges due to the Moon's gravitational pull.

At first glance, this deduction is considerably self-contained, but it misses a crucial point that the Earth's gravitational attraction on these particles at both $P_{1}$ and $P_{2}$ is not considered. Actually, the attractive mechanism is treating a relative motion of these particles and the centre of the Earth. We suppose, if these particles and the Earth are not attracted to each other, the tide producing force may lead these particles to displace away from the centre of the Earth. The problem is that the Earth's mass is focused on the centre of the Earth and that these particles and the Earth are gravitationally fixed together, accordingly to Newton's universal gravitation. I think the tidal producing force provides these particles just a tendency of being displaced away from the centre of the Earth. In fact, in another work of the attractive mechanism the effect of the tide producing force is also considered a tendency [4]. A tendency means an event may or may not occur. This is determined by a physical condition. For instance, if we use force to push a building. Under the effect of this force the building has a tendency to displace. But whether the building really moves away is determined by whether this force is greater than the ground's friction acting on the building. Also, if we use force to lift an object, the force we exert on the object is necessarily greater than the Earth's gravitational attraction on the object. Go back to the event of the Moon's gravitational attraction on these particles at both $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$, to make these particles really displace away from the centre of the Earth, at least, the tidal producing force should be greater than the Earth's gravitational attraction on these particles. But according to the established parameters,
the tidal producing force gives these particles an acceleration of about $11.2 \times 10^{-8} \mathrm{~g}$, largely less than the acceleration $g$ that the Earth's gravitational attraction gives. Thus, the particles at both $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ cannot be displaced away from the centre of the Earth. In addition to those, from the view of an earthly observer, the asserted displacement of these particles is indistinguishable. This is because that the observer does not lie at the centre of Earth and also not lie outside the Earth, usually, he stands at/near seaside watching sea water, and that the tidal producing force also displaces the observer as it displaces sea water, physical knowledge shows that, within an identical reference frame, the observer cannot discern the motion of these particles.
In fact, many people had found the problem of attractive mechanism we demonstrated above. As the Moon's attraction on a particle of the Earth may be further decomposed into vertical component and horizontal (tangential) component, because of the disability of the vertical component fighting against the Earth's attraction on the particle, the horizontal component is used to drive the particle. In consideration of a globally water covering, the driven water particles will tend to pile up towards the line of the Earth and the Moon, two symmetric bulges are formed. This is the final core of the attractive mechanism that is currently accepted by scientific community. In the following section we further demonstrate why the two bulges are still infeasible.

## Expectation

The attractive mechanism finally results in two bulges and asserts that the effect of the Moon's declination is to produce an asymmetry between the two high and the two low water levels as a site rotates on the surface of solid Earth within the two bulges. Namely, a site at $P$ is experiencing a much higher water level than it will experience about 12 hours later when the Earth's rotation brought it to $P^{\prime}$ (Figure 2). This is absolutely not the matter, because the two bulges further generate an equality of two low water levels when they generate the inequality of the two high water levels. For example, we conclude that, for a globally water covering, the two bugles are in the line of the Earth and the Moon, the middle of the two bulges will form a belt (marked with dashed lines) of the lowest water level, the site at $P$ will inevitably pass through this belt at $P_{1}$ and $P_{2}$ per day as the Earth rotates about its axis, two low water levels of same size are eventually formed. From a view of the globe, other sites ( $N$ and $Q$, for instance) at different latitudes also will pass through this belt of the lowest water level (at $N_{1}, N_{2}$, and $Q_{1}, Q_{2}$ ) to form two low water levels of same size per day. On the whole, the attractive mechanism will request the high water levels of these sites to be different in size but the low water levels to be same in size. This is evidently against the observed tides.


Figure 2. Showing how the unequal semidiurnal tides are determined under the attractive mechanism. Dashed line represents the lowest water level out to the two bugles. $O$ is the Earth's centre. Note the Earth's deformation is exaggerated.

Refer to Pugh's work [1], the attractive mechanism deduces a formula for the equilibrium model to describe the elevation of the sea surface because of the Moon's gravitational pull.

$$
\begin{aligned}
& H_{\mathrm{m}}=a\left(M_{\mathrm{m}} / M_{\mathrm{e}}\right)\left[C_{0}(t)\left(3 \sin ^{2} \varphi_{\mathrm{p}} / 2-1 / 2\right)+C_{1}(t) \sin 2 \varphi_{\mathrm{p}}+C_{2}(t) \cos ^{2} \varphi_{\mathrm{p}}\right] \\
& C_{0}(t)=\left(a / R_{\mathrm{m}}\right)^{3}\left(3 \sin ^{2} d / 2-1 / 2\right) \\
& C_{1}(t)=\left(a / R_{\mathrm{m}}\right)^{3}\left(3 \sin 2 d \cos C_{\mathrm{p}} / 4\right) \\
& C_{2}(t)=\left(a / R_{\mathrm{m}}\right)^{3}\left(3 \cos ^{2} d \cos 2 C_{\mathrm{p}} / 4\right)
\end{aligned}
$$

where $H_{\mathrm{m}}, a, M_{\mathrm{m}}, M_{\mathrm{e}}, \varphi_{\mathrm{p}}, R_{\mathrm{m}}, d$, and $C_{\mathrm{p}}$ are respectively the elevation of the sea surface, the Earth's radius, the Moon's mass, the Earth's mass, the latitude of a particle at the sea surface, the distance of the Earth and Moon, the declination of the Moon, and the hour angle of the particle.
Replace $M_{\mathrm{m}}$ and $R_{\mathrm{m}}$ respectively with $M_{\mathrm{s}}$ (the Sun's mass) and $R_{\mathrm{s}}$ (the distance of the Earth and Sun), the elevation of the sea surface $H_{\mathrm{s}}$ due to the effect of the Sun's gravitational pull may be obtained. $H_{\mathrm{m}}+H_{\mathrm{s}}$ thus represents the total elevation of the sea surface because of the combination of the Moon and the Sun. Figure 3 compares the expected tides based on equilibrium model with the observed tides. It can be found that there is a significant discrepancy of morphology between them. The equilibrium mode requests the tide to be asymmetric. This point includes two aspects: 1) the tide amplitude variation between two successive high tides is greater than that between two successive low tides; and 2) two successive high tides are reversely developed, namely, the size of one high tide is increased (decreased) whereas the size of another is decreased (increased). This may be concluded from Figure 2. We suppose, the latitude of site $N(Q)$ is always greater than the Moon's declination $18.3^{\circ}\left(-18.3^{\circ}\right)$, the water level at $N^{\prime}\left(Q^{\prime}\right)$ is increased (decreased) whereas the water level at $N$ $(Q)$ is decreased (increased) whereas the bulges turns toward pole. Inversely, the water level at $N^{\prime}\left(Q^{\prime}\right)$ is decreased (increased) while the water level at $N(Q)$ is increased (decreased) as the bulges turns toward equator. Contrary to this, the observed tides perform highly symmetric.


Figure 3. A morphological comparison of the expected tide from the equilibrium mode (left) and the observed tides (right) out to lunar declination. The tidal data is from GLOSS database University of Hawaii Sea Level Center.

On the whole, the structure of the two bulges, as deduced from the horizontal (tangential) component, will lead diurnal tides to occur dominantly in higher latitude regions and semidiurnal tides to dominantly occur in lower latitude regions. We examined thousands of observed tides around the globe and found no evidence to support this expectation. The distribution of the semidiurnal and diurnal tides, as shown in Figure 4, is nearly random .


Figure 4. Distribution of tidal patterns around the globe. Data is from U.S. NOAA.
The arguments above mean that the attractive mechanism needs to be given up. However, various evidence shows a strong coupling of the phase of the Moon and Sun and the tide. So, what's the matter about it?

## 2 An analytic treatment of the deformed Earth and the resulting tide

The Earth is an oblate spheroid that is widely thought to be from a centrifugal effect of the Earth's rotation about its axis. This suggests that the Earth may be deformed by a centrifugal effect. It is already known that the Earth orbits about the centre of mass of the Earth-Moon system and the Earth-Moon system orbits about the Sun, these two curved movements generate two centrifugal effects $F_{1}$ and $F_{2}$ for the Earth (Figure 5(A)). If we define the centrifugal effect of the Earth's rotation about its axis as $F_{3}$, then the ratio between $F_{1}, F_{2}$, and $F_{3}$ will be 1:178:505 based on established parameters. Practically, $F_{2}$ is far larger than $F_{1}$, but the working point of $F_{2}$ (the barycenter of the Earth-Moon system) is not at the Earth's centre, compared to the working point of $F_{1}$ (the barycenter of the Earth) that is just at the Earth's centre. Even so, the centrifugal effect $F_{2}$ still will contribute a component $F_{2}|\cos \theta|$ directed to the line of the Earth-Moon system. In consideration of the fact that the working point of $F_{2}$ is not at the Earth's centre, we suppose the effective part of $F_{2}$, which has able to stretch the Earth, is relatively small. The tidal range of Betio at the time of new Moon or full Moon is about 6 times the tidal range at the time of first quarter or first quarter, the tidal range of Diego Garcia at the time of new Moon or full Moon is about 6.5 times the tidal range at the time of first quarter or first quarter. This ratio may reach 4.0 times at Ecuador, 5.6 time at Mombasa, and 7.5 times at Marshall Islands. From the view of a compromise, we assume the effective part of $F_{2}$ to be $4 F_{1}$. Therefore, the combined centrifugal effect that is in the line of the Earth-Moon system may be expressed as $F=F_{1}+4 F_{1}|\cos \theta|$, where $\theta$ is the angle between the Moon and the Sun relative to the barycenter of the Earth-Moon system and may be approximately represented by the angle between the Moon and the Sun relative to the Earth. Here we suppose, this combined centrifugal effect may generate a component $\left(F_{1}+4 F_{1}|\cos \theta|\right) \cos \alpha$ towards the direction of equator to disturb the centrifugal effect of the Earth's rotation about its axis, the oblate spheroid is therefore deformed (Figure 5(B)).


Figure 5. Combined centrifugal effects for solid Earth and the resulting extension. $F_{1}$ and $F_{2}$ are the centrifugal effects that solid Earth undergoes because of curved motions around the centre of mass of the Earth-Moon system and around the Sun. $O_{1}, O_{2}, M$, and $S$ are the Earth's centre, the barycenter of the Earth-Moon system, the Moon, and the Sun, respectively. $\theta$ is the angle between the Moon and the Sun relative to the barycenter of the Earth-Moon system. $v_{1}$ and $v_{2}$ are respectively the velocity of the Earth orbiting the barycenter of the Earth-Moon system and the velocity of the Earth-Moon system orbiting the Sun, which generate the centrifugal effects $F_{1}$ and $F_{2}$.

About $71 \%$ of the Earth's surface is covered with water, this makes the Earth look like being submerged in the water. Because of the Earth's daily rotation, the deformed Earth makes its each part regularly rise from and fall into the water, this results in the fall and rise of water level. To better run the following deduction, we consider a global water covering of same depth and cut the oblate spheroid to form two sections $\mathrm{A}_{0} \mathrm{E}_{0} \mathrm{C}_{0} \mathrm{~F}_{0}$ (including section AECF), which is the equatorial plane, and $\mathrm{H}_{0} \mathrm{~B}_{0} \mathrm{G}_{0} \mathrm{D}_{0}$ (including section HBGD), the section HBGD passes through a site $M$ that is at the surface of the oblate spheroid. We use the generated component $\left(F_{1}+4 F_{1}|\cos \theta|\right) \cos \alpha$ to stretch the oblate spheroid along the line $\mathrm{AC}\left(\mathrm{A}_{0} \mathrm{C}_{0}\right)$. The two sections $\mathrm{A}_{0} \mathrm{E}_{0} \mathrm{C}_{0} \mathrm{~F}_{0}$ and $\mathrm{H}_{0} \mathrm{~B}_{0} \mathrm{G}_{0} \mathrm{D}_{0}$ then become sections $\mathrm{A}_{0}{ }^{\prime} \mathrm{E}_{0}{ }^{\prime} \mathrm{C}_{0}{ }^{\prime} \mathrm{F}_{0}{ }^{\prime}$ and $\mathrm{H}_{0}{ }^{\prime} \mathrm{B}_{0} \mathrm{G}_{0}{ }^{\prime} \mathrm{D}_{0}$, the sections AECF and HBGD then become sections $A^{\prime} E C{ }^{\prime} F$ and $H^{\prime} B G^{\prime} D$, site $M$ also turn to the site M' (Figure 6).


Figure 6. A global water covering the deformed Earth. I: an equatorial section of the Earth. Black dashed line is the original water level while black real line is the water level after the oblate spheroid is extended in the line AC. Purple dashed line presents the Earth's deformation due to the centrifugal effect while purple real line presents the original shape. Ellipse A'EC'F represents the Earth's shape in the direction of equator. II: a longitudinal section of the Earth that passes through polar axis, which represents a perspective view of the oblate spheroid. $O_{1}$ is the Earth's centre.

These adjustments yield a globally water reallocation that occurs dominantly in the direction of line $E_{0}{ }^{\prime} \mathrm{F}_{0}$ '. This is because the Earth's extension in the direction of line AC makes water
gravitationally flow towards the direction of line $\mathrm{E}_{0}{ }^{\prime} \mathrm{F}_{0}{ }^{\prime}$, in addition, the Earth's extension is rolled from east to west due to the Earth's rotation, this makes water reallocation in the direction of line $\mathrm{B}_{0} \mathrm{D}_{0}$ become somewhat slight. With the Earth's rotation around its axis, site $M$ (if not polar) will continuously pass through these regions of different water depth. We suppose, another site P of water surface turns to site P ' when water reallocation takes place and that both site P and site M are with same latitude and longitude. According to the geometry of ellipse, the water depth of site $M$ may be expressed as

$$
\begin{align*}
& \mathrm{Q}=\mathrm{P}^{\prime} O_{1}-\mathrm{M}^{\prime} O_{1}  \tag{1}\\
& \mathrm{P}^{\prime} O_{1}=\left[\mathrm{H}_{0} O_{1}^{2} \cos ^{2} \gamma+\mathrm{B}_{0} O_{1}^{2} \sin ^{2} \gamma\right]^{1 / 2}  \tag{2}\\
& \mathrm{M}^{\prime} O_{1}=\left[\mathrm{H}^{\prime} O_{1}^{2} \cos ^{2} \gamma+\mathrm{B} O_{1}^{2} \sin ^{2} \gamma\right]^{1 / 2}
\end{align*}
$$

Where
$\mathrm{H}_{0}{ }^{\prime} \mathrm{O}_{1}$ and $\mathrm{B}_{0} \mathrm{O}_{1}$ are respectively the semi-minor axis and semi-major axis of ellipse $\mathrm{H}_{0}{ }^{\prime} \mathrm{B}_{0} \mathrm{G}_{0}{ }^{\prime} \mathrm{D}_{0}, \mathrm{H}_{0}{ }^{\prime} O_{1}=\mathrm{A}_{0}{ }^{\prime} O_{1}=\mathrm{F}_{1} O_{1}, \mathrm{~A}_{0}{ }^{\prime} O_{1}$ may be further obtained by an area formula $\pi \mathrm{x}$ $\mathrm{A}_{0} O_{1} \times \mathrm{F}_{0} O_{1}-\pi \times \mathrm{AO}_{1} \times \mathrm{FO}_{1}=\pi \times \mathrm{A}_{0}{ }^{\prime} O_{1} \times \mathrm{F}_{0}{ }^{\prime} O_{1}-\pi \times \mathrm{A}^{\prime} O_{1} \times \mathrm{FO}_{1}, \mathrm{~A}_{0} O_{1}=\mathrm{F}_{0} O_{1}$, $\mathrm{A}_{0} O_{1}=\mathrm{A} O_{1}+h, \mathrm{~F}_{0} O_{1}=\mathrm{F} O_{1}+h, \mathrm{~B}_{0} O_{1}=\mathrm{BO}_{1}+h, \mathrm{~A}^{\prime} O_{1}=\mathrm{A} O_{1}+k$, where $h$ is the mean water depth of global water, and $k$ is the extension of the oblate spheroid in the line $A C$ that is due to the combined centrifugal effect, $\beta$ is the angle of section ABCD and HBGD , which is equal to the difference in longitude between the Moon and site M .
$\mathrm{H}^{\prime} O_{1}$ and $\mathrm{BO}_{1}$ are respectively the semi-major axis and semi-minor axis of ellipse $H^{\prime} \mathrm{BG}^{\prime} \mathrm{D}, \mathrm{H}^{\prime} O_{1}$ may be obtained by a formula $\mathrm{H}^{\prime} O_{1}=\left[\mathrm{A}^{\prime} O_{1}{ }^{2} \cos ^{2} \beta+\mathrm{FO}_{1}{ }^{2} \sin ^{2} \beta\right]^{1 / 2}$;
$\gamma$ is the angle of line $\mathrm{MO}_{1}$ and equatorial plane AECF , which is equal to the latitude of site M;
We further suppose, an island that stands on the position of site $M$ to protrude the water surface, and then the water level variation of a site at the shore of this island may be represented by the water depth variation of site M , which is $\mathrm{Q}-h$. The related parameters are considered as follows: the equatorial and polar radius of the Earth are respectively 6378.00 km and 6357.00 km , the difference between them is $21.00 \mathrm{~km}[5,6]$. The ratio of centrifugal effect $F_{3} / F_{1}$ is $505: 1$. A rough estimation based on this value is the centrifugal effect $F_{1}$ may generate an extension of about 42.00 m . However, the Earth's oblate spheroid is likely to be resulted from an accumulating effect of Earth's rotation around its axis during a time scale of billions of years, thus, the centrifugal effect $F_{1}$ at instant could give rise to only a slight extension. Here we assumed the extension due to the centrifugal effect $F_{1}$ at instant to be 0.10 m when the Moon's declination is zero, the resulting extension $k$ in the line AC may be expressed by formula $k=(0.1+4 \times 0.1|\cos \theta|) \cos \alpha$, where $\alpha$ is the Moon's declination. And therefore, within these ellipses (as shown in Figure 6) there will be $\mathrm{AO}_{1}=\mathrm{FO}=6378.00$ $\mathrm{km}, \mathrm{BO}=6357.00 \mathrm{~km}$. We assume $h$ to be the reference water depth and value it as 3.60 km . In consideration of the fact that the oceans cover approximately $71 \%$ of Earth's surface, the total water volume is estimated to be $1.32 \times 10^{9} \mathrm{~km}^{3}$. This amount approaches the established threshold $1.34 \times 10^{9} \mathrm{~km}^{3}$ [7]. A site $\left(30^{\circ} \mathrm{N}\right)$ is selected to run the simulation (Figure 7). It can be found that the variation of water depth at site M (representing the variation of water level) is semidiurnal in a timely manner, and within one lunar month the variation experiences two cycles. The high water and the low water are symmetrically developed. The highest high water (the so-called spring tide) occurs at the times of full Moon and new Moon, the lowest low water (the so called neap tide) occurs at first quarter and last quarter. This is because at times of spring tides the combined centrifugal effect $\left(F_{1}+4 F_{1}|\cos \theta|\right)$ in the line of the Earth-Moon system becomes maximal, and thus in the direction of equator the resulting centrifugal effect $\left(F_{1}+4 F_{1}|\cos \theta|\right) \cos \alpha$ becomes maximal, and further the deformation of the oblate spheroid is the most serious, but at times of neap tides the combined centrifugal effect in the line of the Earth-Moon system becomes minimal, and further the deformation is the slightest. As $\alpha$, the Moon's declination, is usually between $-18.3^{\circ}$ and $18.3^{\circ}$, this makes the Moon's individual contribution in deforming the oblate spheroid not important.


Figure 7. The expected tide out to the phase of the Moon and Sun. Time span is from 2014-08-01 00:00:00 to 2014-08-30 23:00:00. The lunar and solar ephemeris are from JPL HORIZONS system.

Let's consider the influences of these factors such as the distribution of continent, topography, ocean width, and so on. The continents and oceans are randomly distributed around the globe. We cut the Earth along equator to form a section. This section ideally includes two continents and two oceans, and within one of the continents there is an enclosed sea/lake. Because of the effect of the centrifugal effect, the Earth is extended in the direction of line AC (refer to Figure 8(left)). The elevation in the direction of line AC leads local water to gravitationally flow towards the direction of line EF. With the Earth's rotation about point $O_{1}$, these sites ( $a$, $b, c, d, g$, for instance) located at the shores of continents and islands will continuously pass through the deep water in the direction of line EF and the shallow water in the direction of line AC. Two high waters and two low waters are therefore experienced by them per day. It should be noted that, even if ocean one is short, latitudinally less than $90^{\circ}$, with the Earth's rotation, the oscillation of ocean bed $a^{\prime} b$ ' still makes water flow back and forth between site $a$ and site $b$. The high and low waters are accordingly generated for these two sites. This mechanism also adapt to the matter of enclosed sea/lake. The amplitude of high (low) water at one end of the sea/lake is determined by the difference of the rise (fall) of another end and the fall (rise) of this end. If we select two sites respectively from the east end and from the west end of Black sea and use the formula (2) above to estimate, Black sea will hold a tide of about 6.0 cm . Similarly, a tube of water ( 20 m in length) horizontally located at equator will experience a tide of about $9.3 \times 10^{-4} \mathrm{~mm}$, an imperceptible amount. This means that, any vessel, such as swimming pool, water cup, water bowl, and so on, because its size is too short, will not experience an perceptible tide. Most of continental shores are bent, they are either convex or concave. These features may largely boost the amplitudes of tides. We cut a little part from the equatorial section to form a plane area (Figure 8(right)). At the area the Earth's extension in the direction of line AC makes water flow towards continent one if site $c$ (representing the shore of continent one ) moves to the direction of line EF. Suppose the extension is $h_{1}$, from
the viewpoint of energy conservation, the expelled water will then hold a movement of $v_{1}=\left(2 g h_{1}\right)^{1 / 2}$. If there is no loss of energy and the shore is vertical to the movement of water, the water will accumulate at shore $c$ with an amplitude of $h_{1}$ on the assumption that shore $c$ is straightforward. As shore $c$ is often concave, this creates an effect of narrow to enlarge the speed of water. This effect of narrow fits to a simple relation: $v_{1} L_{1}=v_{2} L_{2}$, where $L_{1}$ and $L_{2}$ are respectively the width of section $c_{1} C_{2}$ and section $c_{1}{ }^{\prime} C_{2}$ ', namely, the flow velocity is inversely proportional to the width of passage. Thus, the water of velocity $v_{2}$, if accumulated at section $c_{1}{ }^{\prime} c_{2}^{\prime}$, may theoretically form a water of amplitude $h_{2}=\left(L_{1} / L_{2}\right)^{2} h_{1}$. In contract, the shores of those islands (represented by $d$ ) that are isolated in the deep oceans are too short and not vertical to the movement of water. The expelled water cannot be effectively accumulated and may bypass. This determines larger tides to occur at the shores of coastal seas and smaller tides to occur at the shores of the islands in the deep oceans.


Figure 8. A realistic distribution of global water and solid Earth. Left: an equatorial section of the Earth. Black dashed line is the water level after the Earth is extended in the line AC while black real line the original water level. Green dashed line represents the deformation of the Earth due to the centrifugal effect. Black arrows represent the Earth's rotation around $O_{1}$ (the Earth's centre); Right: an interaction of moving water and uneven shore.

There are some shores that are concave, nearly semi-enclosed, but with openings connecting to the oceans. These regions include the Mexico Gulf, the Persian Gulf, the Karumba Gulf, and so on. The east end and west end of these areas may oscillate as the Earth is continuously deformed from east to west, at the same time the north end and south end of these areas may also oscillate because the extension of the Earth is gradually decreased from equator to pole. These may give rise to respectively a latitudinally water movement and a longitudinally water movement. As these openings are connected to the oceans, the water within these areas will be greatly disturbed by the water that is from the oceans, the variation of water level therefore becomes irregular, this may generate the so-called diurnal or mixed tide.

## 3 Discussion

The Earth's deformation results in mainly an oscillation of ocean water. The driven water in travel, if constrained by the narrowness of strait, may form swift current, like that in the Cook Strait. For the various features of the tides in the Atlantic and in the North West Europe shelf seas, as the Earth is progressively stretched in a manner of from east to west, the Earth's rising part around West Africa firstly forces the water of East Atlantic to flow towards north, west, and south. With the passage of time, the Earth's extension moves to middle Atlantic and continues to force water to flow towards east, north, west, and south. Finally, the extension moves to West Atlantic and further forces water to flow towards north, east, and south. The westerly water may reach the eastern coastline of America nearly at the same time and leaves no difference of tidal phase. The tides from Florida to Nova Scotia are the case. The northerly water may form a progression of tidal phases in the North Atlantic. In particular, a large body
of northeasterly water may enter the strait of Gibraltar and cross the Celtic Sea, from where it continues to run into the English Channel and other related regions. A series of progressive tides along the shores of these regions are determined. On the other hand, the southerly water may be depressed by the import of the water from the southern ocean and becomes less observable. As the extended Earth progressively rolls water to flow from east to west. This generates a generally westward propagation of the tides.
Galileo in his Dialogue Concerning the Two Chief World Systems described the tides in the Mediterranean (translated by Stillman Drake): "three varieties of these hourly changes are observed: in some places the waters rise a fall without making any forward motions; in others, without rising or falling they move now toward the east and again run back toward the west; and in still others, the height and the course both vary. This occurs here in Venice, where the waters rise in entering and fall in departing. ......, elsewhere the water runs to and fro in its central parts without changing height, as happens notably in the Straits of Messina between Scylla and Charybdis, where the currents are very swift because of the narrowness of the channel. But in the open Mediterranean and around its islands, such as the Balearics, Corsica, Sardinia, Elba, Sicily (on the African side), Malta, Crete, etc., the alterations of height are very small but the currents are quite noticeable, especially where the sea is restrained between islands, or between these and the continent." We at first consider the water movement of an oscillating vessel. As shown in Figure 9, let the right side of water box rise, the water of the right side flows towards the left side. If line MN is a reference level, the water level of site M rises while the water level of site N falls. And then we restore the right side to its former level and let the left side rise, the water of the left side flows towards the right side, the water level of site M falls while the water level of site N rises. Repeat the rise and fall of these two ends continuously, the water level of sites M and N accordingly vary. Compared to sites $M$ and $N$, another site $S$, which is in the middle of the vessel, holds the minimal variation of water level. Here we conclude that, although the oscillating ocean water can hardly exert influence to the Mediterranean since the latter connects to the Atlantic by means of a narrow Gibraltar Strait, the regularly Earth's deformation still may give rise to an oscillation for the sea bed of the Mediterranean, which looks like a big vessel. As a result, the greatest alternation of water level occurs between the two ends, while the smallest does in the open area. The oscillating water, when being constrained by the straits, forms swift current, like that in the open Mediterranean and around its islands.


Figure 9. Modelling the water movement of an oscillating vessel. Line MN represents the reference level.

Most people could be confused because the attractive mechanism is thought to be competent for the tidal prediction. Refer to Pugh's work [1], one may see that the attractive mechanism provides a reason why the Earth undergoes daily water movements of rise and fall, whereas tidal prediction is based on a tidal analysis method that is fully experienced. The prevailing tidal analysis method is the harmonic method, considering tide as the sum of a finite number of harmonic constituents, $H_{\mathrm{n}} \cos \left(\sigma_{\mathrm{n}} t-g_{\mathrm{n}}\right)$, a symmetrically cosine function. The attractive mechanism, however, as we demonstrated in section 2 , inevitably generates an asymmetric result that is against the observed tide. But by means of a series of mathematical transformations, the so-called expansion, this discrepancy is blurred. The major problem of the attractive mechanism is it develops a great deal of mathematics and supporting scenarios (standing wave and resonance, for instance) that are highly complicated enough to prevent general people making further investigation. This could be the reason why many people continue to consider tidal analysis and prediction as a black art. On the other hand, as
demonstrated in section 2, if don't examine the evolution of high and low water, we cannot find the problem of two bugles of the attractive mechanism. In fact, Pugh had pointed out [1], "The Equilibrium Tide bears no spatial resemblance to the real observed ocean tide". Only a few know this, most of people are likely to be unclear.
Two methods are provided to differentiate this theory from the attractive mechanism.
A) observe the weight variation of an object. This theory requests the Earth to be extended along the direction of equator. For an object that stands at the Earth's surface (neither at equator nor at pole), with the Earth's rotation around its axis, the object will pass through two high altitudes of same size and two low altitude of same size per day, its weight will change as follows: given the Moon's declination is not zero, it is lighter when hour angle of the Moon is $0^{\circ}$ or $180^{\circ}$ than when hour angle is $90^{\circ}$ or $270^{\circ}$, and it is same when hour angel is $0^{\circ}\left(90^{\circ}\right)$ as when hour angle is $180^{\circ}\left(270^{\circ}\right)$. In contrast, the attractive mechanism requests the Earth to be extended along the direction of the Earth-Moon system, the object will pass through two high altitudes of different size and two low altitude of same size per day as the Earth revolves about its axis, its weight will change as follows: it is lightest when hour angle of the Moon is $0^{\circ}$ but weighter when hour angle of the Moon is $90^{\circ}$ or $270^{\circ}$.
B) observe the movement of a pendulum. This theory requests the Earth to be extended along the direction of equator. For a pendulum standing at the north hemisphere, with the Earth's rotation around its axis, given the Moon's declination is not zero, the pendulum per day will turn to north pole when hour angle of the Moon is $0^{\circ}$ or $180^{\circ}$ and return when hour angle of the Moon is $90^{\circ}$ or $270^{\circ}$. The amplitude of the pendulum is the same when angle hour is $0^{\circ}$ and $180^{\circ}$. In contrast, the attractive mechanism requests the amplitude of the pendulum to be different when angle hour is $0^{\circ}$ and $180^{\circ}$. Note the string of the pendulum should be long enough.
Any of the two above may be used to value the deformation of the oblate spheroid.
Acknowledgements I am honestly pleased to thank University of Hawaii Sea Level Center and NASA's JPL, and express great thanks to Mr. Walter Babin and Mr. Thierry De Mees for suggestive discussion.

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