Abstract. The numerical value of \( G \) has been derived in terms of electron substructure and the Coulombic field, by using action principles. Theoretical values are within experimental error:
\[
G = 6.673 \, 7846 \times 10^{-11} \, \text{m}^3\text{kg}^{-1}\text{s}^{-2}, \quad \text{and} \quad (e / m)^2 / G = 4.165 \, 9308 \times 10^42.
\]

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1. Introduction

Until now, the gravitational constant \( G \) has not been related to the corresponding electromagnetic constant, nor has Einstein’s theory of geometrical space-time curvature with its black-hole singularities been unified with the standard model of quantum theory and its quark/electron singularities. However, Einstein’s equations of general relativity can be solved in terms of real energetic gravitons which preclude the concept of space-time curvature and singularities, see Wayte, [1], [2]. In addition, particles can be described as having real structure to eliminate singularities in quantum theory, see Wayte, [3], [4], [5], [6], [7]. Then gravitation can be related to electromagnetism through the properties of realistic particles and quanta. Other investigators have derived alternative expressions for \( G \); see Brandenburg [8], and Casey [9].
Here in this work, gravitons are regarded as independent quanta, separate from the electromagnetic Coulomb field quanta; so gravity is not an aspect of that field, though gravitons can behave in a similar way, see Wayte, [10, Section 3.3]. If every particle emits a number of identical gravitons in proportion to its mass, then the radial flux density of gravitons emitted by a spherically symmetric mass will decrease as an inverse square law with distance to produce Newton’s law of force; and $G$ will be a universal constant. A graviton also has an energy density along its length decreasing as $r^{-2}$, in order to produce an overall gravitational field energy density around the mass decreasing as $r^{-4}$. Gravitons travel at the velocity of light, in agreement with Einstein’s theory, and appear to have spin 2. They are electromagnetic, possessing energy and momentum, to be compatible with their particle sources. Gravitons from one particle interact with the field of another particle to produce the gravitational force by momentum exchange; they do not sink into other particles, as sometimes depicted resembling iron filings around magnets, [1, Section 7].

2. Calculation of $G$

The electric force between two electrons compared with the gravitational force may be calculated as:

$$\left(\frac{e^2}{G m^2}\right) = \frac{E}{G} = 4.16589(50) \times 10^{42},$$

where the latest measured values are taken from http://physics.nist.gov/constants:

$$e/m = \sqrt{E} = -1.758 \, 820 \, 088(39) \times 10^{11} \, \text{C/kg}^{-1}, \quad G = 6.673 \, 84(80) \times 10^{-11} \, \text{m}^3 \text{kg}^{-1} \text{s}^{-2}.$$  

This huge ratio can be explained in terms of particle properties, by continuing an earlier investigation into the accelerating universal expansion, see Wayte, [11]. Thus it will be assumed arbitrarily that the electron’s Coulomb field is restricted in range because the energy originally beyond radius $r_m$ has been transformed into the electron’s gravitational field. This is given by integrating the classical energy density from $r_m$ to infinity:

$$\int_{r_m}^{\infty} \frac{1}{8\pi} \left(\frac{e^2}{r^4}\right) \times 4\pi r^2 \, dr = \frac{1}{2} \frac{e^2}{r_m^2}. \quad (2.2)$$
The electron’s total gravitational energy, emitted from an effective internal source radius \( r_s \), is determined by integrating energy density:

\[
\int_{r_s}^{r_m} \frac{1}{8\pi} \left( \frac{Gm^2}{r^4} \right) \times 4\pi r^2 dr \approx \frac{1}{2} \frac{Gm^2}{r_s}. \tag{2.3}
\]

Consequently, by equating Eqs.(2.2) and (2.3), the ratio in Eq.(2.1) may be expressed as a unique range \( (r_m/r_s = E/G) \), for the electron’s Coulomb and gravitational field quanta.

Previously, such large ratios have been interpreted in terms of an action principle operating within particle mechanisms, [6] [7]. So the aim is to understand this in terms of electron and proton substructure, with regard to the Coulomb field action from source \( r_s \) to range \( r_m \). Based on the electron paper [3], we will propose:

\[
\ln(r_m/r_s) = \ln(E/G) \approx 2\pi \ln(137 \times 37.7 \times 24 \times 50) \approx 2\pi \ln(\xi), \tag{2.4}
\]

where the fine structure constant is \( (\alpha = 1/137.035 \ 999 \ 074) \), the pearl structure constant is defined as \( (\delta = 1/12\pi \approx 1/37.7) \), grain structure constant \( (\epsilon = 1/24) \), and mite structure constant \( (\mu = 1/16\pi \approx 1/50) \). The number of fundamental charge elements in an electron core-segment accounts for the factor \( \xi = (137\times37.7\times24\times50) \). In a proton [5], Eq.(2.4) also holds for the number of elements in a trineon gluonic-loop \( (24\times37.7\times137\times50) \).

After differentiating and introducing the basic electron expression \( (e^2/c = mcr_o) \) and charge for one core-segment \( (e/137) \), we get a double integral representing action in the Coulomb field:

\[
\int_{r_s}^{r_m} \frac{(e/137)^2}{c} dr \approx 2\pi \int_0^{\xi O_4} \left( \int_0^{O_4} \frac{(e/137)^2}{c} dx \right) d\theta. \tag{2.5a}
\]

Here on the right, the inner integral represents potential energy action for the elements constituting one core-segment, \( (O_4 \text{ is the circumference of an element, } [3]) \). After integration, this term acts as a weighting factor for equivalent kinetic energy action:

\[
\int_{r_s}^{r_m} \frac{(e/137)^2}{r} dt \approx 2\pi \ln(\xi) \times \int_0^{m \cr_o} \frac{m}{(2\times137^2)} dr_o d\theta. \tag{2.5b}
\]

Ultimately, the integral on the left represents potential energy action of the emitted exterior Coulomb field from one core-segment, over its complete extent to radius \( r_m \).

This range for the electron has an extraordinary value of \([((E/G)(2\pi r_o) = 7.376\times10^{28}m = 7796Glyr] \), and for the proton it is 38.21Glyr, [11].
Given this approximate analysis, it is possible to gain more understanding of the Coulomb field emission mechanism by refining Eq.(2.4):

\[
\left( \frac{e^2}{c^2} \right) \times \ln \left( \frac{E}{G} \right) \approx \left[ \frac{m_{137}^2}{c^2} c r_0 \right] \times \left[ \frac{1}{1 + (1/2 \pi e_n)^2} \right]^{1/2} \times 2\pi \ln(\xi) \ . 
\] (2.6)

This expression yields a much improved value of G, within 0.15% of that measured, so the new quotient should represent an essential part of the emission mechanism. If the curly bracket has the appearance of \{(angular momentum) x \cos \phi \}, then there could exist an associated orthogonal expression for \{(angular momentum) x \sin \phi \}:

\[
\left( \frac{e^2}{c^2} \right) \times \left[ \frac{\pi}{e_n} \ln(137) \right] \approx \left[ \frac{m_{137}^2}{c^2} c r_0 \right] \times \left[ \frac{(1/2 \pi e_n)}{1 + (1/2 \pi e_n)^2} \right]^{1/2} \times 2\pi \ln(\xi) \ . 
\] (2.7)

The new expression on the left side may be differentiated and interpreted as potential energy action within a core-segment of an electron spin-loop, (or the gluonic-loops in a proton-trineon):

\[
\frac{\pi}{e_n} \times \int_0^{137} \frac{(e/137)^2}{z} \frac{dt}{z} = \left( \frac{e/137}{c} \right)^2 \frac{\pi}{e_n} \ln(137) \ . 
\] (2.8)

This is clearly an important part of the emission process for the Coulomb field, in spite of its relatively small magnitude \((1/2\pi e_n \approx 0.059)\).

The choice of this factor \((1/2\pi e_n)\) is connected with the optimum emission/transmission of quanta inside the electron. Another paper (Wayte, [12, Section 4] ) showed how material in large galaxies tends to orbit at 201kms\(^{-1}\) because its orbital gravitational guidewave emission is then matched to the impedance of free space \(Z_0\) and its related velocity \((V_0 = c/4\pi)\). Thus here, if the angular momentum term in Eq.(2.6) is proportional to velocity \(c\), then that in Eq.(2.7) is proportional to \((V_{\sin} = c/2\pi e_n)\).

Consequently, \((V_{\sin} /V_0 = 2/e_0)\) is equivalent to an impedance ratio \((Z_0/Z_{\sin} = 2/e_0)\), which corresponds very well with good impedance matching in waveguides and exponential horn profiles, (Glazier & Lamont [13, pp167, 176, 193]).
It is now possible to refine Eq.(2.6) further, to get a satisfactory fit to the empirical value of \((E/G)\) in Eq.(2.1):

\[
\left(\frac{e^2}{c 137^2}\right) \ln \left(\frac{E}{G}\right) \approx \left\{ \frac{m}{137^2 e_0} \times \frac{1}{\left[1 + \left\{\frac{1}{2} / 137\pi\right\}/2\pi e_n^2\right]^{1/2}} \right\} \times 2\pi \ln(\xi). \tag{2.9}
\]

Calculated values from this expression are now within experimental error:

\((E/G = 4.165 \times 10^{42})\), and \((G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2})\). \hspace{1cm} \tag{2.10}

The extra term in the denominator on the right of Eq.(2.9) has an interesting interpretation because the purpose of it is to reduce velocity \(V_{\text{sin}}\) slightly:

\[
\frac{V_{\text{sin}}}{V_0} = \left( 1 - \frac{2}{137\pi} \right) e_n/2 = \left[ \frac{2}{e_n} \right] \left[ 1 + \left( \frac{2}{137\pi} \right) + \left( \frac{2}{137\pi} \right)^2 + \left( \frac{2}{137\pi} \right)^3 + \cdots \right]. \tag{2.11}
\]

Here, the series format of the square bracket will be interpreted as the sum of a decaying electromagnetic field, wrapped many times around the 137 core-segments of an electron spin-loop, (or the 137 gluonic-loops in a trineon). For each turn around the spin-loop, the electromagnetic current has decreased by factor \((2/137\pi)\). Then, in addition to an external radial Coulombic field quantum, each elemental-charge emits an internal tangential component which decays in strength exponentially as it propagates around the spin-loop many times. This means that the very latest emission is overlapped by weak decaying previous emissions, which increase the total current at that point; and this tends to decrease the wave impedance below the optimum value \(Z_0(e_n/2)\). Consequently, the sole purpose of the \((1 - 2/137\pi)\) term is to restore impedance matching by decreasing velocity to compensate for the increased current. This decaying electromagnetic field can be understood by analogy with a radar echo box, see [13, p243].

The value of the factor \((2/137\pi)\) is explained as follows. Let the quantum current decay with instantaneous distance \(x\) around the spin-loop (of length, \(x_e = 137 \times 2\pi r_o\)) as:

\[
I = I_0 \left( \frac{2}{137\pi} \right)^{x/x_e} = I_0 \exp \left[ \frac{x}{x_e} \ln \left( \frac{2}{137\pi} \right) \right] = I_0 \exp[-5.3718 \left( \frac{x}{x_e} \right)]. \tag{2.12}
\]
This decay is due to energy being continuously radiated away, at a rate proportional to:

\[ \frac{dl}{dx} = \frac{I}{x_e} \ln \left( \frac{137\pi}{2} \right) \tag{2.13} \]

The logarithmic term looks like a weighting factor, as in Eq.(2.5b), resulting from a double integral:

\[ \int_{I_0}^{I} \frac{dl}{I} \approx \ln \left( \frac{I}{I_0} \right) = -\frac{x}{x_e} \ln \left( \frac{137\pi}{2} \right) = -\frac{1}{m_c r_o} \times \left( \int_{0}^{x} \left( \frac{(137\pi/2)}{1} \right) \frac{e^2}{c} \frac{dy}{y} \right) \frac{dx}{x_e} \tag{2.14} \]

Here on the right, the inner integral represents action of potential energy around the pearly circumference of an electron core-segment (or around gluonic-loops in a trineon). The outer integral may then be solved to get Eq.(2.12), for any number of core-segments \((137\times x/x_e)\) around the electron’s spin-loop (or around the proton’s spin-loop).

**Conclusion**

A theoretical value of G has been calculated using action principles in terms of electron properties and the electric field. In a classical sense, a key assumption was made that the electric field and gravity are both finite in range because the originally distant Coulomb energy has been transformed into the entire gravitational field energy. Ratio \(E/G\) could then be interpreted in terms of electromagnetic action in the electric field, with regard to electron substructure to the finest detail. Previous work on astronomical properties of gravity and other work on fundamental particle properties were brought together. According to this analysis, \(E/G\) must be a universal constant for electrons; however, \(G\) varies like \((e/m)^2\), where \(m\) is the relativistic mass. If this were not so, then the deflection of a double-pith-ball electroscope [14] would depend on its velocity, and the experimenter could determine his own velocity from the deflection.

**References**

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