

# E8 Physics Fermions: 3 Conformal Generations

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The E8 Lie Algebra of the E8 Physics Model contains two D4 Lie subalgebras:

248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8

120-dim D8 = 28-dim D4 + 28-dim D4 + 64-dim D8 / D4xD4

**One of the D4** contains an A2 = SU(3) Lie subalgebra that represents the Color Force of the Standard Model.

The Weak and Electromagnetic Forces are produced by a Batakis mechanism

(see Class. Quantum Grav. 3 (1986) L99-L105 by N. A. Batakis) in which spacetime is 8-dimensional Kaluza-Klein M4 x CP2.

Color Force SU(3) acts globally on CP2 = SU(3) / SU(2)xU(1) and,

due to Kaluza-Klein structure, acts as local gauge group on M4 Minkowski spacetime.

Local gauge group action of Weak SU(2) and Electromagnetic U(1) Forces comes from their being local isotropy groups of the symmetric space CP2.

Casimir Operators describe some physical properties of the Standard Model Forces:

A0 Lie algebra U(1) has trivial Weyl Group 1

and trivial Casimir of degree 1

so that the Photon carries no charge.

A1 Lie algebra SU(2) has Weyl Group S2 of order 2! = 2

and quadratic Casimir of degree 2 representing isospin

so that SU(2) Weak Bosons can carry Electromagnetic Charge.

A2 Lie algebra SU(3) has Weyl Group S3 of order 3! = 6

and two Casimir Operators of degrees 2 and 3:

a quadratic Casimir representing 2 { + , - } isospin charge states and

a cubic Casimir representing 3 { red, green, blue } colors

so that SU(3) Gluons can carry Electromagnetic Charge and Color Charge.

**The other D4** contains an A3 = D3 Conformal Lie subalgebra that represents

Gravity by a generalized MacDowell-Mansouri mechanism (see section 14.6 of Rabindra Mohapatra's book "Unification and Supersymmetry", 2nd edition, Springer-Verlag 1992).

The Conformal Group in the form SU(2,2) = Spin(2,4) is described by

Robert Gilmore in his books "Lie Groups, Lie Algebras, and Some of Their Applications", Wiley 1974, and "Lie Groups, Physics, and Geometry", Cambridge 2008.

The Conformal Group has a Weyl Group of 2<sup>2</sup> x 3! = 24 elements

and has 3 Casimir Operators of degrees 2 and 4 and 6/2 = 3.

**The Conformal degree 3 Casimir represents the 3 Generations of Fermions**

(instead of the 3 colors as in the case of the Standard Model D4 of E8).

In its D3 Spin(2,4) form the Conformal Lie algebra can be represented as a 6x6 antisymmetric real matrix:

$$\begin{array}{cccccc}
 0 & J_1 & J_2 & M_1 & A_1 & G_1 \\
 -J_1 & 0 & J_3 & M_2 & A_2 & G_2 \\
 -J_2 & -J_3 & 0 & M_3 & A_3 & G_3 \\
 -M_1 & -M_2 & -M_3 & 0 & A_4 & G_4 \\
 -A_1 & -A_2 & -A_3 & -A_4 & 0 & G_5 \\
 -G_1 & -G_2 & -G_3 & -G_4 & -G_5 & 0
 \end{array}$$

$\{J_1, J_2, J_3\}$  form a Spin(0,3) subalgebra of Spin(2,4) and produce a quadratic Casimir Operator that represents an Angular Momentum Operator.

Adding  $\{M_1, M_2, M_3\}$  forms a Spin(1,3) subalgebra of Spin(2,4) and produces a second quadratic Casimir Operator that represents a Laplace-Runge-Lenz Operator.

Adding  $\{A_1, A_2, A_3\}$  and  $\{A_4\}$  forms a Spin(2,3) AntiDeSitter subalgebra of Spin(2,4) with a quartic Casimir Operator that is a combination of  $\{M_1, M_2, M_3\}$  and  $\{A_1, A_2, A_3\}$ .  $\{A_1, A_2, A_3\}$  represents a Momentum Operator and  $\{A_4\}$  represents an Energy/Mass Operator.

Adding  $\{G_1, G_2, G_3\}$  and  $\{G_4\}$  and  $\{G_5\}$  forms the full Spin(2,4) and produces a cubic Casimir Operator for representation of 3 Generations of Fermions. The  $\{G_1, G_2, G_3\}$  represent the 3 Higgs components giving mass to Weak Bosons.  $\{G_4\}$  and  $\{G_5\}$ , like  $\{A_4\}$ , each represent an Energy/Mass Operator.

The Higgs as a Fermionic Condensate gives mass to Fermions. The fundamental Fermion Particles are those of the First Generation:

{neutrino, red down quark, green down quark, blue down quark;  
electron, red up quark, green up quark, blue up quark}

They can be represented as basis elements  $\{1, i, j, k, E, I, J, K\}$  of Octonions O.

Each of  $\{A_4\}$  and  $\{G_4\}$  and  $\{G_5\}$  can represent the mass of Fundamental Fermions.

The {A4} Conformal substructure

$$\begin{array}{cc} 0 & A4 \\ -A4 & 0 \end{array}$$

represents First Generation Fermion Particles as Octonion basis elements O.

The {A4} plus {G4} Conformal substructure

$$\begin{array}{ccc} 0 & A4 & G4 \\ -A4 & 0 & \end{array}$$

represents Second Generation Fermion Particles as Octonion Pairs OxO.

The {A4} and {G4} plus {G5} Conformal substructure

$$\begin{array}{ccc} 0 & A4 & G4 \\ -A4 & 0 & G5 \\ -G4 & -G5 & 0 \end{array}$$

represents Third Generation Fermion Particles as Octonion Triples OxOxO.

Fermion AntiParticles are represented in a similar way.

Combinatorics of O and OxO and OxOxO produce realistic Fermion masses, as calculated in detail in viXra 1108.0027

The Third Generation Truth Quark (Tquark) is by far the most massive Fermion so the Higgs as a Fermionic Condensate is effectively a Tquark Condensate.