Confined Helical Wave Structure of Electron and the Dirac Equation

V.A.I.Menon, M.Rajendran, V.P.N.Nampoori
International School of Photonics, Cochin University of Science and Technology,
Cochin 680022, India.

The authors after showing that a confined helical wave possesses rest mass, electric charge and half spin [1][2] now proves that such a wave is a solution of the Dirac equation. Using the confined helical wave representation of electron, the authors explain in a simple manner why the eigen value for Dirac's velocity operator is ± c while the average value of the velocity of the wave packet formed by its positive energy solutions is the particle velocity. Further, the confined helical wave representation is seen to be consistent with van der Waerden equation and the zig-zag picture of electron proposed by Pen Rose.

PACS numbers: 03.30, 03.65-w, 14.60-z, 41.20jb

Key Words: Structure of electron, Zig-zag picture of electron, Solutions of Dirac equation, Alternative to string theory

1 Introduction

We have earlier shown that Maxwell's equations allow the electromagnetic waves to have oscillations not only in the electromagnetic field but also in spatial displacement **[3]**. Such a plane polarized electromagnetic wave on acquiring spin angular velocity was seen to get confined resulting in the creation of the particle-antiparticle pair having rest mass, electric charge and half spin[1][2]. If such confined waves are to represent electron-positron states, it is necessary to show that they satisfy the Dirac equation. We shall examine this issue in detail in this paper.

To begin with let us take the helical wave confined along z-axis. The forward component of such a wave can be expressed as [1]

$$\phi^{L_1} \quad = \quad (\mathbf{j}-i\mathbf{i})\,\eta_o \;\; e^{i\theta} e^{i\hbar^{-1}E(z'-vt')/c} \;\; e^{-i\hbar^{-1}(Et-\text{þ}\, z)} \tag{1}$$

where $\sin\theta = -i\gamma\beta,\;\; \gamma \;= (1-\beta^2)^{-\frac{1}{2}}$, β being equal to v/c. Here ϕ^L denotes that the wave is rotating anticlockwise when viewed head on. Note that due to the periodic nature of the function '$e^{i\hbar^{-1}E(z'-vt')/c}$' the vector (**j**-i**i**) stands for rotation in the anticlockwise direction in the internal coordinates. This rotation can very well be denoted by the unit vector **k** in the z-direction. But then we know that such a vector will always coincide with the direction of the momentum and therefore we may drop the unit vector **k** altogether with the understanding that the spin can be either in the direction of the momentum or opposite to that depending on the fact that the rotation is either right handed or left handed. Using the relation E = ½(E_1+E_2) and þ = ½(p_1-p_2), where E_1, p_1, E_2 and p_2 denote the energy and momentum of the forward wave and the reverse wave respectively, we may express (1) as

$$\phi^{L_1} \quad = \quad u^1 w^{2*} \phi \tag{2}$$

where $$u^1 \; = \; \eta_o^{\frac{1}{2}}\, e^{i\frac{1}{2}\theta}\, e^{i\frac{1}{2}\hbar^{-1}A_1'}, \quad w^2 \; = \; \eta_o^{\frac{1}{2}}\, e^{i\frac{1}{2}\theta}\, e^{\,i\frac{1}{2}\hbar^{-1}A_2'} . \tag{3}$$

Here $A_1' = -(E_1t' - \text{þ}_1z')$, $A_2' = -(E_2t' + \text{þ}_2z')$ and $\phi = e^{-i\hbar^{-1}(Et-\text{þ}z)}$. Note that since θ is an imaginary number, exp(iθ/2) is a real number and remains unchanged in complex conjugation. As we are interested only in the spin of the particle, we may drop w^{2*} conveniently from the expression for ϕ^{L_1} [1] and re-express (2) as

$$\phi^{L_1} \quad = \quad u^1\phi \tag{4}$$

Similarly we may express the reverse wave as

$$\phi^{L_2} \quad = \quad u^2 w^{1*}\phi \quad \equiv \quad u^2\phi \tag{5}$$

where
$$u^2 \;=\; \eta_o^{\frac{1}{2}}\, e^{-i\frac{1}{2}\theta}\, e^{i\frac{1}{2}\hbar^{-1}A_2{}'}, \quad w^1 \;=\; \eta_o^{\frac{1}{2}}\, e^{-i\frac{1}{2}\theta}\, e^{i\frac{1}{2}\hbar^{-1}A_1{}'} \tag{6}$$

In the above representation, although the spin of the system is well expressed it is at the cost of losing sight of its internal structure.

Till now we have been representing a particle in terms of a confined helical wave having a specific value for its energy and momentum. On confinement the helical wave constituted by the electromagnetic waves is seen to transform itself into a plane wave which is no more a wave with spatial amplitude. Interestingly, the transverse displacement aspect of the electromagnetic waves appears to get compacted into the internal coordinates. It can be seen that the plane wave representation followed in quantum mechanics has its origin in the confined helical structure of particle as described above. Let us now using (2) and (5) represent the left handed confined helical wave as

$$\phi^L \quad = \quad \phi^{L_1} + \phi^{L_2} \quad = \quad (w^{2*}u^1 + w^{1*}u^2)\phi \tag{7}$$

Here the amplitude ϕ^L given by "($w^{2*}u^1+w^{1*}u^1$) " is defined in the internal coordinates of the particle and accounts for its spin. Therefore, the amplitude of ϕ^L in the external coordinate can to be taken as unity. Since we were dealing with a given state of the particle, it is obvious that the probability of finding the particle in that state has to be unity. But if the plane wave states formed by the confined helical waves are occupied in a virtual manner by the principle of quantum superposition, then in the place of unity, it becomes logical to introduce the probability amplitude. Let us now denote the plane wave state by ϕ_i where "*i*" denotes its energy state which may take values from 1 to n. If the i^{th} state is occupied "a_i" number of times in the process of superposition, then we may express it as

$$\psi \quad = \quad \Sigma\,\psi_i \quad = \quad \Sigma\, a_i\, \phi_i \tag{8}$$

Here the function $\psi_i^*\psi_i = a_i^* a_i$ denotes the probability that the i^{th} state is occupied. Note that here the spin aspect of the particle is not expressed explicitly.

It is now quite clear that the wave function as defined in quantum mechanics is a logical extension of the function representing the confined helical wave. An important property of Ψ as defined in quantum mechanics is that it treats the eigen state as the fundamental state which is not required to be analyzed any further. But we saw here that the plane wave state can be attributed a sub-structure in terms of the confinement of electromagnetic wave which also explains the creation of the electric charge of the particle. It becomes quite clear now that when we represent a particle by a plane wave, we are attributing it the structure of a confined helical wave. We shall now examine if this structure is compatible with the Dirac equation.

2 Introducing the Spin Matrices

We saw that ϕ^{L_1} and ϕ^{L_2} taken together represents a particle in the spin up state. They represent eigen state of the particle with a specific value for its energy-momentum and spin. Note that u^1 and u^2 have half the frequency of the respective forward and reverse waves. This means that when the plane wave denoted by ϕ undergoes a phase change of 4π, the phase of u^1 and u^2 which represent the spin of the particle varies by 2π. Since u^1 and u^2 stand for the spin up and spin down states, in the rest frame of reference of the particle we may denote them by u_o^1 and u_o^2 where

$$u_o^1 \;=\; \begin{bmatrix}1\\0\end{bmatrix} \qquad \text{and} \qquad u_o^2 \;=\; \begin{bmatrix}0\\1\end{bmatrix} \tag{9}$$

This is the simplest way of representing the spin state. We may now express ϕ^{L_1} given in (4) and ϕ^{L_2} in (5) as

$$\phi^{L_1} \;=\; u^1\phi \;=\; u_o^1\, e^{i\frac{1}{2}\theta}\, \phi \tag{10}$$

$$\phi^{L_2} \;=\; u^2\phi \;=\; u_o^2\, e^{-i\frac{1}{2}\theta}\, \phi \tag{11}$$

Here we have dropped the functions $e^{i\frac{1}{2}\hbar^{-1}A_1'}$ and $e^{i\frac{1}{2}\hbar^{-1}A_2'}$ from the expressions in spite of the fact that they actually determine the spin of the forward and reverse waves [1]. This is because now that spin has already been represented by the two matrices, the need to retain these functions does not exist anymore. Besides, as the phases these luminal waves $\frac{1}{2\hbar}A_1'$ and $\frac{1}{2\hbar}A_2'$ are relativistic invariants, they can be conveniently equated to zero. Note that we have to retain the exponential functions containing θ as they determine the relativistic transformation.

We may now express the forward and the reverse components of the confined helical wave as

$$\phi^{L_1} \;=\; e^{i\frac{1}{2}\theta}\, u_o^1\, \phi \qquad \text{and} \qquad \phi^{L_2} \;=\; e^{-i\frac{1}{2}\theta}\, u_o^2\, \phi \;. \tag{12}$$

It should be noted that we started by using u^1 to represent a helical wave in the internal space which is spinning in the counterclockwise direction when viewed head on while u^2 represented a helical wave which is spinning in the clockwise direction when viewed head on in the internal space. But once u^1 and u^2 were replaced by the two component matrix in (9), the connection of spin with the internal structure of the particle gets completely lost and spin becomes an abstract entity defined in the internal coordinates of the particle.

The justification for the above representation of spin rests on the fact that the forward wave and the reverse wave do not interfere with each other. Remember that we are dealing with a confined helical wave with half spin where the forward wave and the reverse wave move on the surface of an imaginary cylinder as a closed loop. For such a system the reverse wave has the opposite helicity compared to the forward wave and the introduction of two matrices defined above is the most simple way for accounting these properties.

Let us now take the confined helical wave representing the antiparticle. We know that the spin of the antiparticle will be in the clockwise direction and therefore we shall replace L by R in the super-fix. The corresponding forward wave will be given by

$$\phi^{R_1*} \;=\; w^1\phi^* \;=\; e^{-\frac{1}{2}i\theta}u_o^2 \qquad \text{and} \qquad \phi^{R_2*} \;=\; w^2\phi^* \;=\; e^{\frac{1}{2}i\theta}u_o^1 \tag{13}$$

Note that with the modifications introduced, the differences between u^1 and w^2 as also u^2 and w^1 get lost. In the next section we shall show that this simplification of the expressions results in the confusion regarding the interpretation of the solutions of the Dirac equations regarding spin.

3 Confined Helical wave as a solution of the Dirac Equation

In section 1 we had taken sin θ = -iγv/c and cos θ = γ such that tan θ = -iv/c. This means that θ is an imaginary number. We shall now introduce θ′ where θ′= -iθ so that θ′ can be taken as a real number. In that case (12) may be written in the form

$$\phi^{L_1} \quad = \quad (\cosh\tfrac{1}{2}\theta' - \sinh\tfrac{1}{2}\theta')\, u_o^1 \; e^{-i\hbar^{-1}(Et-pz)} \tag{14}$$

$$\phi^{L_2} \quad = \quad (\cosh\tfrac{1}{2}\theta' + \sinh\tfrac{1}{2}\theta')\, u_o^2 \; e^{-i\hbar^{-1}(Et-pz)} \tag{15}$$

Note that the minus sign before "$\sinh\frac{1}{2}\theta'$" implies that it represents the forward wave just as the plus sign applies to the reverse wave. We may now combine (14) and (15) and express them as

$$\chi_L \quad = \quad \phi^{L_1} + \phi^{L_2} \;=\; \sum\nolimits_m \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cosh\tfrac{1}{2}\theta' - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \sinh\tfrac{1}{2}\theta' \right\} u_o^m \; e^{-i\hbar^{-1}(Et-pz)} \tag{16}$$

where m takes values 1 and 2. Note that for m = 1 we obtain (14) and for m = 2, we obtain (15). Equation (16) can be written in a compact fashion as

$$\chi_L \quad = \quad \sum\nolimits_m (\, I\cosh\tfrac{1}{2}\theta' - \sigma_3 \sinh\tfrac{1}{2}\theta')\, u_o^m \; e^{-i\hbar^{-1}(Et-pz)} \tag{17}$$

Here χ_L represents a closed helical wave that is spinning in the counterclockwise direction when viewed head on. This means that the forward wave and the reverse wave have opposite helicity although the direction of the spin of both waves with regard to the translational motion is the same. In other words, the system as a whole is a left handed one which is reflected in the use of χ_L to represent the system. We may now define a function χ_R given by

$$\chi_R \quad = \quad \phi^{R_1 *} + \phi^{R_2 *} \tag{18}$$

We should keep in mind that $\phi^{R_1^*}$ and $\phi^{R_2^*}$ are not obtained just by taking the conjugation of the function given in (14) and (15). It also involves the reversal of the spin and therefore, the spin matrices will also undergo change and we have to express $\phi^{L_1^*}$ and $\phi^{L_2^*}$ as

$$\phi^{R_1 *} \quad = \quad e^{\frac{1}{2}\theta'}\, u_o^2 \; e^{i\hbar^{-1}(Et-pz)} \;\; ; \;\; \phi^{R_2 *} \quad = \quad e^{-\frac{1}{2}\theta'}\, u_o^1 \; e^{i\hbar^{-1}(Et-pz)} \tag{19}$$

Now following similar steps as taken in the case of χ_L , we obtain

$$\chi_R \quad = \quad \sum\nolimits_m [I\cosh\tfrac{1}{2}\theta' + \sigma_3 \sinh\tfrac{1}{2}\theta']\, u_o^m \; e^{i\hbar^{-1}(Et-pz)} \tag{20}$$

We may combine (17) and (20) together in the matrix form as

$$\psi \quad = \quad \begin{bmatrix} \chi_L \\ \chi_R \end{bmatrix} \quad = \quad \sum\nolimits_{r',m} \begin{bmatrix} \cosh\tfrac{1}{2}\theta' & -\sigma_3 \sinh\tfrac{1}{2}\theta' \\ \sigma_3 \sinh\tfrac{1}{2}\theta' & \cosh\tfrac{1}{2}\theta' \end{bmatrix} u_o^{r'm} \; e^{-i\varepsilon_r \hbar^{-1}(Et-pz)} \tag{21}$$

Note that for r′ = 1 and 2, $u_o^{r`,m}$ = $\begin{bmatrix} u^m \\ 0 \end{bmatrix} and \begin{bmatrix} 0 \\ u^m \end{bmatrix}$ respectively. In fact, we may combine the indices r′ and m into one single index r which can take values from 1 to 4. In that case u^r will represent four-component spinor with which we are familiar. We may now express (21) as

$$\psi \quad = \quad \Sigma_r \begin{bmatrix} \cosh\frac{1}{2}\theta' & -\sigma_3 \sinh\frac{1}{2}\theta' \\ \sigma_3 \sinh\frac{1}{2}\theta' & \cosh\frac{1}{2}\theta' \end{bmatrix} u_o^r \; e^{-i\varepsilon_r \hbar^{-1}(Et-pz)} \tag{22}$$

Here $\varepsilon_r = 1$ for r = 1, 2 and $\varepsilon_r = -1$ when r = 3, 4. The components of Ψ can be expressed as

$$\psi^{\,r} \quad = \quad \cosh\tfrac{1}{2}\theta' \begin{bmatrix} 1 & -\sigma_3 \tanh\frac{1}{2}\theta' \\ \sigma_3 \tanh\frac{1}{2}\theta' & 1 \end{bmatrix} u_o^r \; e^{-i\varepsilon_r \hbar^{-1}(Et-pz)} \tag{23}$$

Note that we had taken sin θ = -iγv/c and cos θ = γ. Since θ′ = -iθ, it can be easily shown that cosh $\frac{1}{2}$θ′ = √[(1+γ)/2] and tanh$\frac{1}{2}$θ′ = (v/c)/[1+√(1-v^2/c^2)]. In these relations, by multiplying the nominator and the denominator by mc^2, we obtain, cos$\frac{1}{2}$θ′ = √[(E+mc^2)/2mc^2] and tanh$\frac{1}{2}$θ′ = þc/(E+mc^2). If we assume that the translational motion is along the x -axis, then we have

$$\psi^{\,r} \quad = \quad \cosh\tfrac{1}{2}\theta' \begin{bmatrix} 1 & -\sigma_1 \tanh\frac{1}{2}\theta' \\ \sigma_1 \tanh\frac{1}{2}\theta' & 1 \end{bmatrix} u_o^r \; e^{-i\varepsilon_r \hbar^{-1}(Et-px)} \tag{24}$$

This is the familiar four vector which is a solution of the Dirac equation [4]. For a particle moving in an arbitrary direction, this can be expressed in a compressed form as

$$\psi^{\,r} \quad = \quad e^{\frac{1}{2}\alpha\theta'} \; u_o^r \; e^{-i\varepsilon_r \hbar^{-1}(Et-\mathbf{þ.r})} \tag{25}$$

Let us now examine what u_o^1 and u_o^2 actually stand for in the four component representation of spinor. Here we should keep in mind that in the Dirac picture, the spin up state of the electron is represented by the forward wave with positive helicity (denoted by u_o^1) while the spin down state is represented by the reverse wave with negative helicity (denoted by u_o^2). It is obvious that u_o^1 and u_o^2 represent spin of the forward and the reverse half waves respectively and not the spin of the confined helical wave as a whole or the particle it represents. This is an important point and should be well understood. But with regard to the translational velocity, the spin of both the forward and the reverse waves are in the same direction and therefore their average value represents the spin of the confined helical wave as a whole and the particle it represents. Similarly, u_o^3 and u_o^4 represent respectively the forward and the reverse waves of another confined helical wave representing the antiparticle. Recall that when the confined helical wave is created, simultaneously second confined helical wave representing the antiparticle is also created **[1]**.

Since the spin of the forward wave is the same as that of the particle (which is taken to be the spin up state) treating u_o^1 as representing the particle as a whole does not create any problem. Similarly we may use u_o^2 to represent a particle in the spin down state. But when we do this, we make an erroneous assumption that the spin up and the spin down states belong to separate spin eigen states of the particle. This leads to a serious problem which till date has remained unresolved. The problem arises from the fact that in the Dirac's theory the eigen value of the velocity operator "cα" is ± c while the average velocity of the wave packet formed by the two positive energy solutions is equal to the particle velocity, v. If u_o^1 and u_o^2 are representing two separate eigen states of spin, it is not clear why the velocity operator should give values ± c, since a particle with mass cannot travel at the luminal velocity. To make matters worse, the average velocity of the wave packet formed by the two positive energy solutions is equal to v. This problem gets resolved logically in the present approach since u_o^1 and u_o^2 represent the spin states of the forward and the reverse wave states constituting the confined helical wave. To put it briefly, the confined helical wave structure of the electron gives us a simple and consistent physical picture behind the currently accepted solutions of the Dirac equation.

4 The Confined Helical Wave and the van der Waerden Equation

We shall now examine the van der Waerden equation given below in the light of the confined helical wave representation of electron **[5]**.

$$\left(i\hbar\frac{\partial}{\partial x_o}+i\hbar\boldsymbol{\sigma}.\nabla\right)\left(i\hbar\frac{\partial}{\partial x_o}-i\hbar\boldsymbol{\sigma}.\nabla\right)\chi \quad = \quad (mc)^2\chi \quad . \tag{26}$$

Here χ is a two component wave function. This equation can be split into two first order equations each of which acts on two separate functions as given below.

$$i\hbar[\sigma.\nabla-(\partial/\partial\tau)]\chi_L \quad = \quad -mc\chi_R \tag{27}$$

$$-i\hbar[\sigma.\nabla+(\partial/\partial\tau)]\chi_R \quad = \quad -mc\chi_L \tag{28}$$

Here χ_L and χ_R are by convention taken to describe respectively the left handed and the right handed states of the spin $\frac{1}{2}$ particle. We shall now show that χ_L can be identified with the confined helical wave representing the particle state that is spinning in the anticlockwise direction while χ_R can be identified with the confined helical wave representing the antiparticle spinning in the clockwise direction.

In the approach followed by us, we saw that the confined helical wave representing the particle state ϕ^{L_1} is left handed while the reverse wave ϕ^{L_2} is right handed. But we know that the confined helical wave formed by these two waves together can be taken as the left handed system denoted by χ_L. Similarly, if we take the confined helical wave representing the antiparticle, it will be a right handed system and we may denote it by χ_R. We know from (17)

$$\begin{aligned}\chi_L \quad &= \quad \textstyle\sum_m(\,I\cosh\frac{1}{2}\theta' - \sigma_3\sinh\frac{1}{2}\theta')\,u_o^m\; e^{-i\hbar^{-1}(Et-pz)}\\ &= \quad \textstyle\sum_m \cosh\frac{1}{2}\theta'\,[I-(\sigma_3.p)/(E/c+mc)]\,u_o^m\; e^{-i\hbar^{-1}(Et-pz)}\end{aligned} \tag{29}$$

$$\begin{aligned}\chi_R \quad &= \quad \textstyle\sum_m[I\cosh\frac{1}{2}\theta' + \sigma_3\sinh\frac{1}{2}\theta']\,u_o^m\; e^{i\hbar^{-1}(Et-pz)}\\ &= \quad \textstyle\sum_m \cosh\frac{1}{2}\theta'\,[I+(\sigma_3.p)/(E/c+mc)]\,u_o^m\; e^{i\hbar^{-1}(Et-pz)}\end{aligned} \tag{30}$$

where $\cosh\frac{1}{2}\theta' = \sqrt{[(E/c + mc)/2mc]}$ and $\tanh\frac{1}{2}\theta' =$ þ/(E/c + mc). On substitution it can be easily seen that χ_L and χ_R given in (29) and (30) respectively satisfy the van der Waerden equations given in (27) and (28). This confirms our assumption that a particle like electron has a confined helical wave structure.

In the currently accepted interpretation, the right handed and the left handed waves are taken to represent two spin states. The sub-quantum nature is brought out by the confined helical wave structure of electron. A detailed study of the Dirac equation and the van der

Waerden equations shows that the later ones are more consistent with the confined helical wave structure of electron than the former one. The Dirac equation identifies the spin of the forward wave for the spin of the entire particle itself which leads to interpretational problems already discussed.

5. Discussion

Apart from these two we have another representation of the electron in the 2-spinor formalism proposed by Penrose. According to Penrose, the Dirac electrons can be pictured in terms of a pair of 2-spinors **[6]**. He calls the state denoted by one of the 2-spinors as the 'zig' particle and that by the other as the 'zag' particle. He treats these as massless particles traveling with the speed of light, more like 'jiggling' backwards and forwards where the forward motion of the zig is continuously being converted to the backward motion of the zag and vice versa. He uses this picture of zig and zag particles to explain what is termed as 'zitterbewegung' of the electron. According to him each ingredient has a spin about its direction of motion with a magnitude of $\frac{1}{2}\hbar$. The spin is left handed in the case of zig and right handed for the zag. Although the velocity keeps reversing, the spin direction remains constant in the electron's rest frame. He proposes that the zig particle acts as the source for the zag particle and the zag particle as the source of the zig particle, the coupling strength being determined by M which is the rest mass of the particle. He observes that the average rate at which this zig-zag motion takes place is equal to the de Broglie frequency of the electron. Note that the picture that emerges from Penrose's 2-spinor formalism coincides with the one proposed here based on the confined helical wave structure of electron except that he has chosen the negative helicity wave as the forward wave instead of the positive helicity one we had taken in our approach.

Penrose wonders whether the zig and zag particles are 'real' or if they are the artifacts of a particular mathematical formalism that he has been adopting for the description of the Dirac equation for the electron. He comments " So are these zigs and zags (particles) real? For my own part, I would say so: they are as real as the 'Dirac electron is itself real- as an idealized mathematical description of one of the most fundamental ingredients of the universe" [6]. We now know that the zig and zag particle picture represents the basic structure of electron with the forward component of the confined helical wave playing the role of the zig particle and the reverse component playing the role of the zag particle. In fact, the Dirac spinor turns out to be just a convenient mathematical approximation that represents a spin $\frac{1}{2}$ particle. In that sense, what Penrose assumed to be two mutually exclusive approaches turns out to be just two ways of looking at the most basic confined helical wave structure of particle. From the above discussion we should not have the notion that a particle is represented by a single confined helical wave. Actual state of a particle like electron is composed of a vast number of such individual confined helical waves existing in all directions due to what is known as quantum superposition. This also would provide the spatial symmetry which is the basic requisite for any structure that represents a particle. Therefore, the observed electron will be some sort of the average of these states.

These three pictures of electron appear to be partial pictures as they do not take into account all aspects of the case. For example, the picture emerging from the Dirac equation wrongly identifies the forward wave state of the confined helical wave with the particle state resulting in the confusion regarding the eigen value of the velocity operator. The van der Waerden equation gives a better picture by representing the particle in terms of the right handed and left handed two-component solutions. But without the concept of the confined helical wave, it becomes difficult to understand what these states stand for. The problem is made more confusing when the solutions of van der Waerden's equations are read in conjunction with those of the Dirac equation. Remember that in the conventional approach the first two components of the Dirac spinor represent two spin states which belong to two different particle states. The Penrose picture seems to be closest to the confined helical wave picture as they explain the spin states in terms of the forward and reverse waves.

5 Conclusion

From the above discussion, the representation of electron by a confined helical wave appears to be a viable proposition particularly as it is found to satisfy the Dirac equation. Besides, it provides us with a new insight into generation of mass, electric charge and the spin of electron. We are now able to get a clear physical picture of the spinor in terms of the spinning motion of the forward and reverse half waves having frequency half that of the plane wave. Needless to say one important advantage of this approach is that it gives a simple explanation for Pauli's exclusion principle [1] and the non-classical behavior of the spin angular momentum of the particles.

In quantum mechanics the state of a particle is represented by a plane wave which is an eigen state of the four-momentum in the coordinate representation. Note that the eigen state is taken as the ultimate level of reality in quantum mechanics, beyond which no measurement is assumed to be possible. In relativistic quantum mechanics spin is introduced as an internal degree of freedom and is not directly related to the plane wave representation of the particle. But in the approach followed in this paper, it is observed that the confinement of the electromagnetic wave leads to the vector nature of the spatial component getting compacted into the inner coordinates where the spin of the particle is defined while the time dependent component which is defined in the laboratory coordinates becomes the plane wave. Therefore, we are effectively assuming that the plane wave representation is not the dead end in the investigation into the structure of the particle. We are attributing an inner structure to the plane wave.

The confined helical wave structure proposed for electron may be appropriate for other leptons like muons and tau particles. In fact, even for particles like quarks it may be possible to attribute a similar confined helical wave structure. The only difference will be that while in the case of electron the wave that gets localized is the electromagnetic wave, for quarks, the corresponding wave might be a more complex one having oscillations in the electromagnetic as well as the strong field. It is quite possible that the SU(2) symmetry and SU(3) symmetry that determine the properties of the elementary particles may be traced to such inner structures.

The present approach is based on the assumption that the confinement of the helical wave is effected by a pair of mirrors kept facing each other. This is obviously an artificial construct and needs to be replaced by the interactions with some field. In the next paper, we shall examine this aspect in detail.

References

1. V.A.I.Menon, M.Rajendran, V.P.N.Nampoori, vixra:1211.0083(quant-ph), (2012)
2. V.A.I.Menon, M.Rajendran, V.P.N.Nampoori, vixra;1211.0112(quant-ph), (2012)
3. V.A.I.Menon, M.Rajendran, V.P.N.Nampoori, vixra:1211.0004(cl-ph), (2012)
4. James D. Bjorken and Sidney D. Drell: Relativistic Quantum Mechanics, McGraw-Hill Book Company, New York (1964).
5. J.J.Sakurai: Advanced Quantum Mechanics, Addison-Wesley Longman Inc., Second Indian reprint (1999)
6. Roger Penrose: The Road to Reality, Jonathan Cape, London (2004).