Proof of the Invalidity of the Black Hole and Einstein's Field Equations

Letter to Professor Martin Rees, Astronomer Royal, Concerning his Public Lecture at the University of Sydney on 9th November 2012

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ABSTRACT

Professor Martin Rees, Astronomer Royal, gave a public lecture in the Great Hall at the University of Sydney on 9th November 2012, 6:30pm to 8:00pm. The lecture was titled 'BIG BANGS, BIOSPHERES AND THE LIMITS OF SCIENCE'. The website announcing the lecture is:

http://sydney.edu.au/sydney_ideas/lectures/2012/professor_martin_rees.shtml

In the preamble on the aforementioned website we find the words, "*But there are intimations that physical reality is hugely more extensive than the domain our telescopes can probe. Indeed we may inhabit a 'multiverse' – living in the aftermath of one among an infinity of 'big bangs'.*" This short Letter proves that the black hole does not exist because it violates the physical principles of General Relativity and that General Relativity violates the usual conservation of energy and momentum and is therefore invalid. Many of Professor Rees' arguments in his public lecture are based on the assumption of the validity of General Relativity and are therefore untenable. Professor Rees has also written much in various papers on the existence and properties of black holes. What followers is the content of a letter sent to Professor Rees in advance of his public lecture in Sydney.

(1) The Invalidity of the Black Hole

Recall that according to Einstein his Principle of Equivalence and his laws of Special Relativity must hold in sufficiently small finite regions of his gravitational field and that these regions can be located anywhere in his gravitational field [1]. Recall also that both The Principle of Equivalence and Special Relativity are *defined* in terms of the *a priori* presence of multiple arbitrarily large finite masses and photons [1]. Now Einstein maintains that his field equations (in covariant components)

$$R_{\mu\nu} = 0$$

are "*The Field Equations of Gravitation in the Absence of Matter*" [2]; they pertain to the so-called 'static vacuum field'. "*Einstein made the assumption that in empty space* $R_{\mu\nu} = 0$" [3]. "*The Einstein equations in the absence of matter are* $R_{\mu\nu} = 0$" [4]. In contravariant components, "*… the empty space field equations are* $R^{\mu\nu} = 0$" [5]. "*The law* $G_{\mu\nu} = 0$ *in empty space, is chosen by Einstein for his law of gravitation*" [6]. Note that in the case of [6] Eddington writes the Ricci tensor $R_{\mu\nu}$ as $G_{\mu\nu}$, but the meaning is precisely the same.

It is therefore *impossible* for the Principle of Equivalence and Special Relativity to manifest in the spacetime of $R_{\mu\nu} = 0$ since $R_{\mu\nu} = 0$ contains no matter by mathematical

construction. Therefore $R_{\mu\nu} = 0$ violates the physical principles required by Einstein for his gravitational field and is therefore inadmissible, having no physical meaning. Therefore the solution for $R_{\mu\nu} = 0$ (the so-called 'Schwarzschild solution' actually due to David Hilbert) also violates the physical principles of General Relativity and so also has no physical meaning. But it is from Hilbert's solution to $R_{\mu\nu} = 0$ that the black hole was spawned. Indeed, we note that,

"*Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time.*" [7]

Thus the black hole violates the physical principles of General Relativity and so it too has no physical meaning. In other words, General Relativity does not predict the black hole. The black hole thus fails to have any theoretical justification by means of General Relativity.

(2) The Invalidity of Einstein's Pseudo-Tensor and Hence the Invalidity of His Field Equations

Recall that Einstein's field equations,

"*... couple the gravitational field (contained in the curvature of spacetime) with its sources.*" [5]

So Einstein's field equations are given by,

(1) $$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} .$$

Here $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ the Ricci tensor, R the Ricci curvature, $g_{\mu\nu}$ the metric tensor, κ a coupling constant, and $T_{\mu\nu}$ the energy-momentum tensor. Equation (1) is often written more conveniently as

(2) $$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

The Einstein tensor describes the geometry and curvature of spacetime, and hence the gravitational field. The energy-momentum tensor describes all the material sources of the gravitational field that induce the curvature of spacetime. In relation to Equation (2) Einstein asserts that the sum of the energy and momentum of his gravitational field and its sources is given by $\mathfrak{E} = \left(t_\mu^\sigma + T_\mu^\sigma\right)$ where t_μ^σ is his pseudo-tensor and T_μ^σ is his energy-momentum tensor. Note that this expression is written in mixed form (one superscript and one subscript). Einstein says that the t_μ^σ are the "*energy components … of the gravitational field*" [8]. According to Pauli the components of

Einstein's pseudo-tensor are '*the "energy components" of the gravitational field*' [9]. According to Eddington [10] t_μ^σ denotes "*…potential energy, momentum and stress*", and also says, "*We call t_μ^σ the pseudo-tensor-density of potential energy*". Note that $\mathfrak{E} = \left(t_\mu^\sigma + T_\mu^\sigma\right)$ is **not** a tensor sum since t_μ^σ is not a tensor. For energy and momentum to be conserved the divergence of the expression for the total energy and momentum of the gravitational field must be zero. But the divergence of Einstein's expression for the conservation of energy and momentum is an ordinary divergence, not a tensor divergence, contrary to his requirement that all the equations of physics be tensorial. Indeed, Einstein gives the ordinary divergence of his energy-momentum expression thus [8];

$$\frac{\partial\left(t_\mu^\sigma + T_\mu^\sigma\right)}{\partial x_\sigma} = 0 \tag{3}$$

Einstein [8] says of equation (3),
"Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied."

"… we have to introduce the totality of the energy components of matter and gravitational field."

Now Einstein's allegation that by equation (3) "*… the laws of conservation of momentum and energy are satisfied*" is completely false because Einstein's pseudo-tensor is a meaningless concoction of mathematical symbols and so it **cannot** be used to make any calculations or to represent any physical entity or to model any physical phenomena. Thus, Einstein's energy-momentum expression and the ordinary divergence of it are meaningless both physically and mathematically. Here is the proof. First recall that we can write the components of the metric tensor in contravariant form, thus; $g^{\mu\nu}$. Now Einstein's pseudo-tensor is defined, equivalently, as [9, 10, 11, 12, 13, 14],

$$\sqrt{-g}\, t_\nu^\mu = \frac{1}{2}\left[\delta_\nu^\mu L - \left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\nu}^{\sigma\beta}\right]$$

where

$$L = -g^{\alpha\beta}\left(\Gamma_{\alpha\kappa}^{\gamma}\Gamma_{\beta\gamma}^{\kappa} - \Gamma_{\alpha\beta}^{\gamma}\Gamma_{\gamma\kappa}^{\kappa}\right)$$

$$\Gamma_{bc}^{a} = \frac{1}{2} g^{ad}\left(g_{dc,b} + g_{bd,c} - g_{bc,d}\right) \qquad g_{dc,b} = \frac{\partial g_{dc}}{\partial x^b}$$

We can **contract** a mixed tensor (and Einstein's pseudo-tensor) by setting $\nu = \mu$, to produce an invariant. So contracting Einstein's pseudo-tensor gives the **invariant** t, thus:

$$\sqrt{-g}\, t_\mu^\mu = \frac{1}{2}\left[4L - \left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\mu}^{\sigma\beta}\right] \qquad t_\mu^\mu = t$$

Performing the calculation of the second part inside the brackets gives:

$$\left(\frac{\partial L}{\partial g^{\sigma\beta}_{,\mu}}\right) g^{\sigma\beta}_{,\mu} = 2L$$

Substituting this result into the expression above and rearranging gives the invariant:

$$t = \frac{L}{\sqrt{-g}}$$

where g is the determinant of the metric tensor and hence is composed of the components of the metric tensor. By the definitions of L, Γ^{a}_{bc} and g, we see that t is an invariant that is composed **solely** of the components of the metric tensor and their first derivatives [11]. Tolman [15] also remarks:

"… it will be noted that the value of t^{ν}_{μ} *at any point will be determined by the values of the components of the metrical tensor* $g_{\alpha\beta}$ *and their first derivatives* $\partial g_{\alpha\beta} / \partial x^{\gamma}$ *at that point."*

Now the pure mathematicians G. Ricci-Curbastro and T. Levi-Civita, inventors of the tensor calculus, proved in 1900 that invariants that are composed **solely** of the components of the metric tensor and their first derivatives **do not exist!** [16]. Thus Einstein's pseudo-tensor is totally meaningless and hence his formulation of the usual conservation of energy and momentum totally invalid.

Now the upshot of this is that Einstein's field equations must take the following form on consideration of the conservation of energy and momentum (see equation (3) above) [11, 17],

(4) $$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0$$

The $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Not only is this the necessary form of Einstein's field equations but it also expresses the sum of the energy and momentum of the gravitational field (compare with equation (3) above). The tensor divergence of both sides of this equation is zero so energy and momentum are conserved, but the total energy of Einstein's gravitational field **is always zero**; the $G_{\mu\nu}/\kappa$ and the $T_{\mu\nu}$ **must vanish identically** (so that when $T_{\mu\nu} = 0$ there is no spacetime and hence no gravitational field); there is **no possibility** for the localization of gravitational energy (i.e. **there are no Einstein gravitational waves**). This also means that Einstein's gravitational field **violates** the experimentally well-established **usual** conservation of energy and momentum making them inconsistent with experiment on a deep level and hence invalid. According to Pauli [9], Einstein

"… raised the objection that, with this definition of the gravitational energy, the total energy of a closed system would always be zero, and the maintenance of this value of the energy does not require the continued existence of the system of one form or other. The usual kind of conclusions could not then be drawn from the conservation laws."

But Einstein's objections are futile on account of the failure of his formulation of the usual conservation of energy and momentum, equation (3), in terms of the meaningless pseudo-tensor. Thus, General Relativity is invalid.

Yours faithfully,
Stephen J. Crothers
10th October 2012

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