Newton's Formula of Universal Gravitation is Just Kepler's Third Law

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Abstract

Stepwise analysis reveals that Newton's Formula of Universal Gravitation is just Kepler's Third Law. Newton fully plagiarized Kepler's formula, but added new definitions which were entirely groundless.

I Introduction

The Universal Gravitation Formula is commonly believed to be developed by Newton himself. But in his works, Newton didn't mention where on earth this formula was from.

We can deduce the Universal Gravitation Formula from Kepler's Third Law by using Newton's thoughts, and thus clarify where on earth the Universal Gravitation Formula was from.

II Deduction

Kepler's Third Law is expressed as:
\[
\frac{R^3}{T^2} = K \quad (1)
\]
R is the average orbital radius, and T is the orbiting period.

Because \( T = \frac{2\pi R}{V} \), substitute it into Formula (1), then
\[
V^2 \cdot R = 4\pi^2 K \quad (2)
\]
Put the variables at the left side of the formula and the constants at the right side.
Formula (2) would have immediately reminded Newton of the centripetal acceleration formula: \( \frac{V^2}{R} = a \)

Then transform the left side of Formula (2) to \( \frac{V^2}{R} \), and divide both sides by \( R^2 \), then:

\[
\frac{V^2}{R} = \frac{4\pi^2K}{R^2}
\]

(3)

The core of Newton's mechanics is \( F = ma \), so he would change the left side of Formula (3) into \( F \), namely \( \frac{V^2}{R} \cdot m \).

Multiply both sides of Formula (3) by \( m \) (e.g. the mass of a planet in the solar system), then:

\[
m \cdot \frac{V^2}{R} = \frac{4\pi^2K \cdot m}{R^2}
\]

(4)

The next step is the most miraculous and important. When Newton saw “\( 4\pi^2K \)” in Formula (4), he thought it was too large. Time before Newton's era, people already knew \( k \) is a constant that is only decided by the central celestial body and independent from the orbiting celestial bodies. People also roughly knew a larger central celestial body has a larger \( k \), and they might be in direct proportion. Following this clue, Newton made a very bold hypothesis or guess that “\( 4\pi^2K \)” is the mass of a central celestial body. Then he found “\( 4\pi^2K \)” and mass have different units, so accordingly, he introduced a "constant with unit", which is the gravitational constant \( G \). At this moment, Newton provided at own willingness that:

\[
MG = 4\pi^2K, \text{ where } M \text{ is the mass of the central celestial body. Substitute it into Formula (4), then}
\]

\[
m \cdot \frac{V^2}{R} = G \frac{M \cdot m}{R^2}
\]

(5)

\[
F = ma = m \cdot \frac{V^2}{R} = G \frac{M \cdot m}{R^2}
\]

Formula (5) is exactly the Universal Gravitation Formula that we are familiar with.

We now summarize the whole process. It is a regular transformation from Formula (1) (Kepler's Third Law) to Formula (4). But From Formula (4) to Formula (5), why did he replace “\( 4\pi^2K \)” by \( MG \), Newton didn't provide any reliable or verifiable proof.
In other words, someone else might provide “$4\pi^2K$” as the product of total sun energy and a constant:

$$NG=4\pi^2K$$

$G$ is the constant that balances units, $N$ is total sun energy; substitute into Formula (4), then a "new universal gravitation formula" comes:

$$m\cdot\frac{V^2}{R}=G\frac{N\cdot m}{R^2} \quad (6)$$

$$F=ma= m\cdot\frac{V^2}{R}=G\frac{N\cdot m}{R^2}$$

If we calculate "the orbiting period, linear velocity, distance of celestial bodies, centripetal acceleration" of moving celestials with Formula (6), we can also get the results that are exactly the same as observations. Why? Because this formula is exactly Kepler's Third Law. “$4\pi^2K$” can be replaced by any "letter" without changing the essence of Kepler's Third Law. In this way, Newton replaced “$ma$” by the letter “$F$”. With this "letter replacement", we can create countless forms of Kepler's Third Law. This is just like calling a person by a hundred names with different letters, but he is still the same person.

Both Formula (5) (Newton's Formula of Universal Gravitation) and Formula (6) (new universal gravitation formula) are essentially Kepler's Third Law, because they only undergo some transformation and replacement. Therefore, Newton's Formula of Universal Gravitation and Kepler's Third Law are the same. By means of camouflaging, Newton transformed Kepler's Third Law, which is originally simple and purein, into a gravitation formula, which is complicated and obscure. All achievements in astronomy and astronautics made by Newton's Formula of Universal Gravitation should be attributed to Kepler's Third Law. It is inappropriate to regard Universal Gravitation Formula and Kepler's Third Law as two formulae. When people are calculating celestial bodies and satellites with Universal Gravitation Formula, they don't realize they are using Kepler's Third Law.
III Example

I'll demonstrate the relationship between Universal Gravitation Formula and Kepler's Third Law by using an example. There were two people A and B. A found a formula \( \frac{a}{b} = K \). Many years later, A died. Then B jumped out and said he "found" a formula, which was acquired this way:

\[
\frac{a}{b} = K \tag{7}
\]

\[
ac = bcK \tag{8}
\]

\[
ac = \frac{bc2K}{2} \tag{9}
\]

Then B personally provided \( F = ac \), \( M = 2K \) and substitute into Formula (9), then

\[
F = ac = \frac{bcM}{2} \tag{10}
\]

Then B said the newly "found" formula was Formula (10) and considered this formula greater than Formula (7), because Formula (7) can be deduced from Formula (10). The deduction is:

Formula (10) was transformed into \( \frac{a}{b} \cdot \frac{M}{2} \), namely \( K = \frac{M}{2} \). Then B's followers would say: "Look! M is constant, so is K. So Formula (7) was verified."

Formula (10) can also be transformed into \( M = 2 \cdot \frac{a}{b} = 2K \). Then the followers say:

"Look! We can even get M. How great this formula is!"

It is really funny and annoying.

B obtained Formula (10), but this shouldn't be regarded as a discovery or invention, because B only copied A's formula. B even changed it completely without any foundation, and added some definitions at will. B destroyed the spirits of Formula (7).

Why B didn't tell us how exactly Formula (10) was deduced, but lied that he thought of this formula by himself.

V Conclusions

In Philosophiae Naturalis Principia Mathematica, Newton obtained the preliminary Universal Gravitation Formula by a roundabout math method full of
mistakes. He carelessly added some definitions (two matters with mass are inter-attracting). He only hid the fact he plagiarized Kepler's Third Law.

Hegel thought Kepler who was starved to death in Germany was the real founder of modern celestial mechanics, while Newton's law of gravity was included in Kepler's three laws, especially in the Third Law. Hegel was right.