A Note on Relativity

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A note in favor of the correctness of the relativity, special and general.

Key words: relativity (special and general).

The special relativity gives us the equations: \( E = mc^2, \ E_0 = m_0c^2, \ E = E_0 + T, \ m = \gamma m_0 \) and \( \gamma = (1 - v^2/c^2)^{-1/2}, \) where \( E, E_0 \) and \( T \) are the total, rest and kinetic energies of the particle, \( m \) and \( m_0 \) its moving and rest masses, \( c \) the speed of the light in the vacuum and \( v \) the speed of the particle. For \( \sqrt{c} \ll c, \ E = m_0c^2 + (1/2)m_0v^2, \) which is the correct value for the Newton mechanics, where the kinetic energy is \( T = (1/2)m_0v^2. \) All these equations are obtained without using relativity, but with \( \gamma = (1 - v/c)^{-1}, \) which is not the correct value [1]. In addition, for two inertial systems: \( f'/f = ((1 - v/c)/(1 + v/c))^{1/2} \) (from the relativistic Doppler effect), where \( f' \) and \( f \) are the frequencies of the light in the moving and rest frames, respectively, \( v \) being the moving speed of the primed frame. But also, \( E'/E = ((1 - v/c)/(1 + v/c)) \) (from the Lorentz transformation for the energy), then \( E'/E = f'/f, \ E' = hf' \) and \( E = hf, \) which is the Planck-Einstein equation, \( h \) being the Planck constant [2]. All this is in favor of the correctness of the special relativity.

From the general relativity: \( R_{ik} - (1/2)g_{ik}R = (8\pi G/c^4)T_{ik}, \) \( (i, k = 0, 1, 2, 3), \) where \( R_{ik} \) is the Ricci tensor, \( g_{ik} \) the metric tensor, \( R \) the scalar curvature, \( G \) the universal gravitational constant of Newton and \( T_{ik} \) the energy-momentum tensor. This equation is like the Poisson equation: \( \nabla^2 \phi = 4\pi G \rho, \) where \( V = (\partial \phi/\partial x, \partial \phi/\partial y, \partial \phi/\partial z), x, y \) and \( z \) being the rectangular coordinates, \( \phi \) the gravitational potential and \( \rho \) the mass density. In the vacuum: \( R_{ik} - (1/2)g_{ik}R = 0 \) (Laplace equation: \( \nabla^2 \phi = 0 \)), and \( R_{i} - (1/2)\delta_{i}R = 0, \ R_{i} - (1/2)\delta_{i} = 0, \) \( R - (1/2)4R = 0 \) and \( R = 0 \) (where: \( R_{i} = R, \ \delta_{i} = 1 \) if \( k = i \) and \( \delta_{k} = 0 \) if \( k \neq i \), and \( \delta_{k} = 4 \)); then, \( R_{ik} = 0. \) This equation was solved by Schwarzschild yielding a square space-time interval value of: \( ds^2 = (1 - r_g/r)c^2 dt^2 - r^2(sin^2 \theta d\phi + d\theta^2) - dr^2/(1 - r_g/r), \) where \( r_g = 2GM/c^2 \) is the gravitational (or Schwarzschild) radius, \( M \) being the rest mass of the particle that produces the gravitational field, \( t \) the time, and \( r, \ \theta \) and \( \phi \) the spherical coordinates. Note that \( r > r_g, \) since \( r = r_g \) and \( r = 0 \) would yield \( ds^2 = -\infty \) and \( r < r_g \) would produce a change of sign in time and in the space. Note also that, we may put \( r = 2GM/v^2, \) \( v \) being like a gravitational escape speed \( (E = T + V = (1/2)m_0v^2 - GMm_0/r), V \) being the potential energy, and from \( E = 0, \) the escape velocity would be: \( v = (2GM/r)^{1/2}, \) and as \( r_g = 2GM/c^2, \) it would correspond to a gravitational escape speed of \( c. \) As \( r > r_g, \) \( v < c, \) and there is not black holes. In addition, substituting these values in the interval, we would have that: \( ds^2 = (1 - v^2/c^2)c^2 dt^2 - r^2(sin^2 \theta d\phi + d\theta^2) - dr^2/(1 - v^2/c^2), \) and for given values of \( \theta \) and \( \phi \) \( (\theta = \text{constant}, \ \phi = \text{constant}, \ d\theta = 0 \) and \( d\phi = 0), \) it would be: \( ds^2 = (1 - v^2/c^2)c^2 dt^2 - dr^2/(1 - v^2/c^2) = c^2 dt^2 - dr^2 = ds^2, \) which is a generalization of the special relativity for a radial motion in a gravitational field. Note also that for \( |\phi| \ll c^2, \) which implies that \( v^2 \ll c^2, \) it is recovered the Newton gravitation formula [3]: \( F = -GMM_0/r^2, \) where \( F \) is the gravitational attraction force.
between the masses \(M\) and \(m_0\) separated a distance \(r\) (see the appendix). All this is in favor of the correctness of the general relativity.

Appendix

Newton’s gravitational attraction force from Einstein’s general relativity:

\[
R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}, \quad (i,k = 0,1,2,3)
\]

\[
R_i^i - \frac{1}{2} \delta_i^i R = \frac{8\pi G}{c^4} T_i^i
\]

\[
R_i^i - \frac{1}{2} \delta_i^i R = \frac{8\pi G}{c^4} T_i^i, \quad \left( R_i^i = R, \delta_i^i = 4, T_i^i = T \right)
\]

\[
R = -\frac{8\pi G}{c^4} T
\]

\[
R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right)
\]

\[|\phi| \ll c^2, \quad v^2 \ll c^2\]

\[L = -Mc^2 + \frac{1}{2} M v^2 - M \phi\]

\(L\) being the Lagrangian.

\[L = -Mc \left( c - \frac{v^2}{2c} + \frac{\phi}{c} \right)\]

\[S = \int L dt = -Mc \int \left( c - \frac{v^2}{2c} + \frac{\phi}{c} \right) dt\]

\(S\) being the action.

\[S = -Mc \int ds\]

\[ds = \left( c - \frac{v^2}{2c} + \frac{\phi}{c} \right) dt\]

\[ds = c \left( 1 - \frac{v^2}{2c^2} + \frac{\phi}{c^2} \right) dt\]

\[ds^2 = c^2 \left( 1 - \frac{v^2}{2c^2} + \frac{\phi}{c^2} \right)^2 dt^2\]

\[\xi \ll 1, \quad (1 \pm \xi)^n \approx 1 \pm n \xi\]
\[
\left(1 - \frac{v^2}{2c^2} + \frac{\varphi}{c^2}\right)^2 \approx (1 + \xi)² = 1 + 2\xi = 1 + 2\left(-\frac{v^2}{2c^2} + \frac{\varphi}{c^2}\right) = 1 - \frac{v^2}{c^2} + \frac{2\varphi}{c^2} = 1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}
\]

\[
c^2 \left(1 - \frac{v^2}{2c^2} + \frac{\varphi}{c^2}\right)^2 = c^2 \left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right) = c^2 \left(1 + \frac{2\varphi}{c^2}\right) - v^2
\]

\[v dt = dr\]

\[
c^2 \left(1 - \frac{v^2}{2c^2} + \frac{\varphi}{c^2}\right)^2 dt^2 = c^2 \left(1 + \frac{2\varphi}{c^2}\right) dt^2 - v^2 dt^2 = c^2 \left(1 + \frac{2\varphi}{c^2}\right) dt^2 - dr^2
\]

\[ds^2 = c^2 \left(1 + \frac{2\varphi}{c^2}\right) dt^2 - dr^2\]

\[ds^2 = g_{00} c^2 dt^2 - dr^2\]

\[g_{00} = 1 + \frac{2\varphi}{c^2}\]

\[g_{\alpha\beta} = -1, \quad g_{\alpha\beta} = 0 \quad (\alpha \neq \beta), \quad g_{00} = g_{00} = 0, \quad (\alpha, \beta = 1, 2, 3)\]

\[T^i = \rho c \frac{dx^i}{ds} \frac{dx^k}{dt}, \quad u^i = \frac{dx^i}{ds}, \quad c = \frac{dx^0}{dt}, \quad v^\alpha = \frac{dx^\alpha}{dt}, \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},\]

\[ds = c dt \gamma^{-1}, \quad T_0^0 = \gamma \mu c^2 = \mu c^2, \quad T_\alpha^\alpha = \gamma \mu v^\alpha v_\alpha = \gamma \mu v^2 = \mu v^2\]

\[T = T_i = T_0^0 + T_\alpha^\alpha = \rho c^2 + \rho v^2 = \rho c^2\]

\[R_i^k = \frac{8\pi G}{c^4} \left(T_i^k - \frac{1}{2} \delta_i^k T\right)\]

\[R_0^0 = \frac{8\pi G}{c^4} \left(T_0^0 - \frac{1}{2} \delta_0^0 T\right) = \frac{8\pi G}{c^4} \left(T_0^0 - \frac{1}{2} \rho c^2\right) = \frac{4\pi G \rho}{c^2}\]

\[R_{00} = R_0^0 = \frac{4\pi G \rho}{c^2}\]

\[R_{00} = \frac{\partial \Gamma_{00}^\alpha}{\partial x^\alpha}\]

\[\Gamma_{00}^\alpha\] being a Christoffel symbol.

\[\Gamma_{00}^\alpha \approx -\frac{1}{2} g_{\alpha}^{\alpha\beta} \frac{\partial g_{00}}{\partial x^\beta} = -\frac{1}{2} (-1) \frac{\partial g_{00}}{\partial x^\alpha} = \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(1 + \frac{2\varphi}{c^2}\right)\]

\[\Gamma_{00}^\alpha = \frac{1}{c^2} \frac{\partial \varphi}{\partial x^\alpha}\]

\[R_{00} = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial x^{\alpha\beta}}\]

\[\frac{\partial^2 \varphi}{\partial x^{\alpha\beta}} = 4\pi G \rho\]

3
\[ \nabla^2 \varphi = 4\pi G \rho \]
\[ \varphi = -\frac{1}{4\pi} \int \frac{4\pi G \rho dV}{r} = -G \int \frac{\rho dV}{r} \]

\( V \) being the volume, since \( \nabla^2 (1/r) = -4\pi \delta(r) \), where \( \delta(r) \) is the Dirac delta function: \( \delta(r) = +\infty \) for \( r = 0 \) and \( \delta(r) = 0 \) for \( r \neq 0 \) and \( \int \delta(r) dV = 1 \); and \( \nabla^2 \varphi = -G/\rho \nabla^2 (1/r) dV = 4\pi G/\rho \delta(r) dV = 4\pi G \rho \int \delta(r) dV = 4\pi G \rho \). For a group of \( n \) particles

\[ \varphi = -G \sum_n \frac{M_n}{r_n} \]

\( M_n \) being the masses of the particles and \( r_n \) the distances from them to the field points. And for a single particle

\[ \varphi = -G \frac{M}{r} \]
\[ F = -m_0 \frac{\partial \varphi}{\partial r} \]
\[ F = -G \frac{M m_0}{r^2} \]

\( F \) being the Newton gravitational attraction force between two particles of masses \( M \) and \( m_0 \) separated a distance \( r \).

