Draw the metric!

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December 13, 2012

Abstract

Be sure to use proper tensor rank and orientation. The recent efforts by Professor Kip S. Thorne do not meet this need. The utility of a correct method is remarkably strong. Here's how!

Discussion

The tutorial on forms and exterior derivatives in Chapters 2 to 4 of GRAVITATION is most welcome to this project. It is a shame, though, that ongoing use of this framework was labeled as unimportant in that famous book.

The metric is a set of symmetric products with one instance in that set for each covariant coordinate system. It is defined so that, when we multiply an instance, it being taken as a matrix, by two genuine vectors, we get the scalar product for the two vectors. This scalar product is required to be invariant over the set, so long as the unit of length does not change. Despite choosing one coordinate system for calculation, the scalar product is coordinate-free. And the metric, taken as an abstract set, also becomes coordinate-free.

(Covariant, here, means that a 1-form is used to represent a coordinate. A 1-form is a surface or hypersurface with one preferred direction, such that passing though the surface in one direction signals a positive change of coordinate value, and the other direction negative.)

We can draw each one of the instances of symmetric products everywhere to gain the general result, but lets just choose just one that is especially simple to draw. No utility is lost.

It is possible to choose a covariant coordinate system that is Cartesian orthogonal everywhere, and that has one coordinate in the direction of time at the location of our chosen observer. The off diagonal terms of the symmetric product are then zero everywhere, which makes this much easier to draw. At each location there is at least one orientation that makes this so; just choose one compatible with the next step.

Now make that set of coordinates homogeneous by joining up the edges of the 1-forms between adjacent locations so that no edges are left on any of the four 1-forms that make up the coordinate system. The sparseness or calibration of the

1-forms must be adjusted everywhere to achieve this, but we leave the 1-forms as normally calibrated at the location of our observer. This requirement sets the exterior derivative of each of the four coordinates to be zero everywhere. The exterior derivative geometrically marks boundaries, but we have just now eliminated all such boundaries.

Now we can draw these four coordinates as four orthogonal 1-forms everywhere. (If there is a double value, just pick one.)

On the diagonal of the symmetric product, there is a multiplication of each one of the 1-forms with its complimentary 1-form at each location in spacetime. We must make this compliment with the same orientation as the original, and we also need to give this complimentary 1-form the magnitude that is needed to fulfill the mission of the metric. This second set of coordinate 1-forms generally has boundaries that can be marked off by the exterior derivative.

Now, we can just draw the two sets of coordinate 1-forms, each of the four drawing pairs together with a symbol that denotes the product for each of the four orthogonal directions.

This drawing has the property of general covariance - sparse or scaled versions of it still represent the same physics.

We now have a drawing. But lets make another kind of drawing that is more revealing. Using the same orientation at each location as our coordinate system (that is specialized for our chosen observer but with no loss of utility), simply draw red partitions wherever a perpendicular crossing of that partition in either direction would result in the gain of one unit of spacetime interval as compared to flat spacetime. Draw blue partitions for the loss of one unit. There can be as many as four red or blue partitions at a location, and they are all mutually Cartesian orthogonal.

These drawings also have the property of general covariance. The partitions, red and blue, represent the square root of the product of two 1-forms. So our drawings transform naturally like length to the minus one power.

Flat spacetime would then have none of these partitions. But curved spacetime is now decorated with red or blue. it is now easy for us to construct curved geodesics - that curve inward at the boundary of red partitions, and outward at the boundary of blue.

Applications

We can see a cosine squared directionality of the effects of these partitions. Stationary gravitational fields do not curve space!

The two Bianchi identities that apply to these partitions supply the means to compute the metric by integrating from geometric initial conditions. It is then possible for us to draw the results of the computation. We need to use the exterior derivative for this.

We can easily see how to draw dilation horizons, and we can see how they are conserved by the applicable Bianchi identity. It's no difficulty if they trend in the timelike direction or spacelike.

It can now be clear for us that Kaluza-Klein fields have a two component source - two dilation horizons at a Cartesian right angle to each other.