The quantization in the Bohr’s theory about the hydrogen atom

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ABSTRACT
The article treats that the quantization in the Bohr’s theory about the hydrogen atom. If calculates the electron motion in the hydrogen atom by the classical mechanic, can do the quantization in the Bohr’s theory about the hydrogen atom. In this time, the electron’s orbit velocity \( v \) is non-relativity velocity.

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I. Introduction

The article treats that the quantization in the Bohr’s theory about the hydrogen atom.

If calculates the electron motion in the hydrogen atom by Coulomb’s law in the classical mechanic, in (D.Halliday & R.Resnick, Fundamentals of PHYSICS(John Wiley & Sons,Inc.,1986))

\[ F = ma \]
\[ \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = m \frac{v^2}{r} \]  \hspace{1cm} (1)

\[ m \] is the electron’s mass.

The electron’s kinetic Energy is

\[ K = \frac{1}{2} mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \]  \hspace{1cm} (2)

The potential Energy in the hydrogen atom is

\[ U = V(-e) = -\frac{e^2}{4\pi\epsilon_0 r}, \quad V = \frac{e}{4\pi\epsilon_0 r} \]  \hspace{1cm} (3)

The total Energy in the hydrogen atom is

\[ E = K + U = -\frac{e^2}{8\pi\epsilon_0 r} \]  \hspace{1cm} (4)

The Bohr’s frequency condition is

\[ h\nu = E_k - E_j \]  \hspace{1cm} (5)

By Eq(2), the electron’s the orbit velocity \( v \), the momentum \( p \), the angular momentum \( L \) are

\[ v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}}, \quad p = mv = \sqrt{\frac{me^2}{4\pi\epsilon_0 r}}, \quad L = pr = \sqrt{\frac{me^2 r}{4\pi\epsilon_0}} \]  \hspace{1cm} (6)

II. Additional chapter-I

The Bohr’s hypothesis is

\[ L = n\frac{h}{2\pi} \]  \hspace{1cm} (7) \quad n = 1,2,3,...

By Eq(6), Eq(7), the radius \( r \) is

\[ r = r_n = n^2 \frac{\hbar^2 \epsilon_0}{\pi me^2} \]  \hspace{1cm} (8) \quad n = 1,2,3,...

By Eq(4), Eq(8), the total energy \( E \) is

\[ E = E_n = -\frac{me^4}{8\epsilon_0^2 \hbar^2 n^2} \]  \hspace{1cm} (9) \quad n = 1,2,3,...

By Eq(5), Eq(9), the hydrogen line spectra’s frequency \( \nu \) is
\[ \nu = \frac{me^4}{8\varepsilon_0^2 h^3} \left( \frac{1}{j^2} - \frac{1}{k^2} \right) \quad (10) \quad j, k \text{ is number.} \]

By Eq(6),Eq(8), the quantization of the electron’s the orbit velocity \( \nu \), the momentum \( p \) is in the hydrogen atom

\[ \nu = \nu_n = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{2n\hbar\varepsilon_0}, \quad \nu_1 = \frac{e^2}{2\hbar\varepsilon_0} \ll c \quad (11) \quad n = 1, 2, 3, \ldots \]

\[ p = p_n = mv = \sqrt{\frac{me^2}{4\pi\varepsilon_0 r}} = \frac{me^2}{2n\hbar\varepsilon_0} \quad (12) \quad n = 1, 2, 3, \ldots \]

In this time, the electron’s orbit velocity \( \nu \) in the hydrogen atom is not continuous.

Bohr’s orbit that it’s radius is \( r \) is

\[ n\lambda = 2\pi r \quad (13) \quad n = 1, 2, 3, \ldots \]

Therefore, the quantization of the electron’s wavelength \( \lambda \) is in the hydrogen atom

\[ r = \frac{n\lambda}{2\pi} = \frac{n^2 \hbar^2 \varepsilon_0}{\pi me^2} \]

\[ \lambda = \lambda_n = \frac{2n\hbar^2 \varepsilon_0}{me^2} \quad (14) \quad n = 1, 2, 3, \ldots \]

By Eq(2),Eq(8), the quantization of the electron’s kinetic Energy is

\[ K = \frac{1}{2} mv^2 = K_n = \frac{e^2}{8\pi\varepsilon_0 r} = \frac{e^4 m}{8n^2 \hbar^2 \varepsilon_0^2} \quad (15) \quad n = 1, 2, 3, \ldots \]

By Eq(3),Eq(8), the quantization of the potential Energy in the hydrogen atom is

\[ U_n = V_n (-e) = -\frac{e^2}{4\pi\varepsilon_0 r^2} = -\frac{e^4 m}{4n^2 \hbar^2 \varepsilon_0^2}, \quad V_n = \frac{e}{4\pi\varepsilon_0 r} = \frac{e^3 m}{4n^2 \hbar^2 \varepsilon_0^2} \quad (16) \quad n = 1, 2, 3, \ldots \]

By Eq(1),Eq(8), the quantization of the electric force is in the hydrogen atom is

\[ F = F_n = \frac{mv^2}{r} = \frac{e^4 m}{4n^2 \hbar^2 \varepsilon_0^2} \]

In this time, the electric force in the hydrogen atom is quantized.

**III. Conclusion**
In this time, de Broglie wavelength is in the hydrogen atom

\[ \lambda = \frac{2nh^2}{me^2} = \frac{h}{p} \]  \hspace{1cm} n = 1, 2, 3, ... \hspace{1cm} (18)

In Eq(8), Eq(9), Eq(11), Eq(12), Eq(15), Eq(16), Eq(17), if \( n \to \infty \), it is \( r \to \infty \), \( E \to 0 \), \( v \to 0 \), \( p \to 0 \), \( K \to 0 \), \( U \to 0 \), \( V \to 0 \), \( F \to 0 \). In this case, the state is that it is except the electron from the hydrogen atom.

**Reference**


