

New solution of the cosmological constant problem and superconducted cosmology

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Abstract

Application of a quantum theory of superconductivity to the formation of the primary vacuum at the Planck scale allows us to solve the cosmological constant problem and to obtain the observed value of the vacuum energy density. It is offered the new model of exponential expansion and the hot stage of the Universe. It is followed that the modern evolution of the universe can be viewed as a process of phase transition, and the physical time is an indicator of this transition.

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INTRODUCTION

As it is known, the standard estimations within the limits of the quantum field theory give values of vacuum energy density at $\omega_k^2 = k^2 + m^2$, $k_{max} \gg m$ [1]:

$$\rho_\nu = \frac{1}{2} \int_0^{k_{max}} \frac{4\pi k^2}{(2\pi)^3} dk \sqrt{k^2 + m^2} = \frac{k_{max}^4}{16\pi^2} \approx \frac{E_P^4}{16\pi^2} \approx 2.2 \cdot 10^{71} \text{GeV}^4.$$

where $E_P = (\hbar c^5 / (8\pi G_N))^{1/2}$ is the Planck energy. This value is in 10^{120} times more than the observed one $\rho_{v(obs)} \approx (2 \cdot 10^{-3} \text{eV})^4$. This is one of the most serious problems in physics, because it is also related to the nature of “dark energy”.

By this time the set of variants of the solution of the problem of the cosmological constant and energy of vacuum is offered [1–4]. However the majority of such solutions require the use of specific parameters, exotic variants of the field theory or modification of the gravitation theory. Therefore those directions of this problem solution, which are characterized by maximum physical naturalness and simplicity, have the advantages. It is possible to refer to such directions the attempts to solve the cosmological constant problem by analogy to the superconductivity theory. But within the limits of such approach usually the special assumptions on gravitational interaction between primary fermions, which defines value of vacuum energy, are made [3, 4].

An approach offered in this article does not use such additional hypotheses, and the law of interaction between primary fermions follows from the theory and experimental data. It in turn gives the value of the cosmological constant close to the observed one and explains the nature of “dark energy”. Thus it becomes clear not only the mechanism of formation of modern vacuum, but also variety of the problems related to the theory of a Big Bang and to the cosmological evolution of the Universe [5].

VACUUM STRUCTURE ON PLANCK SCALES AND SUPERCONDUCTIVITY

Usually it is supposed that the space-time on Planck scales has a foamy structure. However, there are calculations of interactions on Planck distances which show that domains, with the masses close to Planck mass, can form regular structures [6]. Such structures can form a regular crystal-like lattice with the cells close to Planck scales $L_P = (8\pi G_N \hbar / c^3)^{1/2}$. As the domains are the quasi self-contained objects, their effective mass is close to zero.

They also can interact among themselves by quadrupole gravitational forces. At such description it is possible to consider the space-time on Planck scales as analogue of a solid body [6, 7]. Collective excitations in such structure play a role of phonons, arising in a crystal-like lattice with the period close to Planck interval.

Let's consider such crystal-like structure not simply as analogue of a solid body, but as the structure similar to structure of metal in which the free primary subfermions (b-fermions) exist. These subfermions can interreact with the spatial crystal-like lattice and, under certain conditions, can couple by means of phonon interaction, it is similar to Bardeen-Cooper-Schrieffer model (BCS) [8] for electrons in metal with the bose-condensate formation. Thus coupling of subfermions can occur near the Fermi surface of such "quasicrystal". Thus the maximum oscillation frequency of crystal-like lattices, as an analogue of Debye frequency, is close to Planck frequency:

$$\omega_D = \frac{\omega_P}{4\sqrt{8\pi}} = \frac{1.85 \cdot 10^{43} \text{Hz}}{4\sqrt{8\pi}} = 9.23 \cdot 10^{41} \text{Hz}. \quad (1)$$

The macroscopical equation, which describes interaction of fermions [9], looks like:

$$\left[i \frac{\partial}{\partial t} - \frac{\nabla^2}{2m} - \zeta(\bar{\psi}\psi) \right] \psi = 0. \quad (2)$$

At the use of pulsing representation of this equation we found for Fermi energy: $E_F = p_F^2/(2m) + \zeta\langle\bar{\psi}\psi\rangle$, where p_F is an impulse of subfermions by Fermi surface,

$$E - E_F = \frac{p^2 - p_F^2}{2m} + \zeta(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle) \quad (3)$$

Raising both sides of the square, averaging and using $|\Psi|^2 = \langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2$, we can make the substitution near the Fermi surface $(p^2 - p_F^2)/(2m) \rightarrow v(p - p_F)$, where $v = p_F/m$ is a velocity of subfermions at this surface, and obtain [9]

$$E - E_F = \pm \sqrt{v^2(p - p_F)^2 + \zeta^2|\Psi|^2}.$$

$2\Delta = 2\zeta|\Psi|$ is an energy gap, related to effect of coupling of fermions, which separates the basic and excited state [9]. As it appears from theory BCS, thus

$$2\Delta = \frac{4\hbar\omega_D}{\sinh\left(\frac{1}{v \cdot M(0)}\right)} = \frac{4\hbar\omega_D}{e^{\frac{1}{\lambda}} - e^{-\frac{1}{\lambda}}} = 3.52k_B T_c \quad (4)$$

where $\lambda = V \cdot M(0)$ is a constant of fermion-phonon interactions, V is a potential, $M(0)$ is a density of fermion states on Fermi surface, T_c is a critical temperature [10].

Let's consider now vacuum energy density, starting from (4). The initial density of the Universe is equal to Planck one:

$$\rho_{v_0} = \rho_P = \left(\frac{\hbar\omega_P}{\sqrt{8\pi}} \right)^4 \cdot \frac{3}{8\pi\hbar^3 c^5} = \frac{3M_P}{(8\pi L_P)^3} = \frac{3(\hbar\omega_P)^4}{(8\pi)^3 \hbar^3 c^5}. \quad (5)$$

In that case the vacuum energy density is defined by density of energy gaps $2\Delta_v$ as binding energies of subfermion pairs, which form a condensate in a Planck crystal-like lattice:

$$\rho_v = E_v^4 \approx (2\Delta_v)^4 = \left(\frac{1}{\sqrt{8\pi}} \cdot \frac{\hbar\omega_P}{e^{\frac{1}{\lambda}} - e^{-\frac{1}{\lambda}}} \right)^4. \quad (6)$$

At $\lambda \gg 1$

$$\rho_v \approx \frac{3}{8\pi G_N \left(8\pi t_P e^{\frac{1}{\lambda}} \right)^2}. \quad (7)$$

If the density of the condensate of energy gaps is close to the up-to-date critical density

$$\rho_c \approx \frac{3}{8\pi G_N} \cdot \left(\frac{1}{8\pi t_P e^{\frac{1}{\lambda}}} \right)^2 = \frac{3}{8\pi G_N} H_0^2, \quad (8)$$

then $H^{-1} = t_H = 8\pi(G_N \hbar / c^5)^{1/2} \cdot e^{\alpha_{em}^{-1}/2} = 8\pi t_P e^{2/\lambda}$.

From here we can calculate a value of interaction constant λ . As $H^{-1} = t_H \approx 1.4 \cdot 10^{10}$ years = $4.4 \cdot 10^{17}$ s,

$$\frac{2}{\lambda} = \ln \frac{t_H}{8\pi t_P} = 137. \quad (9)$$

Thus, the received value of coefficient of fermion-phonon interactions in spatial crystal-like lattice is defined by value of electromagnetic fine-structure constant:

$$\frac{2}{\lambda} = \alpha_{em}^{-1} = \frac{\hbar c}{e^2}, \quad \frac{1}{\lambda} = \frac{\alpha_{em}^{-1}}{2} = \frac{\hbar c}{2e^2}. \quad (10)$$

Let's note that λ is defined also by a ratio of value of a charge of a Dirac magnetic monopole to an electrical charge: $\lambda^{-1} = g/e$. This result naturally confirms our point of view on character of interaction of fermions in spatial crystal-like lattice. Thus, the mechanism of condensation of primary fermions has not the gravitational nature, but is defined by special fermion-phonon interaction. The coefficient of this interaction is the function of the electromagnetic fine-structure constant. The description of this fermion-phonon interaction will be given in separate work. The vacuum energy density as condensate of energy gaps or energies interaction of subfermions in a primary crystal-like lattice is:

$$\rho_v = \Omega_v \frac{3}{8\pi G_N (8\pi t_P \cdot e^{\alpha_{em}^{-1}})^2} = \Omega_v \frac{3}{(8\pi)^3} \cdot \frac{c^{10}}{G_N^2 \hbar} \cdot \frac{1}{e^{2\alpha_{em}^{-1}}}. \quad (11)$$

At $\Omega_\nu = 3/4$

$$\rho_\nu = \frac{9}{4} \cdot \frac{1}{(8\pi)^3} \cdot \frac{c^{10}}{G_N^2 \hbar} \cdot \frac{1}{e^{2\alpha_{em}^{-1}}} = \frac{9}{32\pi G_N t_P^2} \cdot \frac{1}{e^{2\alpha_{em}^{-1}}} \quad (12)$$

$$\Lambda^{-\frac{1}{2}} = (3\Omega_\nu)^{-\frac{1}{2}} \cdot 8\pi e^{\alpha_{em}^{-1}} t_P = \frac{16}{3} \pi e^{\alpha_{em}^{-1}} t_P. \quad (13)$$

Thus the dynamic parameter, the Hubble parameter, is very close to a value of the constant or is equal to it. Such exact coincidence, apparently, is related to affinity of values of density of substance and vacuum in present period.

A fact, that the formula (8) gives the modern energy density of the Universe, allows to assume that “dark energy” and observable substance within the limits of superconducting model can be described as a condensate and supercondensate of primary subfermions.

Therefore, the existence of “dark energy” of the Universe is defined by the process of phase transition — condensation of subfermions into superconducting electron pairs and formation of a Bose condensate of these pairs.

DYNAMICS OF THE MODERN VACUUM DENSITY

Let’s consider the process of formation of the up-to-date vacuum in the hot early Universe. As it is known from quantum electrodynamics, a value of electromagnetic fine-structure constant is function of four-impulse Q^2 : $\alpha_i^{-1} = \alpha_{em}^{-1} - (\beta/(3\pi)) \ln(Q_i/(2m_e))^2$, where m_e is the mass of electron, $\alpha_{em} = e^2/\hbar c$ is the fine-structure constant [11].

Then the effective vacuum energy density as density of energy gaps will make:

$$\rho_{cond} = \rho_\nu \approx \Omega_\nu \cdot \frac{3c^5}{(8\pi)^3 G_N^2 \hbar e^{2\alpha_{em}^{-1}}} \left(\frac{Q_i}{2m_e} \right)^{4\beta/(3\pi)}, \quad (14)$$

where Q_i is a 4-impulse of quanta of radiation and substance in the early Universe.

The energy density of a vacuum condensate is driven by the energy density of radiation and substances. Thus ρ_ν reaches a minimum and becomes a stationary value at $Q = 2m_e c = 1.022 \text{ MeV}/c$. For energies $\mu_{GUT} \approx 10^{15} \text{ eV}$, $\alpha^{-1}(\mu_{GUT}) = \alpha^{-1}(\mu_0) - (b_i/(2\pi)) \ln(\mu_{GUT}/\mu_0)$,

$$\rho_\nu = \frac{(\hbar\omega_p)^4}{(8\pi)^3 e^{2\alpha^{-1}(\mu_0)}} \left(\frac{\mu_{GUT}}{\mu_0} \right)^{2b_i/\pi}. \quad (15)$$

Let’s consider now a question on quantity of α_{em}^{-1} change. At the Planck energies the value for vacuum is $\rho_\nu \approx \rho_P$ and $\alpha_i^{-1} = 1$. Thus the value of an electrical charge equally primary one: $e_0^2 = \hbar c$.

For definition of the law of α_i^{-1} change we will consider some aspects of the Universe formation. If the Universe started from Planck density $\rho_P \sim M_P^4$, it extended in a vacuum-like state till the moment of phase transition under the law

$$\rho_v = \rho_P \left(\frac{1}{e^{\alpha_i^{-1}}} \right)^2 \approx \frac{1}{8\pi G_N} \Lambda_i = \frac{1}{8\pi G_N \tau_v^2}, \quad (16)$$

and the radius of the Universe at the moment of transition from vacuum-like state in a hot state was $R_U = R_v = R_H T_{CMBR} / T_{GUT} = \Lambda_i^{-1/2} = 8\pi L_P e^{\alpha_i^{-1}}$, where $R_H = cH^{-1}$ is the up-to-date Hubble radius, T_{CMBR} is the temperature of cosmic microwave background radiation. Thus τ_v plays a role of parameter of time for phase transition.

The value α_i^{-1} could change from 1 to $\alpha_{GUT}^{-1} \approx 80$. At the moment of phase transition we can estimate the radius of the Universe R_U and, accordingly, value α_i^{-1} at $\rho_{GUT} = 3 \cdot (32\pi G_N t_{GUT}^2)^{-1} \approx (10^{15} \text{ GeV})^4$ depending on value of energy of a vacuum energy gap Δ_v .

So, at $2\Delta_v = 3.22 \cdot 10^3 \text{ GeV}$ and $k_B T_{GUT} = 2.11 \cdot 10^{16} \text{ GeV}$ the Universe radius is $R_U = 2.32 \cdot 10^{-2} \text{ sm}$ at $\alpha_i^{-1} = \alpha_{em}^{-1} / 2 = 68.518$.

At $2\Delta_v = m_{H^0}^* = 246.3 \text{ GeV}$ and $k_B T_{GUT} = 1.35 \cdot 10^{15} \text{ GeV}$ the Universe radius is $R_U = 2.292 \text{ sm}$ at $\alpha_i^{-1} = 73.1$.

At $2\Delta_v = m_{Z^0}^* = 91.18 \text{ GeV}$ and $k_B T_{GUT} = 5.02 \cdot 10^{14} \text{ GeV}$ the Universe radius is $R_U = 16.7 \text{ sm}$ at $\alpha_i^{-1} = 75$.

At $2\Delta_v = m_{W^\pm}^* = 80.4 \text{ GeV}$ and $k_B T_{GUT} = 4.43 \cdot 10^{14} \text{ GeV}$ the Universe radius is $R_U = 21.49 \text{ sm}$ at $\alpha_i^{-1} = 77.22$.

The energy of phonon interaction between primary subfermions can be considered as the special form of energy which differs from observable forms of energy.

Let's note that the value of initial exponential expansion of the Universe corresponds to that which arises also in the inflation theory. Therefore the superconducting mechanism of the Universe's expansion provides causality and homogeneity of the Universe. Moreover, while in the inflationary theory maintenance of the Universe homogeneity is a one-time event, in the superconducting scenario existence of a quantum vacuum condensate continuously provides homogeneity of the Universe. It resolves the Penrose's paradox: R. Penrose repeatedly underlined the lack of mechanism of sync of electroweak phase transition in various points of space, which occurs much more after inflationary dilating [12]

It means that the birth of substance during a Big Bang is defined by presence of special vacuum vector bosons and, possibly, the Higgs vacuum field. The energies of prospective

usual X, Y-bosons with $E \approx 10^{15}\text{GeV}$ correspond to these vacuum bosons with mass $m^* = 2\Delta_v$. Therefore the higher energies $2\Delta_v > 10^3\text{GeV}$ and $E > 10^{16}\text{GeV}$, seemingly, are not realized because the Universe warming up at transition from a vacuum state to radiation-dominating state begins with $E_{GUT} \approx 1.35 \cdot 10^{15}\text{GeV}$. At the same time it is a possibility of Universe expansion not from Planck, but from bigger volume, for example from volume with radius $r = 8\pi L_P e^{\frac{1}{2}\alpha_{em}^{-1}} = 2.32 \cdot 10^{-2}\text{sm}$. Such volume corresponds to the greatest possible packaging of the Planck domains which number is $N_P \approx e^{\frac{3}{2}\alpha_{em}^{-1}} = 1.86 \cdot 10^{89}$.

As far as the quantity of fundamental particles in the Universe is close to this number, it is obvious that phonon oscillations of the lattices, formed by domains of spatial quasi-crystal, at phase transition have served as the cause of formation of these particles. Let's note that the presence of a vacuum condensate with $\rho_v = (3.22 \cdot 10^3\text{GeV})^4$ provides homogeneity of such volume and there is no necessity to care of existence of 10^{89} causally unrelated Planck volumes: all of them prove to be related to a vacuum coherent subfermion condensate. It follows that the Universe birth could occur in 2 stages: the first was the formation of quasi-stable vacuum-like state with $r_v = e^{\frac{1}{2}\alpha_{em}^{-1}} 8\pi L_P$ and the second was the expansion of this volume for $t_{GUT} \approx 10^{-37}\text{s}$ in 90–100 times. At exterior energy action the vacuum-like Universe could begin transition into radiation-dominated state with an energy liberation $E = E_{GUT}$. It caused the phase transition and the beginning of the hot Universe. Thus, according to (14), a reorganisation of structure of vacuum happened because of α^{-1} changes. At $\alpha_i^{-1} = \alpha_{0em}^{-1}$ a radius of vacuum curvature $\Lambda^{-\frac{1}{2}}$ becomes close to $R_H|_{z=0}$: $\Lambda^{-\frac{1}{2}} \sim R_H$ at $Q_i = 2m_e c$.

At the second scenario phase transition with condensate formation looks natural, because the certain value of parameter of interaction $\lambda = 2\alpha$ already existed.

PHYSICAL TIME AS THE FUNCTION OF THE PHASE TRANSITION

Within the limits of superconducting cosmology the radius $\Lambda^{-1/2}$ and the Hubble radius are nearly equal. The cosmological time t_U is $t_U \approx t_H = H^{-1} = \Omega_v^{-1/2} R_\Lambda / c = 8\pi t_P e^{\alpha_i^{-1}}$. Now, at $z = 0$, $\alpha_{em}^{-1} = \alpha_j^{-1}$. Then it is possible to present the evolution of the critical density of the Universe energy also in the form of α_j^{-1} evolution:

$$\rho_c = \frac{3}{8\pi G_N t_H^2} = \frac{3}{8\pi G_N (8\pi t_H e^{\alpha_{em}^{-1}})^2} \left(\frac{Q_j}{2m_e} \right)^{4\beta/(3\pi)}, \quad (17)$$

where Q_j is a 4-impulse of quanta of radiation and substance, which are so far unknown and, probably, related to other dimensions.

Thus, current Hubble time t_H and observable evolution of the Universe can be described as process of phase transition with change of parameter of interaction between fermions and phonons $\alpha_j^{-1} = \ln(t_H/(8\pi t_P))$, similar to α_{em} . So a hierarchy of parameters of evolution or times appears, it relates to the hierarchy of the energies of the fermions condensates, each of them plays a role of vacuum for overlying level.

VACUUM ENERGY DENSITY

Occurrence and destruction of condensates in the hot Universe correspond to the law (17). Therefore usually calculated vacuum of electroweak interactions and a QCD vacuum, which arose during phase transitions in the early Universe [2], are distinct from real vacuum. They, possibly, represent false vacuums in relation to the vacuum condensate, considered above. During cooling of the hot Universe they disappear. It is easy to understand if to take into consideration that the hypothetical vacuum should stop evolution at level of energy density of gluon condensate $\rho_v \approx (0.2 \text{ GeV})^4$ or π -mesons $(0.14 \text{ GeV})^4$ [2]. This value is greater than the modern vacuum energy density $\rho_v \approx (2.3 \cdot 10^{-3} \text{ eV})^4$ to 44 order, but there are no observable phase transitions to the modern vacuum density.

However the Universe cooling to the up-to-date temperature gives the chance to display the true vacuum condensate. It is thus obvious that the average density $\langle \rho_f \rangle$ of real vacuum energy of other fields of the observable Universe does not exceed the energy of this vacuum condensate: $\langle \rho_f \rangle \leq \rho_v$, as the frequency of phonon excitations in the crystal-like space lattice is less than the Debye frequency $\omega_i \leq \omega_D$ (see (1)). Apparently, the true vacuum of fields both in hot and in the up-to-date Universe coincides with a vacuum evolving condensate as the inferior energy level.

CONCLUSION

1. Within the limits of the theory of superconductivity for crystal-like space on Planck distances the real value of density of vacuum energy as density of energy gaps $(2\Delta)^4$ of phonon interactions of a primary subfermions condensate is received. Subfermions,

apparently, do not give a contribution to observable energy density. The contribution to observable forms of energy is given only by phonon excitation of a primary lattice — boze- and fermi-phonons.

2. Initial exponential expansion of the vacuum-like Universe within the limits of superconducting cosmology allows to provide the birth of the hot Universe and solves the same problems which are solved by an inflationary cosmology. Thus in the initial Universe at least two components exist: the first component is, probably, the supercondensat, it breaks up and generates the hot Universe with temperature $T_{GUT} \approx 1.35 \cdot 10^{15}$ GeV; the second component with much lower energy plays a role of vacuum for the hot Universe.
3. Origin of cosmological time $t_U \approx t_H$ becomes clear: in the observable Universe time is a consequence of proceeding phase transition of II kind, which is similar to the phase transition, which has created the up-to-date vacuum energy density with change and fixing of a fine-structure constant $\alpha_j = \ln(t_H/(8\pi t_P))$.
4. Transition into a superconducting state at $T < T_c$ is accompanied by entropy reduction, because entropy of a superconducting state S_S is less than entropy of a normal state S_N : $S_S - S_N < 0$. It means that during the Universe expansion process both entropy of a vacuum condensate ($\partial S_V/\partial t < 0$) and entropy of the Universe ($\partial S_{\mu_0 t}/\partial t < 0$) at its cooling decrease. Let's note that it corresponds to Nernst's theorem, because $S \rightarrow 0$ at $T \rightarrow 0$. It explains forming of complex structures, including live beings, together with observers, during the Universe evolution, contrary to the "thermal death" concept.
5. If occurrence of the observer of terrestrial type is the indicator of the termination of phase transition and the beginning of a new stage of the Universe evolution, it explains the cause of applicability of Anthropic principle: the determined phase transition of the Universe in a certain state generates the observable Universe with observers [13].
6. Existence of a coherent condensate of primary fermions with one wave function ψ_c , interacting with the crystal-like space lattice on Planck scales, eliminates a problem of homogeneity of the Universe at all stages of its evolution.

Thus, the vacuum condensate, defining the primary Universe expansion, defines also modern dynamics of its evolution.

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