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On the Expanding Earth and Shrinking Moon

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ABSTRACT
Exactly 100 years ago, German scientist Alfred Lothar Wegener (1880 – 1930), sailed against the prevailing wisdom of his day when he posited that not only have the Earth's continental plates receded from each other over the course of the Earth's history, but that they are currently in a state of motion relative to one another. To explain this, Wegener setforth the hypothesis that the Earth must be expanding as a whole. Wegener's inability to provide an adequate explanation of the forces and energy source responsible for continental drift and the prevailing belief that the Earth was a rigid solid body resulted in the acrimonious dismissal of his theories. Today, that the continents are receding from each other is no longer a point of debate but a sacrosanct pillar of modern geophysics. What is debatable is the energy source driving this phenomenon. Herein, we hold that continental drift is a result of the Earth undergoing a secular radial expansion. An expanding Earth hypothesis is currently an idea that is not accepted on a general consensus level. Be that it may, we show herein that the law of conversation of angular momentum and energy entail that the Earth must not only expand as a consequence of the secular recession of the Earth-Moon system from the Sun, but invariably, that the Moon must contract as-well. As a result, the much sort for energy source driving plate tectonics can (hypothetically) be identified with the energy transfers occurring between the orbital and rotational kinetic energy of the Earth. If our calculations are to be believed – as we do; then, the Earth must be expanding radially at a paltry rate of about 1.50 mm/yr while the Moon is contracting radially at a relatively high rate of about −410 mm/yr.

Key words: astrometry – celestial mechanics – ephemerides – planets and satellites: formation.

1 INTRODUCTION

As is well known since about the late nineteenth century (*e.g.* Mantovani 1909, 1889), that; if the solid Earth did expand, one would *in principle* be able to explain the relative motion of the Earth's continental plates. The relative motion of the Earth's continental plates is technically and commonly referred to as *plate tectonics*[1]. In order that this expansion actually explain successfully the relative motion of the Earth's continental plates, it [expansion] would unambiguously have to be of the right magnitude capable of explaining the observed relative motion of the Earth's continental plates. In the this study – where the present reading is the first in a three part series; we consider the possibility that the Earth might be expanding radially as a whole solid body. This expansion is hypothesised to be driven by the observed recession of the Earth from the Sun *via* spin-orbit interaction. The laws of conservation of angular momentum and energy, when applied to the problem of the recession of the Earth-Moon system from the Sun; the pointer, strongly appears to point to the seemingly *difficult-to-dismiss* fact that the Earth must expand radially as a result, at a nominal rate of $\sim 1.50\,\mathrm{mm/yr}$ – this is about the average growth rate of human nails (see *e.g.* Zhi-Wei et al. 2012; Yaemsiri et al. 2010). This same phenomenon must also occur to the Moon as-well. We find that instead of expanding, the Moon must be contracting. Clearly, because we here infer the expansion of the Earth (and contraction of the Moon) from the cherished conservation laws (of angular momentum and energy), this study removes forthwith, the Expanding Earth Hypothesis (EEH) from the realm of speculation and pseudo-science, to that of plausibility. Unequivocally and unambiguously, we here put the EEH on a credible physical basis warranting further investigation.

[1] *Tectonics* is the field of geology which studies the processes which deform the Earth's crust.

1.1 Background

Since about 1596, scientists have long recognised that the opposing margins of continents fit together (see *e.g.* Romm 1994) in a manner suggesting that in the past, they formed one landmass, a hypothetical supercontinent popularly known as *Rodinia*[2] which is thought to have formed about 1 billion years ago and to have embodied most if not all of Earth's continents. This hypothetical supercontinent is believed to have broken up into eight continents some 600 million years ago (see *e.g.* Li et al. 2008). These eight continents later re-assembled into another supercontinent called *Pangaea*. Later, Pangaea broke up into *Laurasia* (which became North America and Eurasia of today) and *Gondwana* (which became the remaining continents).

Though the idea of plate tectonics had been known since about 1596, in its modern form, it was given a new birth certificate – *alas*, an almost stillborn birth certificate, by the German astronomer, turned meteorologist Alfred Lothar Wegener ($1880-1930$). Though widely rejected by his contemporaries, Wegener ($1912a$,2)'s idea has become the *headstone* and *chief-corner-stone* of modern geology and geophysics. Wegener's mobilist idea of drifting continents contrasted sharply with the contemporary view of fixed and rigid continents, and a shrinking Earth due to thermodynamic cooling.

While generally not accepted, Wegener's ideas did attracted a handful credible supporters such as the renowned Afrikaner geologist, Alexander Logie *du* Toit ($1878-1948$) from South Africa (whose work is summed up in the book *du* Toit 1937) and English geophysicist, Arthur Holmes ($1890-1965$) from England. At its inception, Wegener pieced together evidence from diverse fields to prop his theory. Prior to this, as early as 1910, Wegener – like others before him; realized that the continents had probably been together in the distant past, they must have drifted apart to form the arrangement that we see today. In his lifetime, Wegener wrote and published three books on the idea (Wegener 1915, 1922,9), were in each book, he sort to expound on the idea, each time making the endeavour to improve the theory in each version of the three editions.

Despite Wegener's ideas being generally criticized on both geophysical and geological grounds, he presented a large and significant body of very strong ponderable evidence in support of continental drift, but the mechanism driving these plates remained elusive. To this day, the energy source driving continental drift is not known and its search is one of the "*holy grails*" of modern plate tectonics theory. What is the energy source driving the motion of plates? For example, planets generally consist of concentric shells of matter, except for the Earth which has its unique, two-component surface, comprising of about 41% continental rock and with the reminder being quite different ocean floor basalt. To the present day, there has been no satisfactory explanation for the partial, crustal continental rock layer, except by assuming that in the distant past, the Earth was smaller and subsequently expanded (Hilgenberg 1962, 1933).

As already afore-stated, the principal impediment and shortcoming to this otherwise interesting idea of an expanding Earth is and has always been the lack of knowledge of a mechanism that could provide the necessary energy (Beck 1982; Cook and Eardley 1961) to drive this expansion without departing from the known physical laws of nature (Jordan 1971). To this conundrum and quagmire, Adrian E. Scheidegger (1982) stated concisely the view:

> *"Thus, if expansion on the postulated scale occurred at all, a completely unknown energy source must be found."*

This view, is as valid today as it was in 1982 when Scheidegger made this pronouncement. To that end, amongst others, this reading seeks to furnish the energy source driving the expansion. This energy must be coming from the locked-up gravitational binding energy that is being unlocked in the energy transfers occurring between the orbital angular momentum and the spin angular momentum of the Earth.

In-closing the present section; from Scheidegger (1982)'s comment above, it appears he (and many others) envisage a once-off expansion. As we will see herein that – like star formation is an ongoing process and not a once-off event where stars are created at a special and particular moment in the history and evolution of the Universe; the expansion of the Earth is an ongoing process that must have began at the beginning once the Earth and the Moon where formed and began their secular recession from the Sun.

1.2 Problem Statement and Substantiation

While it is well known that an EEH can *in principle* explain a number of currently inexplicable geophysical phenomenon such as plate tectonics (see *e.g.* Carey 1975, and references therein), current *scientific consensus* rejects forthwith, any expansion or contraction of the Earth. In very recent times, the EEH is rejected on the solemn grounds of observations because on a level accuracy of $+2.00\,\mathrm{mm/yr}$, current observations using modern high-precision geodetic techniques (Fukumori 2011) indicate that the Earth's radius has remained constant throughout its ponderable history. Before dismissing the EEH, a most logical and natural question to ask is, what is the expansion rate of the Earth that is required to explain plate tectonics?

If the expansion of the Earth required to explain plate tectonics is significantly smaller than the observed $+2.00\,\mathrm{mm/yr}$ or is of the same order of magnitude as the sensitivity of the measurement, then, the current position that the Earth is not expanding requires much more sensitive measurement before sending this idea to the "*Science Museum of Great but Failed Ideas*". Or, if the expansion rate of the Earth is significantly less than Fukumori (2011)'s measurement of $+2.00\,\mathrm{mm/yr}$, then – at best, Fukumori (2011)'s measurements place an upper limit to the Earth expansion and not the conclusion that they reached, that the Earth is not expanding.

As will be demonstrate herein, it appears that Fukumori (2011)'s threshold of sensitivity is within the same range or order of magnitude as the signal. An application of the sacrosanct and embellished laws of conservation of momentum and energy to the Solar receding Earth-Moon system leads us to conclude that the Earth must be expanding on a scale of about $+1.50\,\mathrm{mm/yr}$. In comparison, $+1.50\,\mathrm{mm/yr}$ is

[2] This word is derived from the Russian word *rodit* which means "*to give birth*" (see *e.g.* Li et al. 2008).

not only about the same order of magnitude[3] as Fukumori (2011)'s threshold of sensitivity, the signal to sensitivity ratio is about $1 : 0.75$ – simple, the threshold of sensitivity very low. There is thus need to improve the signal-to-sensitivity ratio to about $1 : 10$ measured upper limit of $+2.00\,\mathrm{mm/yr}$, thus making Fukumori (2011)'s measurements inadequate to rule out an Expanding Earth Hypothesis since they are not sensitive enough to discern the implied expansion of the Earth. Placing our confidence in the law of conservation of momentum and energy, this is the position that we take.

Therefore, the Earth may very well be expanding and the source of the energy of this expansion being the lost orbital angular momentum of the Earth-Moon system which is a result of the secular recession of this system from the Sun. If the Earth is expanding, then, evidence of this expansion should come not only in the form of relative motion of the landmasses, but also in the form of a Hubble-type motion of the landmasses. Therefore, this study is a preparatory study of a possible future study that will seek to quantity whether or not the continents are undergoing a Hubble-type flow from one another. If this can be ascertained, it would be the most revealing evidence yet that indeed, the Earth is undergoing an expansion.

Last and most important of all is that, current plate tectonics theory is incomplete in that it provides no appreciable and adequate energy source for geodynamics and it critically depends on the assumption of mantle convection. There is need to identify the energy source. It is the goal of this study to suggest or identify a plausible energy source driving the continents apart.

1.3 Hypothesis

That the continents are moving relative to each other is no longer a point of debate. What is debatable is the energy source driving this phenomenon. Herein, we hold that continental drift is a result of the Earth undergoing radial expansion. This expansion is driven or powered by energy transfer occurring between the rotational kinetic of the Earth's spin and the rotational orbital angular momentum kinetic energy. It is well established fact that the Moon is receding from the Earth at a rate of about $38.247 \pm 0.004\,\mathrm{mm/yr}$ (Williams and Boggs 2009; Chapront et al. 2002), this must lead to a loss of orbital angular momentum for both the Earth and the Moon as individual bodies. In addition to this, the Russian astronomers Krasinsky and Brumberg (2004) and the American astronomer Standish (2005) reported that the mean Sun-Earth-Moon distance known as the Astronomical Unit (denoted $\mathrm{AU} = 1.49598000 \times 10^9\,\mathrm{m} \pm 3\,\mathrm{m}$) is undergoing a secular positive change of $\delta\mathrm{AU} = 150.00 \pm 3.00\,\mathrm{mm/yr}$ and $\delta\mathrm{AU} = 70.00 \pm 2.00\,\mathrm{mm/yr}$ respectively. To up-hold the law of conservation of angular momentum (*i.e.*, the sum total of the orbital and rotational angular momentum), this lost orbital angular momentum can not be lost into the oblivious; it must be transferred to the spin of the respective bodies, thus leading to changes in the sizes and the spin periods of these celestial bodies since the spin depends on the size and the spin period.

[3] If the signal-sensitivity-ratio is less the $1 : 10$, the signal is said to be of the same order of magnitude as the sensitivity.

Corollary

If the Earth is expanding globally, then, evidence of this expansion should come not only in the form of relative motion of the landmasses, but also in the form of a Hubble-type motion of the landmasses. That is, if one has a sphere of radius R that undergoes a global radial expansion at a rate $\dot{R}$, then, any two points on the surface that are seperated by a distance $\mathcal{D}$ will undergo relative motion such that their relative rate of separation $\dot{\mathcal{D}}$ will be given by:

$$\dot{\mathcal{D}} = \left(\frac{\dot{R}}{R}\right)\mathcal{D}. \tag{1}$$

Therefore, a direct verification or refutation of a globally expanding Earth is to undertake a study that seeks to quantify whether or not the continents are undergoing a Hubble-type flow relative motion. A Hubble-type flow of the landmasses is but the clearest signature yet, of an expanding solid Earth. If this can be ascertained, it would be the most revealing evidence yet, that indeed, the Earth is undergoing an expansion and this expansion is causing the continents to drift apart. Hence, this work serves as a precursor or preparatory work for a future quantitative study of global plate tectonics with the aim of a verifying or refuting the hypothesis of a globally expanding Earth.

Other than a radial expansion of the Earth, it is possible that other geodynamic forces may come into play and contribute to plate tectonics such as mantle conventional currents which are thought to be the major driving force of plate motion. Lateral density variations in the mantle are believed to result in these convectional currents which if they exist, will lead to the angular displacement ($\Delta\theta$) of the plates. Whatever the cause of the mantle currents, if they exist, they can be taken into account in (1) by adding an angular displacement term, that is:

$$\dot{\mathcal{D}} = \left(\frac{\dot{R}}{R} + \frac{\dot{\theta}}{\theta}\right)\mathcal{D}. \tag{2}$$

If a graph of $\dot{\mathcal{D}}$ *vs* $\mathcal{D}$ were to be plotted, for those plates where conventional currents are not present, these will lie on a main straight line graph whose slope is $\dot{R}/R$ and passes through the origin, while for those plates where conventional currents are present – depending on the magnitude of the currents; these plates will lie off-set from the main straight line. Thus, the effect of the mantle currents on the graph $\dot{\mathcal{D}}$ *vs* $\mathcal{D}$ is to produce a graph with scatter of points about the main straight line graph.

Before we leave this section, it is perhaps important to make a glimpse into what to expect for the expansion rate of the Earth – we have to make a crude calculation. We know that current plate tectonic measurements indicate that the plates are moving at a rate of $\sim +10 - 160\,\mathrm{mm/yr}$. According to our proposed Hubble type expansion theory as laid down above, if we assumed that the angular component is negligible, *i.e.* $\dot{\theta} = 0$, and given that the average distance between continental margins should be about $1/12^{th}$ of the circumference of the Earth (*i.e.*, $\mathcal{D} \simeq 5 - 10,000\,\mathrm{km}$, which corresponds an angular displacement $\Delta\theta \sim 30^\circ$), from this crude information and the Hubble type expansion model of the Earth, the expected expansion rate of the Earth must be of the order of $+1 - 100\,\mathrm{mm/yr}$. So, as we workout the implied expansion rate from the conservation of energy and

angular momentum, we shall keep at the back of our mind the afore-calculated expansion rate – this will act more as a guide.

1.4 Aim

This rather brief study explores the viable possibility of a secular expansion of the Earth. This secular expansion is hypothesised to be a direct engender of the observed secular recession of the Earth from the Sun. The supreme and paramount aim of the study is to suggest or posit that the observed continental drift may very well be a result of an expanding Earth.

1.5 Objectives

The objectives of this study (as enshrined in the three part series of readings) are as follows:

(i) Judiciously apply the law of conservation of angular momentum to the Earth-Moon system - *posteriori* justified; the sole aim of which is to unearth a plausible astrophysical link to global geo-plate tectonics.

(ii) Calculate the implied radial expansion and or contraction of the of the Earth and the Moon.

(iii) Show that the rotational kinetic of the Earth-Moon system is a viable, potent and plausible source energy capable of driving global plate tectonics.

(iv) By way of a literature study, set the stage for further work on the possibility of an astrophysical link to global geo-plate tectonics.

(v) Present a new plausible theory on the way the Earth-Moon system might have formed. A backward extrapolation of the evolution of the Earth-Moon system for an expanding Earth, contracting Moon and a receding Earth-Moon leads to a very interesting paradigm that might explain many puzzles about the the nature of the Earth-Moon system.

1.6 Justification

Since the idea of plate tectonics was given birth to, the source driving this activity has eluded those that have made the endeavour to decipher the source. Finding this source is not only of great importance to geology and geophysics, but to science in general because it would lead to a better understanding of the forces operating in our terrestrial habitat. Thus, the paramount nature and justification of the present endeavour cannot be under or overstated.

1.7 Approach to Problem

Our present approach to the problem of finding a plausible energy source that is driving the drifting of continental plates is purely a theoretical one; where-after, it is anticipated that data will be gathered (from existing geophysical stations and from Global Positioning System satellites) in the near future to consolidate the ideas propagated herein. We demonstrate that from the embellished, sacrosanct and cherished law of conservation of angular momentum and energy, that, the Earth is expected to expand radially if the Earth-Moon system is receding from the Sun as determined from the observations of Pitjeva (2005); Standish (2005); Krasinsky and Brumberg (2004).

1.8 Motivation

Only until recently, it has become clear that our understanding of the gravitational phenomenon is in serious dearth. The emerging picture was now that where our knowledge of the gravitational phenomenon appeared to be taking a good shape. Alas, the opposite is actually the case. This position has been brought about because of the improvement in our advancement in technology. There has been a recent upsurge of gravitational "anomalies" and this is a direct result of the higher resolution measurements brought by the aforesaid technological advancements. Gravitational anomalies have puzzled the scientific community for quite sometime now. First, was the discovery of the so-called darkmatter by the Swiss astronomer Fritz Zwicky (1933), followed by the Pioneer anomaly in the late 1980's by the United States *of* America's National Aeronautic Space Administration (*NASA*) scientists Anderson et al. (1998, 2002), then come the Earth-flyby anomalies in the early 1990's again by *NASA* scientists (*e.g.* Anderson et al. 2008) and more recently, the secular increase in the mean Earth-Sun and mean Earth-Moon distance by independent groups of American and Russian astronomers (Williams and Boggs 2009; Pitjeva 2005; Standish 2005; Krasinsky and Brumberg 2004). What really is going on with gravitation? What is the matter? Do we really understand gravitation? Why suddenly an upsurge of these gravitational anomalies? For a conscience review of Solar gravitational anomalies, see *e.g.* Anderson and Neito (2009).

The widely accepted and dominate gravitational model in contemporary physics is Einstein (1916)'s General Theory *of* Relativity (GTR). Against the desideratum, Einstein's GTR is unable to deliver non-*ad hoc* and non-*impromptu* solutions to these problems. That the need for new fresh ideas on this front is imminent, is something that few researchers on these frontiers doubt not, but feel strongly that, this is the way forward. Our motivation is thus the desire to understand the gravitational phenomenon by seeking improved models of gravitation that are able to explain most if not all of these gravitational anomalies from a unified standpoint, this is, explain these using one model and not models designed only to address a particular anomaly.

1.9 Synopsis

The present reading is organised as follows. In the subsequent section, we lay down the proposed theory that we put forward as a plausible explanation of global plate motion. It is not a new theory in the traditional sense of bringing new exotic concepts, but merely a direct application of the law of conservation of angular momentum on the Earth-Moon system. In §(3), we make preparatory calculations that are necessary to quantity the 'new' ideas. In §(4), we derive a quantifiable formula giving the expansion rate of the Earth. In §(5), we apply the 'new' ideas to physically measured data. In our application, we check if the 'newly' founded ideas yield reasonable physical quantities that are in reasonable correspondence with experience. Lastly, in §(6), we

give a general discussion, the conclusions drawn thereof and the recommendations that we feel can be taken up by future studies.

2 THEORY

Both Newtonian and Einsteinian gravitation assume that the orbital angular momentum of a planet orbiting the Sun must be a conserved quantity, thus the emergence of the recession of the Earth-Moon system from the Sun and as-well the recession of the Moon from the Earth has come as nothing short of a Newtonian and Einsteinian surprise. Recessional motion of the Earth-Moon system and the Moon from the Earth have implications on the angular momentum. Since the magnitude of the orbital angular momentum $\mathcal{J}$ depends on the mass $\mathcal{M}$, orbital radius $\mathcal{R}$, and the period $\mathcal{T}$, that is $\mathcal{J} = 2\pi\mathcal{M}\mathcal{R}^2/\mathcal{T}$, if $\dot{\boldsymbol{\mathcal{J}}} = 0$, then, either the mass or the orbital period must change so as to compensate for the change in orbital radius. In the case of Solar planet, their masses can be considered to be constant, thus this leaves the period as the only quantity to vary in-order to compensate for the change in orbital radius. As already argued in Nyambuya et al. (2013), data spanning well over 2700 years of the length of the day indicate that the assumption $\dot{\boldsymbol{\mathcal{J}}} = 0$ does not hold. This means Newtonian and Einstein gravitation are inadequate in so far as explaining the he recession of the Earth-Moon system from the Sun and as-well the recession of the Moon from the Earth.

Prima facie, it would then appear as though the sacrosanct law of conservation of angular momentum is here being violated. However, a closer inspection will reveal that this is not the case. In general, what must be conserved is not the orbital angular momentum but the total angular momentum. The total angular momentum ($\boldsymbol{\mathcal{L}}$) includes two kinds of angular momentums, *i.e.* the orbital angular momentum ($\boldsymbol{\mathcal{J}}$) and the spin angular momentum ($\boldsymbol{\mathcal{S}}$), *i.e.* $\boldsymbol{\mathcal{L}} = \boldsymbol{\mathcal{J}} + \boldsymbol{\mathcal{S}}$ is what must be conserved, *i.e.* $\dot{\boldsymbol{\mathcal{L}}} = 0$. This means that $\dot{\boldsymbol{\mathcal{J}}} = -\dot{\boldsymbol{\mathcal{S}}}$; this can be rewritten in a more convenient way as:

$$\frac{\dot{\boldsymbol{J}}}{J} = -\left(\frac{S}{J}\right)\frac{\dot{\boldsymbol{S}}}{S} - \left(\frac{\dot{\mathcal{M}}}{\mathcal{M}}\right)\frac{\boldsymbol{J}+\boldsymbol{S}}{J}, \tag{3}$$

where $\boldsymbol{J}$ and $\boldsymbol{S}$ are the specific orbital angular momentum and the spin - by specific, we mean per unit mass.

In the reading Nyambuya (2010) where an alternative model of gravitation coined the Azimuthally Symmetric Theory of Gravitation (hereafter ASTG-model) has been posited; after the realisation that the ASTG-model required that the orbital angular momentum be not a conserved quantity, the above idea that the total angular momentum is what must be conserved was proposed as a necessary, straight forward and logical manner to preserve the law of conservation of angular momentum. This proposal does not in any way violate any of the known Laws *of* Physics but is wholesomely and completely within the framework and realm of the known Laws *of* Physics. The added and interesting outcome of this is that, it brings into effect the possibility of *spin-orbit interaction*. This idea is the central theme of the present reading.

The next central idea is the conservation of energy. An object orbiting some central massive body is going to have some rotation kinetic energy $K_{\rm orb}$ and as well some gravitational potential energy $U_{\rm orb}$. The total energy associated with its orbit about this central massive body $E_{\rm orb} = K_{\rm orb} + U_{\rm orb}$, is conserved, *i.e.* $\dot{E}_{\rm orb} = 0$. From this is follows that:

$$K_{\rm orb}\left(\frac{\dot{K}_{\rm orb}}{K_{\rm orb}}\right) + U_{\rm orb}\left(\frac{\dot{U}_{\rm orb}}{U_{\rm orb}}\right) = 0. \tag{4}$$

Actually, the assumption of the conservation of the total energy associated with the orbit of test body is central not only to Newtonian gravitational but Einsteinian gravitation as-well, so this idea is sound and valid.

The next conservation is that of the energy associated with the spin of the orbiting and spinning test body. From what has been stated above, the rotational orbital kinetic energy $K_{\rm orb}$ interacts in a conserved manner with the gravitational potential energy $U_{\rm orb}$ of the test body. In the same manner, the spin must interact with the total stored gravitational energy $U_{\rm g}$ *i.e.*, for an object of mass and radius $\mathcal{M}_{\rm obj}$ and $\mathcal{R}_{\rm obj}$ respectively, the total stored gravitational energy $U_{\rm g} = -G\mathcal{M}^2_{\rm obj}/\mathcal{R}_{\rm obj}$. The total energy $E_{\rm spin} = K_{\rm spin} + U_{\rm g}$, must conserved, *i.e.* $\dot{E}_{\rm spin} = 0$. From this is follows that:

$$K_{\rm spin}\left(\frac{\dot{K}_{\rm spin}}{K_{\rm spin}}\right) + U_{\rm g}\left(\frac{\dot{U}_{\rm g}}{U_{\rm g}}\right) = 0. \tag{5}$$

Thus the present theory is summed up in the three conservation equations (3), (4) and (5). Undoubtedly, the foundation of our theory is not without a firm and valid basis.

3 PRELIMINARY COMPUTATIONS

We are now going to apply the ideas of the presiding section to the Sun-Earth-Moon system. Our first port of call is to establish a single value for the recession of the Earth-Moon system from the Sun. The mean distance from the Sun of the Earth-Moon system is referred to as the Astronomical Unit and denoted by the symbol AU. Let us represent the secular change in the Astronomical Unit by δAU. As stated earlier, at present, there are two values for this quantity, that is, the Russian astronomers Krasinsky and Brumberg (2004) find $\delta\text{AU} = +150.00 \pm 3.00\,\text{mm/yr}$, while the American astronomer Standish (2005) finds $\delta\text{AU} = +70.00 \pm 2.00\,\text{mm/yr}$. From these two values we need the best estimate. For this, we need to appeal to statistical methods to find a best estimate.

Assuming that these two measurements are governed by Gaussian statistics and that the errors in the measurements random and independent, then, the best estimate of these two measurements can be obtained by taking the weighted mean of the two values. For example if $(x_i + \delta x_i : i = 1, 2, \ldots n)$ is set of n measurements of a constant quantity x, where x_i is the best value of for the n^{th} measurement and δx_i is its accompanying error margin, then, the best estimate of $x_{\rm best}$ from this set is $x_{\rm best} = \sum w_i x_i / \sum w_i$ where w_i are the weights such that $w_i = 1/(\delta x_i)^2$ and the best estimate in the error margin $\delta x_{\rm best}$ is $\delta x_{\rm best} = (\sum w_i)^{-1/2}$ (see *e.g.* Taylor 1982, p.150). Applying this prescription to the two measurements of Standish (2005); Krasinsky and Brumberg (2004), we obtain:

$$\delta\text{AU} = +95.00 \pm 2.00\,\text{mm/yr}. \tag{6}$$

We shall from heron adopt this value (6) as representative of the change in the mean distance between the Sun and Earth-Moon system.

The maximum distance of the Earth from the Sun $\mathcal{R}_{\rm orb}^{\rm max} = 1.52098232 \times 10^{11}\,{\rm m}$ and minimum distance is $\mathcal{R}_{\rm orb}^{\rm min} = 1.47098290 \times 10^{11}\,{\rm m}$ (Standish and Williams 2010). In our calculation, we need one single value for the mean distance between the Sun and the Earth-Moon system. From $\mathcal{R}_{\rm orb}^{\rm min}$ and $\mathcal{R}_{\rm orb}^{\rm max}$, the best estimate would the average of these two values, that is, $\mathcal{R}_{\rm orb}^{\rm best} = (\mathcal{R}_{\rm orb}^{\rm max} + \mathcal{R}_{\rm orb}^{\rm max})/2$ and the best estimate in the error margin to this value is $\delta\mathcal{R}_{\rm orb}^{\rm best} = (\mathcal{R}_{\rm orb}^{\rm max} - \mathcal{R}_{\rm orb}^{\rm max})/2$, so that the best value for the mean distance between the Sun and the Earth-Moon system is:

$$\mathcal{R}_{\rm orb} = (1.50 \pm 0.03) \times 10^{11}\,{\rm m}. \tag{7}$$

Now, lets compute the spin angular momentum of the Earth-Moon system $S_{\rm em} = 2\pi(\mathcal{M}_{\rm e} + \mathcal{M}_{\rm m})\mathcal{R}_{\rm em}^2/\mathcal{T}_{\rm em}$. The mass of the Earth $\mathcal{M}_{\rm e}$ and the Moon $\mathcal{M}_{\rm m}$ are estimated to $5.9736 \times 10^{24}\,{\rm kg}$ and $7.3477 \times 10^{22}\,{\rm kg}$ respectively. The period $\mathcal{T}_{\rm em}$ which is the mean period of the Moon in its orbit around the Earth is estimated to be $27.321582/365.25\,{\rm yr}$. In order to calculate $\mathcal{S}_{\rm em}$, we are left with the value of $\mathcal{R}_{\rm em}$. At perigee, we have $\mathcal{R}_{\rm em}^{\rm min} = 3.62570 \times 10^{8}\,{\rm m}$ and at apogee, we have $\mathcal{R}_{\rm em}^{\rm max} = 4.05410 \times 10^{8}\,{\rm m}$. Using the same procedure that we used to estimate the best value for the mean distance between the Sun and the Earth-Moon system, we have for the mean distance between Earth and Moon $\mathcal{R}_{\rm em} = (3.80 \pm 0.20) \times 10^{8}\,{\rm m}$.

4 DERIVATION

We here derive the expansion rate formula for the Earth from the two concepts to be used the conservation of angular momentum and energy of the Earth-Moon system. The total angular momentum of the Earth-Moon system comprises six components, that is: (1) the orbital angular momentum of the Earth-Moon system $\boldsymbol{\mathcal{J}}_{\rm orb}$ about Solar center of mass, (2) the reduced mass term of the system $\boldsymbol{\mathcal{J}}_{\rm rm}$, (3) the orbital angular momentum of the Earth about the common center of mass $\boldsymbol{\mathcal{J}}_{\rm e}$, (4) the orbital angular momentum of the Moon about the common center of mass $\boldsymbol{\mathcal{J}}_{\rm m}$, (5) the spin angular momentum of the Earth about its center of mass $\boldsymbol{\mathcal{S}}_{\rm e}$ and lastly, (6) the spin angular momentum of the Moon about its center of mass $\boldsymbol{\mathcal{S}}_{\rm e}$. So, the total angular momentum $\boldsymbol{\mathcal{L}}$ is given by:

$$\boldsymbol{\mathcal{L}} = \boldsymbol{\mathcal{J}}_{\rm orb} + \boldsymbol{\mathcal{J}}_{\rm rm} + \boldsymbol{\mathcal{J}}_{\rm e} + \boldsymbol{\mathcal{J}}_{\rm m} + \boldsymbol{\mathcal{S}}_{\rm e} + \boldsymbol{\mathcal{S}}_{\rm m} = \boldsymbol{i}\mathcal{L}_x + \boldsymbol{j}\mathcal{L}_y. \tag{8}$$

In the about, the components $\mathcal{L}_x$ and $\mathcal{L}_y$ are the x and y components of $\boldsymbol{\mathcal{L}}$ and from Figure (1), one can deduce that these components are such that:

$$\mathcal{L}_x = \mathcal{S}_{\rm m}\sin\vartheta_{\rm m} - \mathcal{S}_{\rm e}\sin\vartheta_{\rm e} + J_{\rm orb}\sin\vartheta_{\rm orb}, \tag{9}$$

$$\mathcal{L}_y = \mathcal{J}_{\rm rm} + \mathcal{J}_{\rm e} + \mathcal{J}_{\rm m} + \mathcal{S}_{\rm m}\cos\vartheta_{\rm m} - \mathcal{S}_{\rm e}\cos\vartheta_{\rm e} + J_{\rm orb}\cos\vartheta_{\rm orb}. \tag{10}$$

For our purpose here, we shall make a simplifying assumption, namely that the axial tilt of both the Earth and the Moon has remained constant, that is, it does not change, this means we shall take $\dot{\vartheta}_{\rm e} = \dot{\vartheta}_{\rm m} = \dot{\vartheta}_{\rm orb} = 0$. It is very much possible and highly likely that the axial tilt angle of the Earth and Moon have changed over the course of their history and that currently, much in the same manner as other Solar secular changes, the axial tilt angle of these celestial bodies is changing. In the present expedition, we find no reason to invoke this possibility, thus, we simple let it to the rest, in which event, the resulting calculations are much easier to handle.

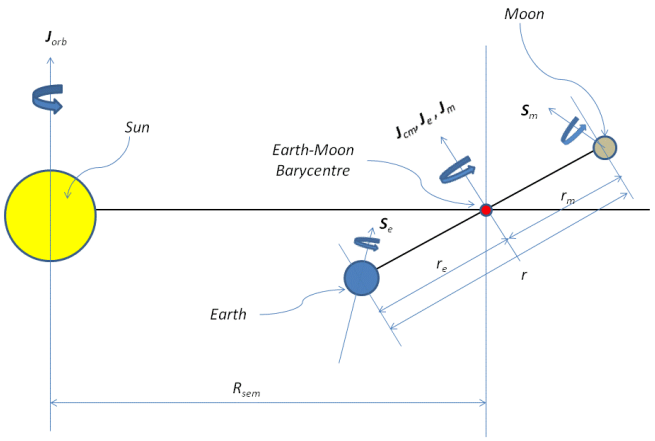

Figure (1). A schematic diagram of the Sun-Earth-Moon System.

Now, let us define the quantity L, which is such that:

$$L_{\rm e} = \cos\vartheta_{\rm m}\mathcal{L}_x - \sin\vartheta_{\rm m}\mathcal{L}_y. \tag{11}$$

Written in full, L is as given in (A.1). From the above assumptions that $\dot{\vartheta}_{\rm e} = \dot{\vartheta}_{\rm m} = \dot{\vartheta}_{\rm orb} = 0$, and as-well from the law of conservation of angular momentum ($\dot{\boldsymbol{\mathcal{L}}} = 0 :\Longrightarrow \dot{\mathcal{L}}_x = \dot{\mathcal{L}}_x = 0$), it follows that $\dot{L} = 0$. Using these facts in conjunction with the law of conservation of energy, the sought for formula is derived overleaf from (A.1) right to (A.6). In the next section, we shall apply this result to compute the predicted expansion rate of the Earth.

5 RESULTS AND ANALYSIS

We give here the results emerging from the derived formulae (A.6) and (A.7) for the Earth and the Moon systems in section (5.1) and (5.2) respectively. In section (5.3), we give an analysis of the results.

5.1 Earth System

Now, from (A.6), the expansion terms can be split into two, that is, Solar and Lunar contribution terms, that is:

$$\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}} = \left(\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}}\right)_{\odot} + \left(\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}}\right)_{\rm L}. \tag{12}$$

By inserting the relevant values, one finds that for the Solar contribution, we will have:

$$\left(\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}}\right)_{\odot} = +(2.40 \pm 0.20) \times 10^{-10}\,{\rm yr}^{-1}, \tag{13}$$

and for the Lunar contribution, we will have:

$$\left(\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}}\right)_{\rm L} = +(5.10 \pm 0.50) \times 10^{-13}\,{\rm yr}^{-1}. \tag{14}$$

Clearly, the Lunar contribution is far weaker than the Solar contribution, it is so weak so much that, we can, for our purposes here, neglect it altogether. From this, we can safely conclude that, whatever force is driving the recession of the Earth-Moon system from the Sun, it is this same phenomenon that should lead to the Earth to expand. Therefore, the recession of the Earth-Moon system from the Sun leads to the expansion of the Earth which in-turn leads to plate tectonics. From (13), the corresponding rate of expansion of the Earth is:

$$\delta R_{\rm e} = +1.50 \pm 0.10\,{\rm mm/yr}. \tag{15}$$

5.2 Moon System

At its conception, this reading never meant to touch seriously on matters to do with the Moon. Actually, after we had calculated and demonstrated that the Earth might be expanding, we felt that was enough, the reading must be sent off for publication. On second thoughts, we felt that the reviewers might ask, "*Since this is within the realm of your calculation, what does your calculation give for the Moon?*". It is only then, that we felt, we must address this matter before we are compelled to do so. We had envisaged a simple reading where we report that the Earth-Moon drift from the Sun, most certainly is the driver to geo-plate tectonics. The same calculation as that for the expansion rate of the Earth, was conducted for the Moon when upon we arrived as (A.7).

Substituting the relevant figures into this formula, one notices that as in the case of the Earth, the Lunar recession's contribution is minute compared to that Solar recession term, that is:

$$\left(\frac{\delta\mathcal{R}_{\rm m}}{\mathcal{R}_{\rm m}}\right)_{\odot} = -(2.35 \pm 0.20) \times 10^{-7}\,{\rm yr}^{-1}, \tag{16}$$

and:

$$\left(\frac{\delta\mathcal{R}_{\rm m}}{\mathcal{R}_{\rm m}}\right)_{\rm L} = +(1.38 \pm 0.20) \times 10^{-10}\,{\rm yr}^{-1}. \tag{17}$$

The Lunar recessional term leads to an expansion of $+0.20\,{\rm mm/yr}$ while the Solar recessional term leads to a contraction of $\sim -410\,{\rm mm/yr}$. Obviously, the expansion is dwarfed by the contraction so that overall, the Moon must be in a state of contracting. Precisely, one find that:

$$\delta R_{\rm m} = -410.00 \pm 0.10\,{\rm mm/yr}. \tag{18}$$

At the time we performed this calculation, our ideas of the geology of the Moon where the pre-2010 ideas; we thought the Moon must be a geological dead piece of rock. Our first reaction to the idea that the Moon might be contracting was that of scepticism. The Moon can not be contracting, especially at such a significant rate of $\sim -410.00 \pm 0.10\,{\rm mm/yr}$.

As any researcher would do nowadays, we "hit" *google*. Much to our surprise, elation and delight, we found out that only recent have Watters et al. (2010) found the first evidence that the Moon is undergoing global contraction. Watters et al. (2010) reported that this evidence has been brought forth by *NASA*'s Lunar Reconnaissance Orbiter Camera (LROC), launched in 2009 aboard the Lunar Reconnaissance Orbiter – which is the first spacecraft to be launched by *NASA* as part of it's "Return to the Moon" initiative.

The LROC photographed Lunar scars known as lobate scarps. Lobate scarps occur when the surface of the geological body experiences a compressional force, causing one part of the upper surface to fold and fracture above the other part. In the absence of significant tectonics on the Moon, Watters et al. (2010) believe this is due to cooling of the Lunar core. If our ideas are correct or prove to be correct in the future, then, this contraction is not due to cooling as Watters et al. (2010) believe, but gravitational contraction caused by the transfer of orbital angular momentum into spin angular momentum and this been a result of the observed secular recession of the Earth-Moon system. As the core of the Moon cooled it also shrunk, applying surface stress to the brittle Lunar crust and causing it to rupture and split.

Lobate scarps have been observed on the surface of the Moon before *i.e.*, from images taken by the panoramic cameras aboard the Apollo 15, 16 and 17 missions (Watters et al. 2010; Binder and Gunga 1985). These earlier missions were confined to the equatorial region of the Moon's surface thus giving not a picture of this being a global phenomenon (Binder and Gunga 1985). Using the LROC, Watters et al. (2010) managed to acquire comprehensive images of the Lunar surface at higher latitudes. The lobate scarps seem to have formed relatively recent – about a billion years or so (Watters et al. 2010) and the most recent analysis by Banks et al. (2012) of these lobate scarps further points to Lunar global contraction.

Other than lobate scarps, the Lunar surface has recently been shown to contain a significant number of *grabens* (Watters et al. 2012). Grabens are tectonic features that form under extension stresses or tension of the landmass. Structurally, they are comprised of two normally intersecting faults, with a down-dropped block between them. Most grabens are found within the Lunar maria near the edges of large impact basins. Despite the presence of these grabens, Watters et al. (2012) believe the Moon should still be undergoing global contraction. Of this (*i.e.*, Lunar contraction), the lead author from the United State *of* America's *Center for Earth and Planetary Studies*, Thomas Watters had this to say:

> *"We think the Moon is in a general state of global contraction due to cooling of a still hot interior. The graben tell us that forces acting to shrink the Moon were overcome in places by forces acting to pull it apart. This means the contractional forces shrinking the Moon cannot be large, or the small graben might never form.*[4] *"*

In conclusion on the matter of Lunar contraction, *i.e.*, the Lunar contraction rate as calculated herein, we have this to say. While we are very skeptical of a Lunar contraction rate as high as $\sim 410\,{\rm mm/yr}$, if we have to learn from history, we have to place aside our skepticism and accept the result pending verification or refutation from observations. This calculation is inferred from the law of conservation of angular momentum and energy, we have no choice but to

[4] Quote from http://newsdesk.si.edu/releases/new-images-show-recent-geologic-activity-moon. Accessed on this day 4/12/2012 15*h*04 GMT+2.

place, not our faith in the calculation, but our confidence in the calculation, confidence that, it is highly likely that this result will in the near future be verified. Our skepticism or any that can be brought forth is nothing but skepticism whose strong foundations is based on prejudice of a scientific nature.

Is it not scientific prejudice that denied Albert Einstein $(1879-1955)$ the monumental and once in a lifetime opportunity to predict that the Universe might be expanding? Is it not scientific prejudice that stalled Wegener's hypothesis of continental drift? Is it not scientific prejudice that might have lead to the rejection of the celebrated French Prince, Louis Victor Peirre Raymond *de* Brogile $(1872-1946)$'s wave-particle duality leading to a great stagnation in the developed of quantum mechanics? While not endless, the list is long. Sometimes, we must accept the bare facts before us, and if *Nature* is to embarrass us, we must be humble enough to accept this as an honour rather than a dishonour. We would rather be 'embarrassed' by *Nature* than by anything else. *Nature* is the greatest teacher of all, we must learn from the best.

5.3 Energy Source for Platetectonics

The energy to drive plate tectonics must come from the energy locked-up in the spin of the Earth. This energy is unlocked when the Earth-Moon system recedes from the Sun. The total energy locked up in the spin is $K^{\rm e}_{\rm spin} = \mathcal{S}^2_{\rm e}/2\mathcal{M}_{\rm e} \sim +5.20\times10^{29}\,{\rm J}$. When the Earth expands, energy is drawn at a rate $\delta K^{\rm e}_{\rm spin} = \mathcal{S}_{\rm e}\delta\mathcal{S}_{\rm e}/\mathcal{M}_{\rm e}$, that is:

$$\delta K^{\rm e}_{\rm spin} = \frac{8\pi^2\mathcal{M}_{\rm e}\mathcal{R}^4_{\rm e}}{\mathcal{T}^2_{\rm e}}\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}} = +2.45\times10^{34}\,{\rm Jyr}^{-1}. \qquad (19)$$

This is at a rate of about $+2.40|\mathcal{L}_\odot|$, that is, the power driving plate tectonics is about two and half times the power of the Sun.

In the case of the Moon which is must be contracting at the rate of $\sim -410\,{\rm mm/yr}$, it must be giving off energy at a rate $\sim -7.10\times10^{22}\,{\rm Js}^{-1} = 0.0002\mathcal{L}_\odot$.

All this energy that driven the expansion of the Earth and the contraction of the Moon is drawn from the lost orbital kinetic energy of the Earth-Moon system $K_{\rm orb} = \mathcal{J}^2_{\rm orb}/2(\mathcal{M}_{\rm e}+\mathcal{M}_{\rm m})$. This orbital kinetic energy of the Earth-Moon system is given of at a rate $\delta K_{\rm orb} = \mathcal{J}_{\rm orb}\delta\mathcal{J}_{\rm orb}/(\mathcal{M}_{\rm e}+\mathcal{M}_{\rm m}) = +4.90\times10^{36}\,{\rm Js}^{-1} = +1.50\times10^{10}|\mathcal{L}_\odot|$, this is the of the order of the luminosity of a typical galaxy! Clearly, there is more than enough energy to drive the Earth's plates apart. This recession of the Earth-Moon system certainly unlock a lot of energy from the orbital kinetic energy.

6 DISCUSSION, CONCLUSION AND RECOMMENDATIONS

Below we present a general discussion, the conclusion drawn thereof, and the recommendations we make for future studies and endeavours toward investigations into the possibility that the Earth might actually be undergoing a secular expansion as inferred herein.

6.1 General Discussion

Judiciously and with great equanimity, allow us to say this at the outset of the present section, namely that, while the ideas propagated herein *spring-forth* from non-exotic nor exogenous ideas but directly from the provinces of the well accepted Physical Laws, if proven correct – as we strongly believe they will; it goes without saying that their implications are nothing short of asymptotically pivotal insofar as our understanding of geo-plate tectonics is concerned. Judging from the rich history of the introduction of new ideas, we should say that, we do not expect a smooth passage and acceptance of the present ideas but more a fierce defence against them.

Be that it may, one thing is however certain – that is, the interpretation and subsequent implication of the recession of the of the Earth-Moon system here conducted, is nothing but a logical and straight forward interpretation despite the nature of the conclusion drawn thereof. To ourself, we merely have made the simplest and logical imaginable endeavour to interpret facts from within the accepted provinces of contemporary physics by applying the sacrosanct and embellished law of conservation of angular momentum. As to whether or not the ideas should be accepted or rejected, this we shall safely leave in the hands of our contemporaries and posterity to decide.

When one traces the history of the Earth as depicted by the present ideas, they can not help but entertain the idea that it is highly likely that all the lands of the Earth most certainly emerged from beneath the waters that once covered all of the Earth – the Earth should have constituted a perfect *geoid*[5]. If one imagines the Earth's crust as comprising a *firmament*[6], and knowing very well that beneath and within the Earth's crust lies some complex waters systems, then, the emergent picture is that where in the beginning, the waters once where above and below the firmament, the firmament once separated the waters from the waters. With the progression of time, a portion of the firmament must have emerged above the waters (which lie above the firmament) to form the lands we inhabit today and so much cherish as a sacrosanct priceless ancestral possession and heritage.

The idea of an expanding Earth, contraction Moon, receding Moon and receding Earth-Moon system has serious implications on the formation of the Earth-Moon system if one where to wind back the hand of time for such a setting. Initially, we had included a section on this but on second thoughts we felt this would lead to a difficult reading. So, we decided to separate this into a stand-alone reading where this issue of the formation of the Earth-Moon system is tackled. This reading on the formation of the Earth-Moon system shall follow the present reading. Thereafter, we shall present the final part of our three part series of papers, where the issue of whether or not data from geophysical stations

[5] If the Earth where completely covered by water, such an Earth is called a *geoid*. In more technical terms, a geoid is the equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level.

[6] A *firmament* is the apparent surface of the imaginary sphere on which celestial bodies appear to be projected.

around the world supports or refutes a Hubble type flow of the *World*'s continental plates.

6.2 Conclusion

Assuming the correctness of the present ideas, then, we hereby make the following conclusions:

(i) Spin-orbit interaction should lead to the expansion (or contraction) of not only the Earth, but other celestial bodies in the Solar system and elsewhere in the Universe. Plate tectonics is thus not expected to be peculiar to our planet but must be abundant in the cosmos for as long as planetary recession from their parent bodies is present.

(ii) The global contraction of the Moon which has been confirmed by *NASA*, may very well be driven by gravitational decompression which comes about as a result of changes in angular momentum occurring due to the loss of orbital angular momentum of the Earth-Moon system as they drift from the Sun – this lost orbital angular momentum be transferred to the spin of the respective bodies leading to changes in their physical volume. This same phenomenon must be causing the Earth to expand, leading to global plate tectonics.

(iii) The lost orbital kinetic energy due to the recession of the Earth-Moon system from the Sun which according to hypothesis made herein, is converted to the kinetic energy of the spin of the Earth-Moon system, most certainly is the driver of plate tectonics, the contraction of the Moon and as-well the observed recession of Moon from the Earth.

6.3 Recommendations

If the ideas propagated herein are correct (as we believe) or are to prove to tend to that end, then, we hereby make the following recommendations:

(i) To improve on the values obtained for the expansion and contraction of the Earth and Moon respectively, it is necessary to drop the assumption that $\dot{\vartheta}_{\rm e} = \dot{\vartheta}_{\rm m} = \dot{\vartheta}_{\rm orb} = 0$. We have to assume that $\dot{\vartheta}_{\rm e} \neq 0$, $\dot{\vartheta}_{\rm m} \neq 0$ and $\dot{\vartheta}_{\rm orb} \neq 0$. For this, one will require a gravitational theory that can handle angular variation in the gravitational potential. Though still under construction, one such theory has been presented in Nyambuya (2010). We are working on this. We hope to provide our improved calculation in the near future. However, we do not expect the values to be significantly far off from what we have found out in the present endeavour.

(ii) Measurements hoping to detect any secular expansion of the Earth must have a threshold of sensitivity of about $+0.1\,\mathrm{mm/yr}$ in order to verify or falsify the present calculation. If the present calculation is proven to be incorrect by these measurements, then, our laws of conservation of angular momentum and energy may be at fault – this is highly unlikely and we wish not to think in this direction. In worst case scenario, one will have to invoke the dark matter hypothesis in-order to save these laws from the embarrassment.

(iii) There is need to collect data from the different stations around the *World* on continental drifts and check whether or not the measured movement of plates does conform to a Hubble-type flow[7]. If these data reveal a Hubble-type flow, it would be a strong indicator that the Earth may very well be expanding.

(iv) It is very much likely that the recession of the Earth-Moon system is not peculiar to this celestial system alone, but is a phenomenon occurring to all the Solar planets. If this were the case, it would be interesting to calculate and most certainly speculate on the possible plate tectonic activities on these planets.

[7] At the time of writing, one of our graduate students (Ms. Jacqueline C. Nyambiya), is working on this. It is hoped that these results will become available in the near future.

APPENDIX A: Calculation

We have:

$$L = \mathcal{J}_{\rm orb}\left(\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}\right) - \left(\mathcal{J}_{\rm rm} + \mathcal{J}_{\rm e} + \mathcal{J}_{\rm m}\right)\sin\vartheta_{\rm m} - \mathcal{S}_{\rm e}\left(\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}\right). \tag{A.1}$$

Since $\delta L = 0$, it follows that:

$$\mathcal{J}_{\rm orb}\left(\frac{\delta\mathcal{J}_{\rm orb}}{\mathcal{J}_{\rm orb}}\right)\left(\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}\right) - 3\mathcal{J}_{\rm rm}\left(\frac{\delta\mathcal{J}_{\rm rm}}{\mathcal{J}_{\rm rm}}\right)\sin\vartheta_{\rm m} - \mathcal{S}_{\rm e}\left(\frac{\delta\mathcal{S}_{\rm e}}{\mathcal{S}_{\rm e}}\right)\left(\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}\right) = 0. \tag{A.2}$$

In the above, we have made use of the verifiable fact that $\delta\mathcal{J}_{\rm rm} + \delta\mathcal{J}_{\rm e} + \delta\mathcal{J}_{\rm m} = 3\delta\mathcal{J}_{\rm rm}$. Now, writing $\delta\mathcal{S}_{\rm e}/\mathcal{S}_{\rm e}$ as the subject of the formula, we will have:

$$\frac{\delta\mathcal{S}_{\rm e}}{\mathcal{S}_{\rm e}} = \frac{\mathcal{J}_{\rm orb}}{\mathcal{S}_{\rm e}}\left(\frac{\delta\mathcal{J}_{\rm orb}}{\mathcal{J}_{\rm orb}}\right)\frac{\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}} - \frac{\mathcal{J}_{\rm rm}}{\mathcal{S}_{\rm e}}\left(\frac{\delta\mathcal{J}_{\rm rm}}{\mathcal{J}_{\rm rm}}\right)\frac{3\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}. \tag{A.3}$$

Since $\mathcal{S}_{\rm e} = 2\pi\mathcal{M}_{\rm e}\mathcal{R}_{\rm e}^2/\mathcal{T}_{\rm e}$, $\mathcal{J}_{\rm orb} = 2\pi(\mathcal{M}_{\rm e} + \mathcal{M}_{\rm m})\mathcal{R}_{\rm orb}^2/\mathcal{T}_{\rm orb}$ and $\mathcal{S}_{\rm m} = 2\pi\mathcal{M}_{\rm m}\mathcal{R}_{\rm m}^2/\mathcal{T}_{\rm m}$ and assuming $\delta\mathcal{M}_{\rm e} \simeq \delta\mathcal{M}_{\rm m} \simeq 0$; using these facts to split the terms $\delta\mathcal{S}_{\rm e}/\mathcal{S}_{\rm e}$, $\delta\mathcal{J}_{\rm orb}/\mathcal{J}_{\rm orb}$ and $\delta\mathcal{S}_{\rm m}/\mathcal{S}_{\rm m}$, it follows that:

$$\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}} - \frac{1}{2}\frac{\delta\mathcal{T}_{\rm e}}{\mathcal{T}_{\rm e}} = \frac{\mathcal{J}_{\rm orb}}{\mathcal{S}_{\rm e}}\left(\frac{\delta\mathcal{R}_{\rm orb}}{\mathcal{R}_{\rm orb}} - \frac{1}{2}\frac{\delta\mathcal{T}_{\rm orb}}{\mathcal{T}_{\rm orb}}\right)\frac{\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}} - \frac{\mathcal{J}_{\rm rm}}{\mathcal{S}_{\rm e}}\left(\frac{\delta r_{\rm em}}{r_{\rm em}} - \frac{1}{2}\frac{\delta\mathcal{T}_{\rm em}}{\mathcal{T}_{\rm em}}\right)\frac{3\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}. \tag{A.4}$$

Now, we have to apply the *Law of Conservation of Energy* as laid down in (4) and (5). The Earth spins with a kinetic energy $K_{\rm spin}^{\rm e} = 2\pi^2\mathcal{M}_{\rm e}\mathcal{R}_{\rm e}^2/\mathcal{T}_{\rm e}^2$. Its total stored gravitational potential energy $U_{\rm g}^{\rm e} = -G\mathcal{M}_{\rm e}^2/\mathcal{R}_{\rm e}^2$. Assuming a non-variable G and mass of the the Earth $\mathcal{M}_{\rm e}$, if the spin kinetic energy $K_{\rm spin}^{\rm e}$ and the total gravitational potential energy of the Earth $U_{\rm g}^{\rm e}$ are conserved ($E_{\rm spin} = K_{\rm spin}^{\rm e} + U_{\rm g}^{\rm e}$, that is $\delta E_{\rm spin} = 0$), then $K_{\rm spin}^{\rm e}\left(\delta K_{\rm spin}^{\rm e}/K_{\rm spin}^{\rm e}\right) + U_{\rm g}^{\rm e}\left(\delta U_{\rm g}^{\rm e}/U_{\rm g}^{\rm e}\right) = 0$, then from this it follows that $\delta\mathcal{T}_{\rm e}/\mathcal{T}_{\rm e} = \left(1 - U_{\rm g}^{\rm e}/2K_{\rm spin}^{\rm e}\right)\delta\mathcal{R}_{\rm e}/\mathcal{R}_{\rm e}$. Applying the very same assumptions to the Earth-Moon system's Solar gravitational potential energy $U_{\rm g}^{\rm em} = -G\mathcal{M}_{\odot}\left(\mathcal{M}_{\rm e} + \mathcal{M}_{\rm m}\right)/\mathcal{R}_{\rm orb}$ the kinetic energy of its spin $K_{\rm spin}^{\rm m} = 2\pi^2\left(\mathcal{M}_{\rm e} + \mathcal{M}_{\rm m}\right)\mathcal{R}_{\rm orb}^2/\mathcal{T}_{\rm orb}^2$, one obtains $\delta\mathcal{T}_{\rm em}/\mathcal{T}_{\rm em} = \left(1 - U_{\rm g}^{\rm em}/2K_{\rm spin}^{\rm em}\right)\delta r_{\rm em}/r_{\rm em}$. The orbital kinetic energy $K_{\rm orb}^{\rm em}$ of the Earth-Moon system is conserved independently from the gravitational potential energy of the Earth-Moon system, that is $\delta K_{\rm orb}^{\rm em} = 0 \Longrightarrow \delta\mathcal{T}_{\rm orb}/\mathcal{T}_{\rm orb} = \delta\mathcal{R}_{\rm orb}/\mathcal{R}_{\rm orb}$. Piecing everything together, we obtain:

$$\frac{1}{\alpha_{\rm e}}\frac{\delta\mathcal{R}_{\rm e}}{\mathcal{R}_{\rm e}} = \frac{\mathcal{J}_{\rm orb}}{\mathcal{S}_{\rm e}}\left(\frac{\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}\right)\frac{\delta\mathcal{R}_{\rm orb}}{\mathcal{R}_{\rm orb}} - \frac{1}{\alpha_{\rm em}}\frac{\mathcal{J}_{\rm rm}}{\mathcal{S}_{\rm e}}\left(\frac{3\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}\right)\frac{\delta r_{\rm em}}{r_{\rm em}}, \tag{A.5}$$

where $1/\alpha_{\rm e} = 1 + U_{\rm g}^{\rm e}/2K_{\rm spin}^{\rm e} = -(2.91 \pm 0.08) \times 10^2$ and $1/\alpha_{\rm em} = 1 + U_{\rm g}^{\rm em}/2K_{\rm spin}^{\rm m} = -(1.319 \pm 0.001) \times 10^5$. The above equation can be written as:

$$\frac{\delta\mathcal{R}_{\oplus}}{\mathcal{R}_{\oplus}} = \overbrace{\alpha_{\rm e}\left(\frac{\mathcal{J}_{\rm orb}}{\mathcal{S}_{\rm e}}\right)\left(\frac{\sin\vartheta_{\rm orb}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}\right)\frac{\delta\mathcal{R}_{\rm orb}}{\mathcal{R}_{\rm orb}}}^{\textbf{Solar Recessional Contribution}} - \underbrace{\frac{\alpha_{\rm e}}{\alpha_{\rm em}}\left(\frac{\mathcal{J}_{\rm rm}}{\mathcal{S}_{\rm e}}\right)\left(\frac{3\sin\vartheta_{\rm m}}{\sin\vartheta_{\rm e}\cos\vartheta_{\rm m} - \cos\vartheta_{\rm e}\sin\vartheta_{\rm m}}\right)\frac{\delta r_{\rm em}}{r_{\rm em}}}_{\textbf{Lunar Recessional Contribution}}, \tag{A.6}$$

where, as usual, $\mathcal{R}_{\oplus} = \mathcal{R}_{\rm e}$ is the current radius of the Earth. If one defines $L_{\rm m} = \cos\vartheta_{\rm e}\mathcal{L}_x - \sin\vartheta_{\rm e}\mathcal{L}_y$ as has been done in (11), then, in exactly the same manner as has been conducted above, if one where to calculate the corresponding formula for the Moon system by making $\delta\mathcal{R}_{\rm m}/\mathcal{R}_{\rm m}$ of the formula, they would obtain:

$$\frac{\delta\mathcal{R}_{\rm m}}{\mathcal{R}_{\rm m}} = \overbrace{-\alpha_{\rm m}\left(\frac{\mathcal{J}_{\rm orb}}{\mathcal{S}_{\rm m}}\right)\left(\frac{\sin\vartheta_{\rm orb}\cos\vartheta_{\rm e} - \cos\vartheta_{\rm orb}\sin\vartheta_{\rm e}}{\sin\vartheta_{\rm m}\cos\vartheta_{\rm e} - \cos\vartheta_{\rm m}\sin\vartheta_{\rm e}}\right)\frac{\delta\mathcal{R}_{\rm orb}}{\mathcal{R}_{\rm orb}}}^{\textbf{Solar Recessional Contribution}} + \underbrace{\frac{\alpha_m}{\alpha_{\rm em}}\left(\frac{\mathcal{J}_{\rm rm}}{\mathcal{S}_{\rm m}}\right)\left(\frac{3\sin\vartheta_{\rm e}}{\sin\vartheta_{\rm m}\cos\vartheta_{\rm e} - \cos\vartheta_{\rm m}\sin\vartheta_{\rm e}}\right)\frac{\delta r_{\rm em}}{r_{\rm em}}}_{\textbf{Lunar Recessional Contribution}}. \tag{A.7}$$

This completes our derivation.

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