Quaternion Spin 2 Field Theory

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Abstract

In this paper I propose solutions to the nature of Dark matter, Dark energy, Matter and the Matter-Antimatter asymmetry. The real spin representations of a 7d complex space are assumed to be the source of a chiral gauge group SU(8)xU(1) and a spin 2 quaternion field. The integral of the probability density of the spin 2 field results in a lower bound for r and consequently the Schwarzschild physical singularity is non-existent. Fermion mass is bounded by a lower and an upper limit. Cosmology of the universe is cyclic with no past or future singularities and the Cosmological density ratios are in agreement with WMAP 7 year data.

Introduction

Cosmological observations has elucidated the need for dark matter and dark energy to explain the rotation curves of galaxies and the accelerating expansion of the Universe. Within the Λ-CDM model of cosmology, WMAP 7 year data has constrained the ratios of Dark matter, Dark energy, Baryonic matter and has also found support for the Inflation hypothesis. However the inflation hypothesis does have issues of its own **[1]**.
Particle physics seeks explanation for the Matter-Antimatter asymmetry, the origin of the Lie gauge groups of the Standard Model of particle physics and the 3 generations of quarks and leptons. Quantum Field Theory and General Relativity both break down as $r \to 0$.
This paper addresses these issues except Inflation, as work on this is not yet completed.

The real spin representations of a n-dimensional complex vector space **[2]** is the starting point of this theory.

It is assumed that

1 Universe is a structure on a 7d dimensionless complex vector space

2 Spin of the Quaternion field is 2

3 The gauge group is SU(8)xU(1)

4 Quantum Field Theory (QFT) is the framework for the behavior of particles

The odd spin representations and the Quaternion Spinor

A 7d complex vector space has a 8d complex spinor χ
The real spin representations are the vector spaces

$$\{\mathbb{R}^{6,\,1},\ \mathbb{R}^{2,\,5},\ \mathbb{R}^{4,\,3}\} \tag{1}$$

The corresponding odd spin representations are

$$\{SO(3,\mathbb{H}),\ SO(3,\mathbb{H}),\ SO(6,\mathbb{R}) \simeq SU(4)\} \tag{2}$$

In analogy with the chirality of a fermion **[3]** which is defined as the real eigenvalues of $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ define chirality of the vector space $\mathbb{R}^{p,\,q}$ as $e_8 = (i)^p(1)^q$
The real spin representations have chirality $e_8 = \{-1, -1, 1\}$ respectively, hence the reason for the signature of the spaces in (1). It follows that there is only one chiral state -1 for the quaternion structures.
The group $SO(3,\mathbb{H})$ acts on 3d quaternion spinors, $H \equiv H_{h\rho\sigma} = -H_{h\sigma\rho}$ where h = 1, 2, 3 The

normalisation of the quaternion spinor is

$$\int H^{\dagger} H \, \mathrm{d}\Omega = 1 \tag{3}$$

where Ω is the dimensionless volume element of $\mathbb{R}^{p,\,q}$

$H^{\dagger}H$ is the scalar product of $H^{\dagger}$ and H and $H^{\dagger} = e_j^{\dagger} H$ where $e_j^{\dagger}$ is defined as

$$e_j^{\dagger} = \{ e_0,\ e_i : i \in [1, 2, 3] \} \tag{4}$$

The assumption that H is spin 2 implies that the Greek indices $\in$(0,1,2,3) and consequently the metric tensor is therefore 4d. Using the First Fundamental form of differential geometry it follows there exists a 4d space of generalised co-ordinates x_{μ}

The group SO$(6, \mathbb{R})$ is isomorphic to SU(4), and has a 4d complex spinor ζ

Matter-Antimatter Asymmetry

Only the spin s=0 states of H can decay into spin s=1/2 fermions due to the Pauli exclusion principle. In addition, since H is real, it is a strictly neutral spinor **[4]** so spin s=0 states of H cannot decay into fermions and antifermions of s=1/2 hence it follows there are no antifermions if H is the source of matter.

The spin 0 states of H form s=1/2 fermions which can couple to χ the 8d complex spinor to form chiral particle multiplets Ψ or form strictly neutral singlets.

Quaternion Field Lagrangian

For each 3d quaternion spinor, the Lagrangian is assumed to be

$$\mathcal{L}_i = \mathrm{D}^{\mu} H_i^{\dagger} \mathrm{D}_{\mu} H_i - \frac{1}{2} V_i\left(H^{\dagger} H\right) \tag{5}$$

$$V_i\left(H^{\dagger} H\right) = a^2 H_i^{\dagger} H_i - b^2 \left(H_i^{\dagger} H_i\right)^2 \tag{6}$$

where a,b are real constants and i=1,2

The total Lagrangian is

$$\mathcal{L}_H = \mathcal{L}_1 + \mathcal{L}_2 \tag{7}$$

Let $a^{-1} = \lambda_{\mathrm{d}} \big/ 2\pi$ be the reduced rest mass of dark matter particle.

D_{μ} is covariant differentiation since H has rank 2 asymmetric tensor components

Each potential $\mathrm{V}_i\left(\mathrm{H}^{\dagger}\mathrm{H}\right)$ has an unstable maximum when $\mathrm{H}_i^{\dagger}\mathrm{H}_i = \mathrm{a}^2 \big/ 2\,\mathrm{b}^2$

The Euler-Lagrange equations of motion for each H are

$$\mathrm{D}_{\mu}\mathrm{D}^{\mu}\mathrm{H} + a^2 \mathrm{H} - 2\, b^2 \left(\mathrm{H}^{\dagger}\mathrm{H}\right)\mathrm{H} = 0 \tag{8}$$

In the rest of this paper it will be assumed that $\mathcal{L}_1 = \mathcal{L}_2$

Matter Field Lagrangian

The Affine Connection

The invariant interval between two points on a metric space is

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{9}$$

The the invariance of ds, requires

$$D_\sigma g_{\mu\nu} = 0 \quad (10)$$

where D_σ indicates covariant differentiation.

For the affine connection to be determined by a metric tensor only, two cases arise:

Case I: The metric and affine connection are both symmetric

$$g_{\mu\nu} = g_{\nu\mu} \, \Gamma_{\sigma\mu\nu} = \Gamma_{\sigma\nu\mu} \quad (11)$$

With the conditions (11) and (10) the affine connection are the Christoffel Symbols **[5]**

$$\Gamma_{\sigma\mu\nu} = \frac{1}{2}\left(\partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\nu\mu} + \partial_\mu g_{\sigma\mu}\right) \quad (12)$$

Case 2: The metric and affine connection are both asymmetric:

$$\overline{g}_{\mu\nu} = -\overline{g}_{\nu\mu} \qquad \overline{\Gamma}_{\sigma\mu\nu} = -\overline{\Gamma}_{\sigma\nu\mu} \quad (13)$$

With the conditions (13) and (10) the asymmetric affine connection is

$$\overline{\Gamma}_{\sigma\mu\nu} = \frac{1}{2}\left(\partial_\mu \overline{g}_{\sigma\nu} + \partial_\sigma \overline{g}_{\mu\nu} + \partial_\nu \overline{g}_{\mu\sigma}\right) \quad (14)$$

A general affine connection can be formed from equations (12) and (14)

$$A_{\sigma\mu\nu} = \Gamma_{\sigma\mu\nu} + i\left(\overline{\Gamma}_{\sigma\mu\nu} + h_{\sigma\mu\nu}\right) \quad (15)$$

Where the imaginary part of the connection is asymmetric in μ and v; it can be shown that using (10) and (15) affine connection reduces to

$$A_{\sigma\mu\nu} = \Gamma_{\sigma\mu\nu} + i\widehat{\Gamma}_{\sigma\mu\nu} \quad (16)$$

where

$$\widehat{\Gamma}_{\sigma\mu\nu} = \left(\partial_\sigma \widehat{g}_{\mu\nu} + \partial_\nu \widehat{g}_{\sigma\mu} + \partial_\mu \widehat{g}_{\nu\sigma}\right) \quad (17)$$

the asymmetric affine connection is completely asymmetric

The Curvature Tensor

The Riemann curvature tensor can be calculated using the commutator of the 2 covariant derivatives of a real vector **[6]**.

Similarly, taking the commutator of 2 covariant derivatives of a complex vector using (16) for the affine connection gives

$$\left[D_\sigma, D_\rho\right]\Psi_\mu = G^\alpha_{\nu\rho\sigma}\Psi_\alpha + iH^\alpha_{\nu\rho\sigma}\Psi_\alpha + i2\widehat{\Gamma}^\alpha_{\rho\sigma}\partial_\alpha\Psi_\alpha \quad (18)$$

where

$$G^\alpha_{\nu\rho\sigma} = R^\alpha_{\nu\rho\sigma} + S^\alpha_{\nu\rho\sigma} \quad (19)$$

$$R^\alpha_{\nu\rho\sigma} = \partial_\sigma \Gamma^\alpha_{\nu\rho} - \partial_\rho \Gamma^\alpha_{\nu\sigma} + \Gamma^\beta_{\nu\rho}\Gamma^\alpha_{\beta\sigma} - \Gamma^\beta_{\nu\sigma}\Gamma^\alpha_{\beta\rho} \quad (20)$$

$$S^\alpha_{\nu\rho\sigma} = \widehat{\Gamma}^\beta_{\nu\rho}\widehat{\Gamma}^\alpha_{\beta\sigma} + \widehat{\Gamma}^\beta_{\nu\rho}\widehat{\Gamma}^\alpha_{\beta\sigma} + 2\widehat{\Gamma}^\beta_{\rho\sigma}\widehat{\Gamma}^\alpha_{\nu\beta} \quad (21)$$

$$H^\alpha_{\nu\rho\sigma} = \partial_\sigma \widehat{\Gamma}^\alpha_{\nu\rho} - \partial_\rho \widehat{\Gamma}^\alpha_{\nu\sigma} + \Gamma^\beta_{\nu\rho}\widehat{\Gamma}^\alpha_{\beta\sigma} + \Gamma^\alpha_{\beta\sigma}\widehat{\Gamma}^\beta_{\nu\rho} - \Gamma^\beta_{\nu\sigma}\widehat{\Gamma}^\alpha_{\beta\rho} - \Gamma^\alpha_{\beta\rho}\widehat{\Gamma}^\beta_{\nu\sigma} + 2\Gamma^\alpha_{\nu\beta}\widehat{\Gamma}^\beta_{\alpha\rho} \quad (22)$$

The curvature scalar is G

$$G = g^{\nu\rho}g^{\alpha\sigma}G_{\alpha\nu\rho\sigma} = g^{\nu\rho}g^{\alpha\sigma}\left(R_{\alpha\nu\rho\sigma} + S_{\alpha\nu\rho\sigma}\right) = R + S \quad (23)$$

By adding 2 $A^{\alpha}_{\rho\sigma} B_{\alpha} \Psi_{\nu}$ to both sides of (18) equating LHS to zero, and lowering α gives the 2 equations

$$G_{\beta\nu\rho\sigma}\Psi_{\alpha} + 2\Gamma_{\beta\rho\sigma}B_{\alpha}\Psi_{\nu} = 0 \tag{24}$$

$$2\Gamma_{\beta\rho\sigma}\left(\partial_{\alpha} + B_{\alpha}\right)\Psi_{\nu} + H_{\beta\nu\rho\sigma}\Psi_{\alpha} = 0 \tag{25}$$

Contracting (25) by ν = β

$$\left(\partial_{\alpha} + B_{\alpha}\right)\Psi_{\nu} - \Gamma^{\mu}_{\mu\nu}\Psi_{\alpha} = 0 \tag{26}$$

In a geodesic frame, partial derivates of the metric vanish, hence (16) reduces to

$$\left(\partial_{\alpha} + B_{\alpha}\right)\Psi_{\nu} = 0 \tag{27}$$

and since $D_{\mu}\left(\Psi_{\nu}\Psi^{*\nu}\right) = \partial_{\mu}\left(\Psi_{\nu}\Psi^{*\nu}\right)$ it can be shown that $B_{\alpha} \rightarrow iB_{\alpha}$

Contracting (24) with β = σ and ν = ρ gives

$$\left(G\Psi_{\alpha} + 2\, i\Gamma^{\sigma\nu}_{\sigma}B_{\alpha}\Psi_{\nu}\right) = 0 \tag{28}$$

The vector B_{α} can be eliminated by using the following relations

$$\gamma^{\mu}B_{\mu} = \frac{1}{\lambda}\beta \quad \text{and} \quad g^{\mu\nu}B_{\mu}B_{\nu} = \frac{1}{\lambda^{2}}$$

where γ^{μ} are the 4x4 Dirac-Gamma Matrices

Equations (26) and (28) after eliminating B_{α} are

$$\left(i\gamma^{\alpha}\partial_{\alpha} - \frac{1}{\lambda}\beta\right)\Psi_{\nu} - i\Gamma^{\mu}_{\mu\nu}\gamma^{\alpha}\Psi_{\alpha} = 0 \tag{29}$$

$$\left(\gamma^{\alpha}G + i\frac{2}{\lambda}\beta i\Gamma^{\mu\alpha}_{\mu}\right)\Psi_{\alpha} = 0 \tag{30}$$

Eliminating the symmetric connection from (29) and (30) gives

$$\left(i\gamma^{\mu}\partial_{\mu} - \left(\frac{1}{\lambda} - \frac{1}{2}G\lambda\right)\beta\right)\Psi_{\alpha} = 0 \tag{31}$$

The Lagrangian is

$$\mathcal{L}_{0} = \overline{\Psi}^{\alpha}\left[i\gamma^{\mu}\partial_{\mu} - \left(\frac{1}{\lambda} - \frac{1}{2}G\lambda\right)\beta\right]\Psi_{\alpha} \tag{32}$$

The components of Ψ are 8d complex spinors. Applying Yang-Mills Theory and the principle of local gauge invariance under SU(8)xU(1) transformations **[7]** to the free particle Lagrangian (32) results in $\partial_{\mu} \rightarrow D_{\mu}$ the gauge covariant derivative, and $\mathcal{L}_{0} \rightarrow \mathcal{L}_{1}$

$$\mathcal{L}_{1} = \overline{\Psi}^{\alpha}\left[i\gamma^{\mu}D_{\mu} - \left(\frac{1}{\lambda} - \frac{1}{2}G\lambda\right)\beta\right]\Psi_{\alpha} \tag{33}$$

where $D_{\mu} = \partial_{\mu} + igA_{k\mu}T_{k}$ and g is the gauge coupling strength and T_k are the generators of SU(8)xU(1)

The Lagrangian for the gauge fields and the gauge field strength tensor are **[8]**

$$\mathcal{L}_{G} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{34}$$

where the gauge field strength tensor is $F_{\mu\nu}$ defined by the relation

$$\left[D_\mu, D_\nu\right]\Psi_\alpha = igF_{\mu\nu}\Psi_\alpha \quad \textbf{(35)}$$

$\mathbb{R}^{4,3}$ has a 4d complex spinor ζ which coupled to the 8d complex spinor χ forms a 32d complex spinor Φ with Lagrangian of the form

$$\mathcal{L}_\Phi = \partial^\mu\Phi^\dagger\partial_\mu\Phi - V\left(\Phi^\dagger\Phi\right) \quad \textbf{(36)}$$

The Lagrangian for matter then follows as

$$\mathcal{L}_M = \mathcal{L}_1 + \mathcal{L}_G + \mathcal{L}_\Phi \quad \textbf{(37)}$$

$$\mathcal{L}_M = \overline{\Psi}^\alpha\left[i\gamma^\mu D_\mu - \left(\frac{1}{\lambda} - \frac{1}{2}G\lambda\right)\beta\right]\Psi_\alpha + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_\Phi \quad \textbf{(38)}$$

Gravitational field equations

The total Lagrangian density is the sum of the matter field, Curvature scalar and the quaternion field

$$\mathcal{L} = \mathcal{L}_M + \frac{1}{\kappa}\left(G + \mathcal{L}_H\right) \quad \textbf{(39)}$$

where the Einstein constant $\kappa = 8\pi G / c^4$

Variation of the Lagrangian with respect to the symmetric metric $g_{\mu\nu}$ **[9]**

$$\delta\int\left(\mathcal{L}_M + \frac{1}{\kappa}G + \frac{1}{\kappa}\mathcal{L}_H\right)\sqrt{-g}\,dx^4 = 0 \quad \textbf{(40)}$$

$$\int\left(\frac{\delta\left(\mathcal{L}_M + \frac{1}{\kappa}\mathcal{L}_H\right)}{\delta g^{\mu\nu}}\sqrt{-g} + \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}\left(\mathcal{L}_M + \frac{1}{\kappa}\mathcal{L}_H\right) + \frac{1}{\kappa}\left(\frac{\delta(G)}{\delta g^{\mu\nu}}\sqrt{-g} + \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}G\right)\right)g^{\mu\nu}dx^4 = 0 \quad \textbf{(41)}$$

Using the results

$$\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}g_{\mu\nu}\sqrt{-g} \text{ and } \frac{\delta(G)}{\delta g^{\mu\nu}} = R_{\mu\nu}$$

Equation **(41)** is

$$\int\left(\frac{\delta\left(\mathcal{L}_M + \kappa^{-1}\mathcal{L}_H\right)}{\delta g^{\mu\nu}} - \frac{1}{2}g_{\mu\nu}\left(\mathcal{L}_M + \frac{1}{\kappa}\mathcal{L}_H\right) - \frac{1}{\kappa}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+S)\right)\right)g^{\mu\nu}\sqrt{-g}\,dx^4 = 0 \quad \textbf{(42)}$$

From which it follows that

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}S = \kappa\left(\frac{\delta\left(\mathcal{L}_M + \kappa^{-1}\mathcal{L}_H\right)}{\delta g^{\mu\nu}} - \frac{1}{2}g_{\mu\nu}\left(\mathcal{L}_M + \frac{1}{\kappa}\mathcal{L}_H\right)\right) \quad \textbf{(43)}$$

Substituting for $\mathcal{L}_H$ and re-arranging gives

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}\sum_i \frac{1}{2} V_i(H^\dagger H) g_{\mu\nu} = -\kappa\left[-\frac{\delta\left(\mathcal{L}_M + \kappa^{-1}\mathcal{L}_H\right)}{\delta g^{\mu\nu}} + \frac{1}{2} g_{\mu\nu}\left(\mathcal{L}_M + \frac{1}{\kappa}\sum_i \left(D^\sigma H^\dagger D_\sigma H\right)_i - \frac{1}{\kappa} S\right)\right] \quad (44)$$

Equation **(44)** can be written in the more familiar form as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda_{eff} g_{\mu\nu} = -\kappa T_{\mu\nu} \quad (45)$$

and are the gravitational field equations where the energy-stress tensor is

$$T_{\mu\nu} = \left[-\frac{\delta\left(\mathcal{L}_M + \frac{1}{\kappa}\mathcal{L}_H\right)}{\delta g^{\mu\nu}} + \frac{1}{2} g_{\mu\nu}\left(\mathcal{L}_M + \frac{1}{\kappa}\sum_i \left(D^\sigma H^\dagger D_\sigma H\right)_i - \frac{1}{\kappa} S\right)\right] \quad (46)$$

and the dark energy is $\Lambda_{eff} = \frac{1}{2}\sum_i \frac{1}{2} V\left(H^\dagger H\right)_i$

Cosmological Constant

The dark energy potential $V\left(H^\dagger H\right)_i$ has an unstable maximum when $(H^\dagger H)_i = a^2 / 2b^2$

Summing over 2 quaternions the dark energy maximum is

$$\Lambda_{eff} = \frac{1}{2}\sum_i \frac{1}{2} V\left(H^\dagger H\right)_i = \frac{a^4}{8b^2} \quad (47)$$

Hence identify the cosmological constant $\Lambda = a^4 / 8b^2$

Cosmological Density Ratios

Let $\mathcal{L}_1 = \mathcal{L}_2$ and ignoring curvature, the dark matter density ρ_d is approximately

$$\rho_d = 2\frac{1}{2\kappa}\partial^0 H_h^\dagger \partial_0 H^h \quad (48)$$

Let $H_1 = H_2 = H_3 = H$

$$\rho_d = \frac{3}{\kappa}\partial^0 H^\dagger \partial_0 H \quad (49)$$

When Dark energy is near its maximum, the equation of motion for H is

$$D_\mu D^\mu H(r, t) = 0 \quad (50)$$

Ignoring the effects of curvature **(50)** reduces to the wave equation $\partial_\mu \partial^\mu H(r,t) = 0$

A general solution $H = H(r) f(ct / \lambda)$. Substituting into **(49)** gives

$$\rho_d = \frac{3}{\kappa}\frac{1}{\lambda^2} H^\dagger(r) H(r)\, g^2\left(\frac{ct}{\lambda}\right) \quad (51)$$

Near the dark energy maximum so ρ_d reduces to

$$\rho_d = \frac{12}{\kappa}\frac{1}{\lambda^2}\frac{\Lambda}{a^2}\left(\frac{g\left(\frac{ct}{\lambda}\right)}{f\left(\frac{ct}{\lambda}\right)}\right)^2 \tag{52}$$

where $g(ct/\lambda)$ is the derivative of f

The dark matter density has the time-independent value

$$\rho_d = \frac{12}{\kappa}\frac{\Lambda}{\lambda^2 a^2} = \frac{3\Lambda}{\pi^2\kappa}\left(\frac{E_\lambda}{E_a}\right)^2 \tag{53}$$

When energy of dark matter particles approaches rest energy $E_\lambda \rightarrow E_a$ and substituting $\rho_\Lambda = \Lambda/\kappa$ (53) simplifies to

$$\rho_d = \frac{3}{\pi^2}\frac{\Lambda}{\kappa} = \frac{3}{\pi^2}\rho_\Lambda \tag{54}$$

Dividing by the critical density ρ_c

$$\Omega_d = \frac{3}{\pi^2}\Omega_\Lambda \tag{55}$$

where $\Omega_\Lambda = \rho_\Lambda/\rho_c$ and $\Omega_d = \rho_d/\rho_c$

In thermal equilibrium baryons (field with total spin 0) has degeneracy $g_b = 1$ and dark matter (spin 2) has degeneracy $g_d = 5$

$$\frac{\Omega_b}{\Omega_d} = \frac{\rho_b}{\rho_d} = \frac{g_b}{g_d} = \frac{1}{5} \tag{56}$$

WMAP data indicates that the total density of the Universe is near its critical density, so ignoring radiation density

$$\Omega_\Lambda + \Omega_d + \Omega_b = 1 \tag{57}$$

It follows that the Cosmological density ratios are

$$\left\{\Omega_b = \frac{3}{5\pi^2+18},\ \Omega_d = \frac{15}{5\pi^2+18},\ \Omega_\Lambda = \frac{5\pi^2}{5\pi^2+18}\right\} \tag{58}$$

$$\left\{\Omega_\Lambda = 0.7327,\ \Omega_d = 0.2227,\ \Omega_b = 0.0445\right\} \tag{59}$$

which are in agreement with the 7 year WMAP data **[10]**

$$\left\{\Omega_\Lambda = 0.725,\ \Omega_d = 0.229,\ \Omega_b = 0.0458\right\}$$

The differences are due to the assumption that dark matter is at rest, so the predicted ratios for dark matter and matter are lower bounds and the predicted ratio for dark energy is therefore an upper bound. Thus using the WMAP data, it follows that the average speed of dark matter particle is approximately 0.2c, thus dark matter particles are currently non-relativistic.

Fermion mass energy bounds

Contracting equation (45) gives

$$- R - 4\Lambda = -\kappa T = -\kappa\rho \Rightarrow R = \kappa\rho - 4\Lambda \tag{60}$$

Let S = 0 and since rest energy E ≥ 0

$$E = \frac{1}{\lambda} - \frac{1}{2} R\lambda \geq 0 \Rightarrow \frac{1}{\lambda^2} \geq \frac{1}{2} R = \frac{1}{2}(\kappa\rho - 4\Lambda) \tag{61}$$

The energy density of fermions in equilibrium is given by Fermi-Dirac statistics

$$\rho = \frac{\beta\hbar^4 c^4}{\lambda^4} \tag{62}$$

where $\beta = g_s 7\pi^2 / 240\hbar^3 c^3$ and g_s is the degeneracy of the fermions. Substituting into **(61)**

$$\frac{1}{\lambda^2} \geq \frac{\kappa\beta\hbar^4 c^4}{2\lambda^4} - 2\Lambda \Rightarrow 4\Lambda\lambda^4 + 2\lambda^2 - \kappa\beta\hbar^4 c^4 \geq 0 \tag{63}$$

Note that $\Lambda\kappa\beta\hbar^4 c^4 \ll 1$

$$\lambda^2 \geq \frac{-2 \pm 2\left(1 + 2\Lambda\kappa\beta\hbar^4 c^4\right)}{8\Lambda} \tag{64}$$

$$\lambda^2 \geq \frac{1}{2}\kappa\beta\hbar^4 c^4 \Rightarrow \lambda \geq \left(\frac{1}{2}\kappa\beta\hbar^4 c^4\right)^{½} \tag{65}$$

Hence fermions have a minimum wavelength.
Equation **(61)** also implies

$$\frac{1}{\lambda^2} \geq 0 \Rightarrow \kappa\rho \geq 4\Lambda \Rightarrow \frac{\kappa\beta\hbar^4 c^4}{\lambda^4} \geq 4\Lambda \Rightarrow \lambda \leq \left(\frac{\kappa\beta\hbar^4 c^4}{4\Lambda}\right)^{¼} \tag{66}$$

Hence fermions have a maximum wavelength.
Combining the 2 inequalities gives the fermion mass energy bounds

$$\hbar c\left(\frac{4\Lambda}{\kappa\beta\hbar^4 c^4}\right)^{¼} \leq E \leq \frac{\hbar c}{\sqrt{\frac{\kappa\beta\hbar^4 c^4}{2}}} \tag{67}$$

Hence fermion mass energy is bounded and always finite.
For $\Lambda \sim 10^{-52}$

$$4.2 \times 10^{-3} g_s^{-¼} eV \leq E \leq 6.4 \times 10^{18} g_s^{-½} GeV \tag{68}$$

The upper bound is less than Planck energy 1.22×10^{19}GeV

Gravitation range lower bound

The probability that a dark matter particle is found in the dimensionless volume element dΩ is $H^\dagger H d\Omega$, hence the following definite integral holds

$$\int H^\dagger H \, d\Omega = \int g_{ii} H^{\dagger i}_{h\rho\sigma} H^{ih\rho\sigma} + g_{00} H^{\dagger 0}_{h\rho\sigma} H^{0\,h\rho\sigma} \, d\Omega \leq 1 \tag{69}$$

Let $H_1 = H_2 = H_3$
It follows that

$$g_{11} + g_{00} \leq 1 \tag{70}$$

The Schwarzschild metric in the [-+++] convention is **[11]**

$$-ds^2 = \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + r^2\left(d\theta^2 + \sin^2\theta d\Phi^2\right) \tag{71}$$

where $r_s = 2\,GM\big/c^2$ is the Schwarzschild radius and mass M>0

Substituting $g_{11} = \left(1 - r_s\big/r\right)^{-1}$ and $g_{00} = -\left(1 - r_s\big/r\right)$ into **(70)** gives the inequality

$$\frac{1}{\left(1 - \frac{r_s}{r}\right)} - \left(1 - \frac{r_s}{r}\right) \le 1 \tag{72}$$

Hence r has a minimum

$$r \ge \frac{2\,r_s}{3 + \sqrt{5}} \tag{73}$$

Since M>0 it follows that $r > 0$ hence the physical singularity at $r = 0$ does not exist.
Consider 2 bosons in thermal equilibrium of equal energy E in a spherical volume of radius r given by **(73)**. Thus it follows that the upper energy density satisfies the equation

$$\frac{6E}{4\,\pi\,a^3 r_s^3} = \frac{g_s}{\hbar^3 c^3}\frac{\pi^2}{30}E^4 \tag{74}$$

Substituting $r_s = 2\,GE\big/c^4$ into **(74)** and solving for E gives

$$E = \sqrt[6]{\frac{90}{8\,\pi^3 a^3 g_s}}\left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} \tag{75}$$

For dark matter $g_s = 5$ hence $E \sim 1.5 \times 10^{19}$Gev is the upper energy of a dark matter particle given by **(75)**

Galaxy Rotation Curves

The Euler-Lagrange equations of motion for H (ignoring space-time curvature) is

$$\partial_\mu \partial^\mu \mathrm{H} + a^2 \mathrm{H} - 2\,b^2\left(\mathrm{H}^\dagger \mathrm{H}\right)\mathrm{H} = 0 \tag{76}$$

Let $\mathrm{H}_1 = \mathrm{H}_2 = \mathrm{H}_3$

When the dark matter density $\rho_d < \frac{3}{2\,\pi^2}\left(\frac{E_d}{E_a}\right)^2 \rho_\Lambda$ equation **(76)** reduces to

$$\partial_\mu \partial^\mu H(r,\,t) + a^2 \mathrm{H} = 0 \tag{77}$$

The general solution is

$$\mathrm{H}(r,\,t) = \frac{k}{r} e^{-ar} e^{ift} \tag{78}$$

Weak Gravitational field approximation is the Poisson equation

$$\nabla^2 \Phi = -\kappa\rho = -\frac{3}{2}\partial^0 \mathrm{H}^\dagger \partial_0 \mathrm{H} = -\frac{3}{2}\left(\partial_0 \mathrm{H}\right)^2 \tag{79}$$

where Φ is the gravitational potential per unit mass. For centrally symmetric field,

$$\frac{1}{r^2}\partial_r\left(r^2\frac{\partial\Phi}{\partial r}\right) = -\frac{3}{2}\left(\frac{k^2}{r^2}e^{-2ar}\right)f^2 e^{i2ft} \tag{80}$$

which has a general solution

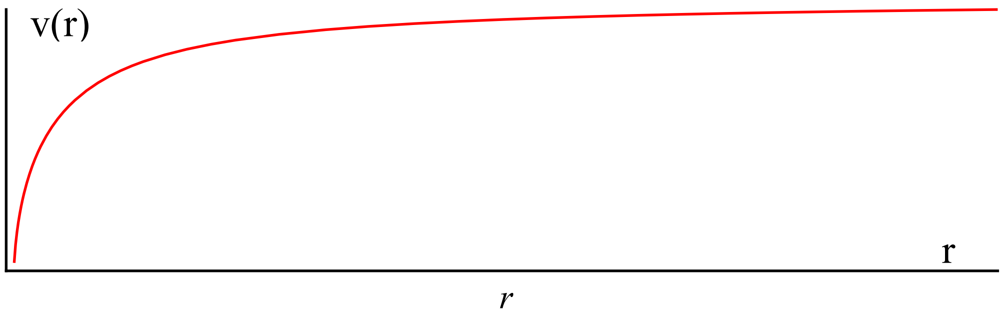
$$\Phi(r) = -\frac{3k^2 e^{-2ar}}{4ar\ln(e)} + \frac{3k^2\,\mathrm{Ei}_1(2ar\ln(e))}{2} - \frac{A}{r} + B \tag{81}$$

where A and B are constants and Ei_1 is the exponential integral.

It follows that the speed of rotation v(r,t) is

$$v(r,t) = \left[-\frac{3k^2 e^{-2ar}}{4ar\ln(e)} + \frac{3k^2\,\mathrm{Ei}_1(2ar\ln(e))}{2} - \frac{A}{r} + B\right]^{½} fe^{ift} \tag{82}$$

A particular graph is



which is similar to the general profile for a galaxy rotation curve with limiting rotational speed **[12a]**.

Using $H^\dagger H \sim \frac{k^2}{r^2}\exp(-2ar) \geq 4\Lambda / a^2$

$$r^2 \leq \frac{k^2}{4\Lambda a^2}\exp(-2ar) \tag{83}$$

and evaluating k when $\rho(r=R) \sim 3H^\dagger H / \lambda^2\kappa$ results in the following upper bound for r

$$r^2 \leq \frac{R^2\rho(R)\lambda^2}{12\rho_\Lambda a^2}\exp(-2a(r-R)) \tag{84}$$

For r<R and $\lambda \to \lambda_a$ **(84)** implies that the dark matter wavelength satisfies the inequality

$$\lambda_a \geq \frac{2R}{\ln\left(\frac{3\rho_\Lambda}{\pi^2\rho(R)}\right)} \tag{85}$$

Since $\lambda_a > 0$ it follows that $\rho(R) < 3\rho_\Lambda / \pi^2 = \rho_d$

The density of dark matter at R is less than the cosmological dark matter density. Consequently the rotational speed v(r>R) falls but then rises as the dark matter density increases to the density in the halo. Hence the galactic rotation curve has a cusp at R, thus R can be identified as the radius of the galactic bulge.

For the spiral galaxy NGC3198 $R \sim 10$ Kpc **[12b]** hence the dark matter wavelength is $\lambda_a > 20$ Kpc, hence the rest mass of a dark particle $E_a > 10^{-27}$eV with average speed approximately > 0.2c

Cosmology

The Robertson-Walker-Friedmann Equations are **[13]**

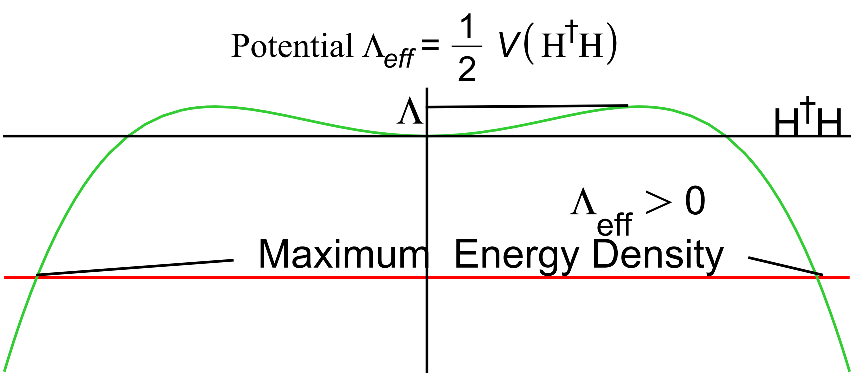
$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G(\rho + 3p)}{c^2} + \frac{\Lambda_{eff} c^2}{3} \tag{86}$$

$$\frac{\dot{a}(t)^2}{a(t)^2} + \frac{kc^2}{a(t)^2} = \frac{8\pi G\rho}{3c^2} + \frac{\Lambda_{eff} c^2}{3} \tag{87}$$

where $\Lambda_{eff} = \frac{1}{2} V\left(H^{\dagger}H\right)$ and ρ is the total matter plus dark matter energy density

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda_{eff} c^2}{3} \tag{88}$$

The graph of $V\left(H^{\dagger}H\right)$ is of the form as shown below where Λ is the maximum



Case I

$\Lambda_{eff} = \Lambda$

The scale factor a(t) increases exponentially

$$a(t) = a(0)\exp\left(\sqrt{\frac{\Lambda}{3}}\, ct\right) \tag{89}$$

Note that in this cosmology dark energy is not a constant, and hence a 'big-rip' scenario is not predicted.

Case II

$\Lambda_{eff} \sim -b^2\left(H^{\dagger}H\right)^2$

From the graph above the potential decreases post exponential expansion. In this phase the potential is dominated by $-b^2\left(H^{\dagger}H\right)^2$

This condition is met when $a^2 / b^2 < H^{\dagger}H$ and results in

$$\Omega_d > \frac{3}{\pi^2}\left(\frac{E_\lambda}{E_a}\right)^2 \Omega_\Lambda \tag{90}$$

where $E_a = m_0 c^2$ is the rest mass of dark matter particle.

Universe contracts when $\Omega \sim \Omega_d + \Omega_\Lambda > 1$ hence the speed v of dark matter particle is

$$v > \left(1 - \frac{3\,\Omega_\Lambda}{\pi^2 \left(1 - \Omega_\Lambda\right)} \right)^{1/2} c \tag{91}$$

When dark matter particles have average speed v>0.442c, the dark matter density exceeds critical density hence the Universe contracts.

With k=0 Equation **(87)** in this phase reduces to

$$\frac{\dot{a}(t)^2}{a(t)^2} = -\frac{b^2\,(H^\dagger H)^2\,c^2}{6} + \frac{8\,\pi G \rho}{3\,c^2} \tag{92}$$

In contracting phase it will be more useful to transform to new variables $a(t) \rightarrow b^{-1}(t)$ and $\rho = \rho_f b^3(t)$ where ρ_f is the energy density after inflation due to

Let $k^2 = 8\,\pi G \rho_f \big/ 3\,c^2$ and since ρ dominates equation **(92)** is

$$\frac{\dot{b}(t)^2}{b(t)^2} = k^2\, b(t)^3 \tag{93}$$

Equation **(93)** is valid for for all ρ and for b(t)>1 because $\frac{\mathrm{b}^2}{6}\left(\mathrm{H}^\dagger \mathrm{H}\right)^2 \mathrm{c}^2 \propto b^2(t)$ and $\frac{8\,\pi G \rho}{3\,c^2} \propto b^3(t)$

A particular solution is

$$b(t) = \left(\frac{2}{3\,k} \right)^{\frac{2}{3}} t^{-\frac{2}{3}} \tag{94}$$

Contracting phase implies the scale factor b(t) is increasing hence t must be decreasing. Thus time has an upper bound $t_{\max}$

$$t_{\max} = \frac{3}{2} \sqrt{\frac{3\,c^2}{8\,\pi G \rho_f}} \tag{95}$$

$$t_{\min} = \frac{3}{2} \sqrt{\frac{3\,c^2}{8\,\pi G \rho_o}} \tag{96}$$

Hence cosmological time is bounded on the interval $t_{\min} \leq t \leq t_{\max}$

Spontaneous Symmetry Breaking of SU(8)xU(1)

The break down of the Non-abelian gauge group is assumed to follow the formalism given by Higgs et al **[14]**

Couple χ the 8d complex spinor to the 4d complex spinor of SU(4), to give a scalar with real representation of 64d

The number of Higgs bosons H is given by

$$H = n - N + M \tag{97}$$

where n is the real dimension of the scalar field, N is dimension of gauge group and M is the dimension of the subgroup.

The orthogonal groups on the Real vector spaces have the sub-groups as follows

$$((6,1) \rightarrow SU(3),\ (2,5) \rightarrow U(1)x\,SU(2)) \tag{98}$$

The gauge group G=SU(8)xU(1) is the same for the 2 quaternion spinors
Let G(p,q) be the gauge group for the multiplet whose quaternion is the spinor on $\mathbb{R}^{p,q}$ where (p,q)∈{(6,1),(2,5)}
Assume that the Higgs scalar bosons form a special unitary group, subscripted by h in the following breakdown of the gauge group G(p,q)

$$G(6,1) \rightarrow SU_c(3) + SU_h(3) + 55 \textit{ massive vector bosons} \quad (99)$$

$$G(2,5) \rightarrow U(1)x\, SU(2) + U(1)_h x\, SU(2)_h + 60 \textit{ massive vector bosons} \quad (100)$$

Since G(p,q) is chiral, these subgroups are also Chiral.

It is noted that the spontaneous symmetry breaking of G(p,q) could lead to different sub-groups than above, which could happen in each cycle of the cosmos.

There are 2x3x2 chiral SU(8)xU(1) multiplets since the 2 quaternion spinors are each 3d and the s=0 state forms 2 spin s=1/2 fermions. consequently after symmetry breaking there are 6 $SU_c(3)$ multiplets and 6 SU(2)xU(1) doublets.

Conclusion

By considering the spin representations on a 7d complex space, the massive quaternion spin 2 field is the proposed source of dark matter, dark energy and matter. The Matter-Antimatter asymmetry is a consequence of the strictly neutral nature of dark matter. The predicted cosmological density ratios for baryonic matter, dark matter and dark energy are in agreement with the WMAP 7 year data. In addition the predicted velocity rotation curve of a galaxy with dark matter as the dominant source of the gravitational field, has broadly the same profile as observed. The Dark matter wavelength is predicted to be of the order of 20Kpc, consequently has an effect on large scale structure formation.
The Cosmological implication of the spin 2 quaternion field is that the universe is cyclic with no beginning or end. Time is predicted to have a maximum determined principally by the duration of the accelerating expansion of the universe due to dark energy. The prediction of a maximum energy density results in time having a minimum. The nature of the matter particles and there interactions could be different in each cycle, depending on the spontaneous symmetry breaking of the SU(8)xU(1) group.
The physical singularity of the Schwarzschild solution is non-existent and since there is an upper energy to fermions and dark matter, there will be no ultra-violet divergences which arise in Quantum Field Theory.
More detailed calculations of the properties of dark matter and cosmology will be undertaken along with the quantum field aspects of the massive quaternion spin 2 field.

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