Smoothening properties related to the Goldbach's Conjecture

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Summary

In this short paper we reveal a sequence of three smoothing procedures related to Goldbach's conjecture. While the cloud of points that represent the number of pairs (p, q), which fulfill the conjecture $p+q=2n$, versus $2n$ occupies a quite broad area of curvilinear triangular shape, suitable averages can reduce it into oscillating lines with progressively decreasing amplitudes.

1. Introduction

As is known, on June 7, 1742, Christian Goldbach in a letter to Leonhard Euler [1] argued that, "every even natural number > 4 can be written as a sum of two primes", namely:

$$2n = p + q \quad \text{where } n>2, \text{ and } p, q \text{ are prime numbers.} \tag{1}$$

We note that Eq(1) has been slightly modified from its original form [1], because the newest definition of all primes $\mathbb{P} = \{2,3,5,7,11,\ldots\}$ excludes the unit.

An extensive literature survey and a deterministic procedure to determine the pairs (p,q) as well as their average number has been recently reported by Markakis et al. [2].

This work continues the research based on results that were obtained using MATLAB® on a common PC for all even numbers between 6 and 360,000. The distribution of all results is shown in **Figure 1**.

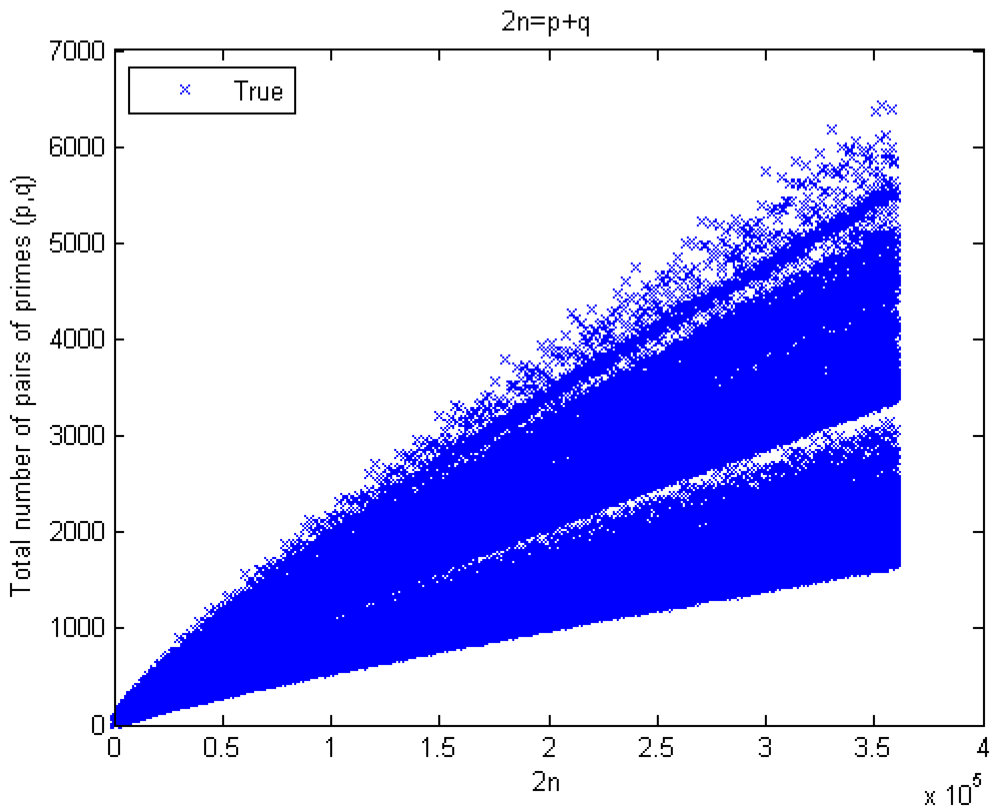

Figure 1: Cloud of points that represent the number of pairs (p, q) that fulfill the condition $p + q = 2n$.

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2. Smoothing procedures

It was previously mentioned in [2] that if we divide the set of natural numbers, $\mathbb{N}$, into cells of the form $(6\lambda-2, 6\lambda, 6\lambda+2)$, then the zone created by only 6λ (in the middle of the cells) is highly correlated with the zone created by the sum of the pairs that correspond to both $(6\lambda-2)$ and $(6\lambda+2)$ (i.e. the ends of the same cell). This is clearly shown again in **Figure 2**.

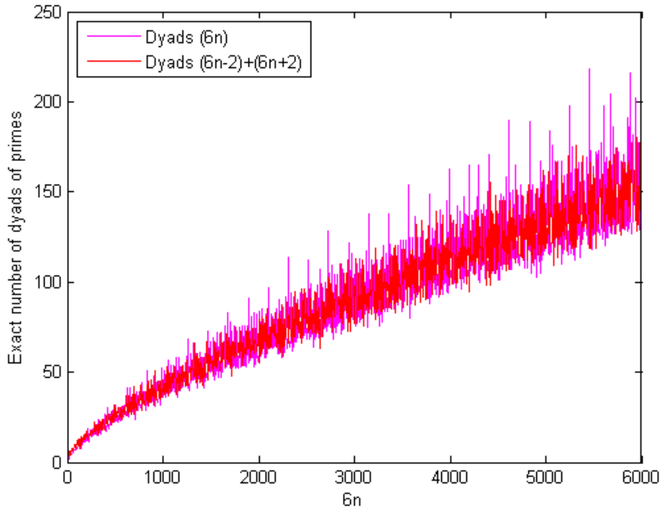

Figure 2: Number of pairs (dyads) of prime numbers that fulfill Goldbach's Conjecture for the lowest 6000 numbers, in two characteristic categories [magenta line: dyads 6*n*, red line: dyads (6*n*-2) and (6*n*+2)].
(Reprinted from Reference [2])

This paper goes further and performs averaging on all three values of the same cell. In more details, the first cell is chosen as (10, 12, 14), the second, cell is (16, 18, 20), and so on. The first smoothing procedure, here called $\mathbf{S_0}$, consists of computing the mean average (one-third of the sum) of the values of pairs at 2*n*=10, 12 and 14 and finally of plotting it in the middle (i.e. at 2*n*=12) of the first cell. Similarly, the mean average of 2*n*=16, 18 and 20 is put in the middle of the second cell (i.e. at 2*n*=18), and so on.

The second smoothing, here called $\mathbf{S_1}$, consists of computing the mean average value of three consecutive values in the new cells that are constructed based on the above-mentioned average values of smoothing $\mathbf{S_0}$. In more details, the first new cell refers to 2*n* = (12, 18, 24), the second to (30, 36, 42), and so on.

The third smoothing, here called $\mathbf{S_2}$, consists of computing the mean average value of three consecutive values in the new cells that are constructed based on the above-mentioned average values of smoothing $\mathbf{S_1}$. In more details, the first new cell refers to 2*n* = (18, 36, 54), the second new cell to (72, 90, 108), and so on.

The graphs of S_0, S_1 and S_2 are illustrated in **Figure 3** in red, yellow and cyan color, respectively. It is clear that the corresponding amplitudes progressively diminish. Obviously, if further smoothing is applied on S_2 so as to obtain S_3, it is anticipated that the latter will lead to a thinner line but it would be very difficult to be identified by naked eye if it were plotted in the same graph.

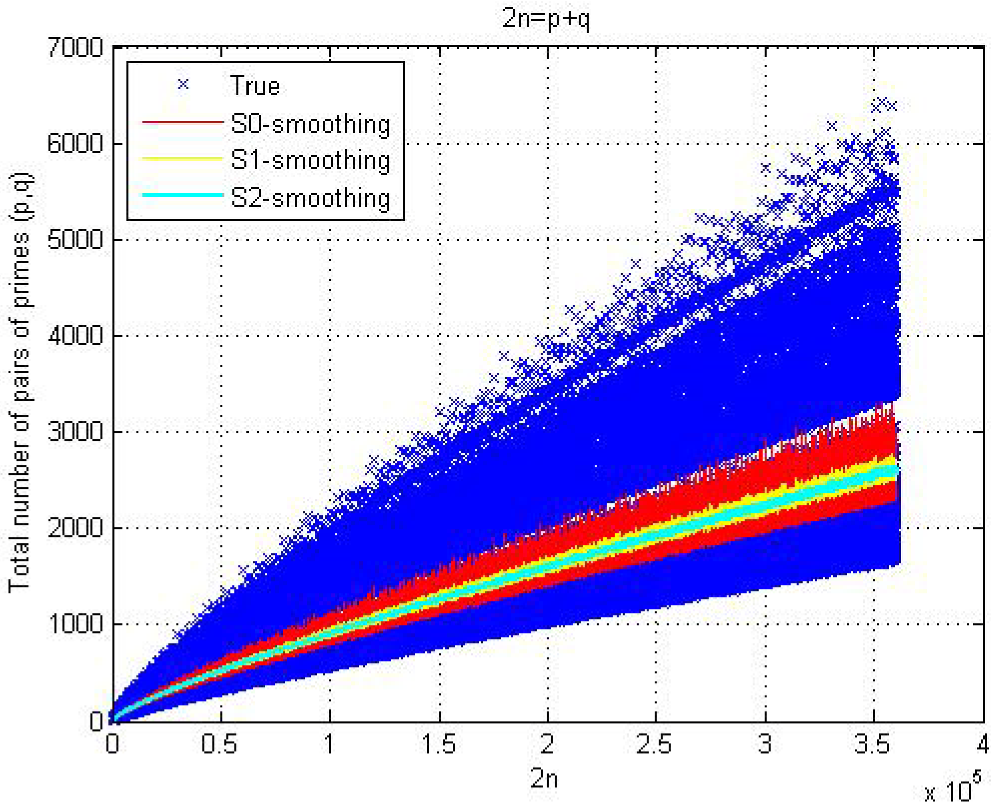

Figure 3: The three smoothing procedures within the cloud of points that fulfill Goldbach's conjecture.

REFERENCES

[1] Christian Goldbach, Letter to L. Euler, June 7, 1742.

[2] E. Markakis, C. Provatidis, and N. Markakis, Some Issues on Goldbach Conjecture, viXra:1205.0108, download from: http://vixra.org/pdf/1205.0108v1.pdf