The Correct Derivation of Magnetism from Electrostatics Based on Covariant Formulation of Coulomb's Law

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Abstract

It is shown , by clear proofs , that , it is impossible to derive magnetism from electricity, from Coulomb's law and Lorentz transformation . It is also shown by a simple thought experiment that a velocity dependant correction to Coulomb's law is a necessary result of Special Relativity . A new method to derive magnetism from electricity based on a modified covariant form of Coulomb's law is given . The method used in derivation and the formulation shown to be consistent with the theory of relativity and the facts of electromagnetic relations .

Keywords

Covariant formulation , Coulomb’s law , Magnetism , Derivation , Relativity .

1.Introduction

Covariant formulation of the fundamental laws of physics attracts growing interest for aesthetic reasons and also because of its possibly important role in reducing the number of the fundamental theories and concepts as an advance on the way to the (dream of an) ultimate unified theory .

2.The Need for Covariant Formulation of Coulomb's Law

The facts that Coulomb's law holds accurately only when the charged particles are stationary , represents a sufficient reason for searching for a covariant generalization for the law so that it holds fully accurately in all movements states of the particles as well as all frames of reference , on condition that it reproduces the usual formulation of Coulomb in the states of slow movement . There is another justification for this searching which is that , a general and covariant formulation will solve the question of the possibility and method of the derivation of magnetism from electricity , because in the presence of such a formulation , as will be shown in details , electricity and magnetism will be totally unified .

3.The Inconsistency of the Derivation Based on Lorentz Transformation

Before introducing the general covariant formulation , it is important to decide about the attempts which have been made to derive magnetism from electricity which is (claimed to be) based on Coulomb's law , Lorentz transformation and charge invariance .

There are at least three arguments each of them is sufficient to certify the impossibility of such a derivation :

Firstly ; in all these attempts [1] , the derivation depends on looking for a source of electrostatic force which appears to an observer in the moving test-charge frame .

This is enough to reject the derivation , because , according to Special Relativity the laws of physics must be applied in all frames *separately* , and it is wrong to search for a source of a force which is observed in certain frame in another frame , *a force observed in a frame must find its source in the same frame* .

Secondly ; the attempts claims the existence of a net charge density in a neutral wire carrying an electric current when observed by an observer who moves along with the wire as a result of length contraction .

Although , the net charge density may be (or may not be) a result if a continuous distribution of electric charge along the wire is assumed , it cannot be so in the real case of quantized form of electric charges , because the number of the positive charges as well as the number of the negative charges in any part of the wire will not be affected by length contraction at all , so there will be no net charge in any frame of reference.

Thirdly ; even the claims of the net charge density cannot be stated in the case of a test-charge moving perpendicular to the current .

4.The Inevitability of Velocity Dependent Correction to Coulomb's law

It can be easily shown by a simple thought experiment that the dependence of force on velocity in any law of physics that describes the interaction between two particles is a necessary consequence of Special Relativity .

Suppose that there are two similar particle (A) and (B) are in interaction according to a physical law without velocity dependence and suppose that (A) is stationary and (B) is moving with a velocity (v) perpendicular to the direction of the distance between the two particles as observed from frame (S1) in which the force of interaction between the two particles is (F) along the direction of the distance between the particles .

Now , when looking from another frame (S2) in which (B) is stationary we will see that (A) is moving with a velocity (v) perpendicular to the direction of the distance between the two particles .

Because of the symmetry and total similarity of the observation of (S1) and (S2) , the force as observed in (S2) must be (F) , but this contradicts what is obtained by Lorentz Transformation in this case which is : ($F\sqrt{1-\frac{v^2}{c^2}}$) .

This contradiction cannot be resolved unless a velocity dependant correction to the law is adopted . The requirements of the resolution of this paradox and the requirements of covariance in the general case are used as a guide to construct the covariant formulation .

5.The General Covariant Coulomb's Law and the Correct Alternative Derivation of Magnetic Force from the Electric Force

5.1.The General Covariant Coulomb's Law

The general covariant formulation which satisfies all the requirements is in the following form :

$$f = k\,\frac{q^o q^1}{{R_o}^2}\sqrt{\frac{c^2 - v^2(\sin\theta)^2}{c^2 - v^2}}\sqrt{\frac{c^2 - u^2\,(\sin\omega)^2}{c^2}}$$

Where :

$\boldsymbol{s^o}$ *is the frame in which* $\boldsymbol{q^o}$ *is stationary.*

$\boldsymbol{R_o}$ *is distance between the two charges as measured in* $\boldsymbol{s^o}$.

($\boldsymbol{R_1}$*which is the distance between the two charges as measured in the rest frame of the charge* $\boldsymbol{q^1}$*is equal to*$\boldsymbol{R_o}$ *because of the symmetry of the two observations and the well-known length contraction equation is not applicable because the two ends of the distance are in relative motion*).

$\boldsymbol{f}$ *is the force acting on the charge* $\boldsymbol{q^o}$*always in the direction of* $\boldsymbol{R_o}$. *observed in the general frame* **S.**

$\boldsymbol{f^o}$ *is the force acting on the charge* $\boldsymbol{q^o}$*always in the direction of* $\boldsymbol{R_o}$. *observed in* $\boldsymbol{s^o}$**.**

$\boldsymbol{v}$ *is the velocity of the charge* $\boldsymbol{q^1}$ *with respect to the frame* $\boldsymbol{s^o}$.

$\boldsymbol{\theta}$ *is the angle between* $\boldsymbol{v}$ *and* $\boldsymbol{R_o}$ *measured in* $\boldsymbol{s^o}$.

$\boldsymbol{u}$ *is the velocity of the general frame* **S** *with respect to the frame* $\boldsymbol{s^o}$**.**

$\boldsymbol{\omega}$ *is the angle between the velocity* $\boldsymbol{u}$ *and* $\boldsymbol{R_o}$ *measured in* $\boldsymbol{s^o}$.

It should be observed that , according to this formulation , the force acting on a particle is not necessary equal to the force acting on the other in the same frame of reference , but *the force acting on a particle as measured in its rest frame is equal to the force acting on the other particle in its rest frame . In the cases of slow movement this statement reduces to classical Newton's third law .*

5.2.The Principle of Correspondence

In the cases of low movement $\boldsymbol{v} \approx \boldsymbol{u} \approx \boldsymbol{0}$

And the formulation reduces to *usual* Coulomb's law**.**

5.3.Covariance Under Lorentz transformation

In the frame $\boldsymbol{S^o}$ the formulation is reduced to the covariant form :

$$\boxed{\boldsymbol{f^o = k\,\frac{q^o q^1}{{R_o}^2}\sqrt{\frac{c^2 - v^2(\sin\theta)^2}{c^2 - v^2}}}}$$

. The Lorentz transformation of the magnitude of force is :

$$f = f^o\sqrt{\frac{c^2 - u^2\,(\sin\omega)^2}{c^2}}$$

Thus , the formulation is a combination of two covariant forms which gives a covariant form .

5.4.Compatibility with the Principle of Symmetry

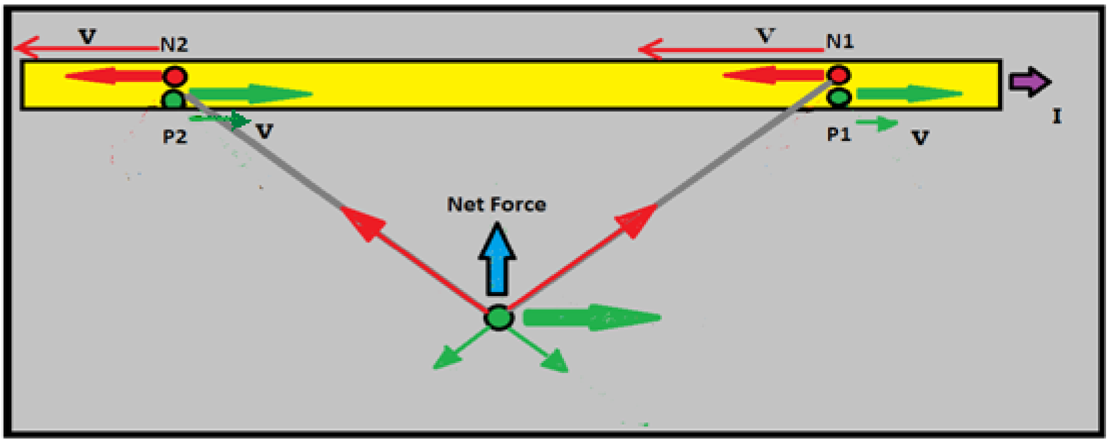
If we reconsider the result of the thought experiment mentioned above using this formulation , it will be consistent with the symmetry requirements . Because , the force experienced by the particle (A) in its rest frame is : $\boldsymbol{(\ F\sqrt{1-\frac{v^2}{c^2}}\ )}$ and the force experienced by the other particle (B) in the same frame is :($\boldsymbol{F}$), now if we apply this formulation in the rest frame of (B) , the force acting on (A) is : ($\boldsymbol{F}$) and the force acting on (B) is : $\boldsymbol{(\ F\sqrt{1-\frac{v^2}{c^2}}\ )}$. This result is consistent with the principle of symmetry and with Lorentz transformation . This consistency is also true in the case of relative motion of charges in the direction of the distance between them.

5.5.Explanation of Magnetism Based on the Formulation

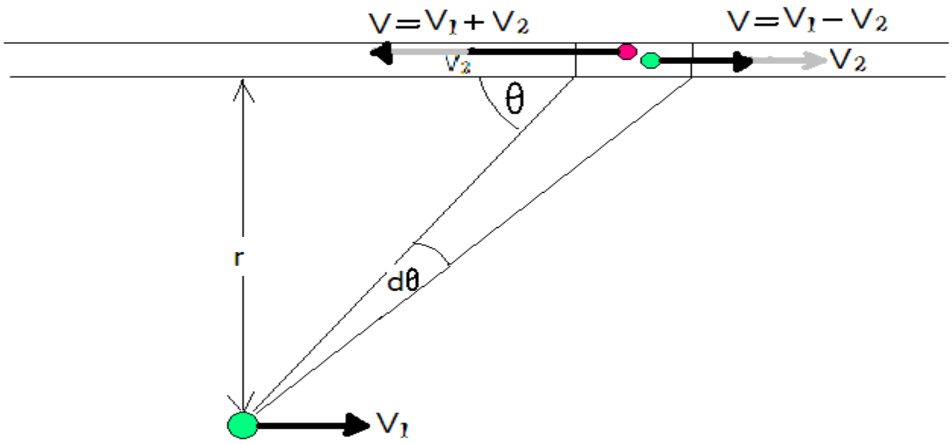
5.5.1.The Magnetic Force on a Test-charge Moving along-side with the Wire

Here as shown in figure (2) the velocity of the test-charge in the direction shown on the figure will cause the relative velocity (V)of the negative charges (N1) and (N2) to increase and the relative velocity of the positive charges (P1) and (P2) to decrease and because of the dependence of the force of interaction on this velocity as given by the formulation , the force from the negative charge will be greater than the force from the positive charge and so a net attraction is experienced by the positive test charge.

Fig (2): **the motion of the test charge along -side with the wire create a difference between the relative velocity of the positive charge carriers and the negative charge carriers and so a net force is obtained.**



The magnetic force of an infinitely long straight wire for non-relativistic speeds can be obtained by applying the formulation:



The force acting on the test charge is :

$$dF = k\,\frac{q\rho A}{r^2}\,2\sin\theta\,d\theta\left(\sqrt{\frac{c^2-(v_1+v_2)^2(\sin\theta)^2}{c^2-(v_1+v_2)^2}} - \sqrt{\frac{c^2-(v_1-v_2)^2(\sin\theta)^2}{c^2-(v_1-v_2)^2}}\right)$$

Where $\boldsymbol{\rho}$ is the charge density , $\boldsymbol{A}$ is the area of the section of the wire.

$$\boldsymbol{F} = \int_{0}^{\pi} \boldsymbol{dF}$$

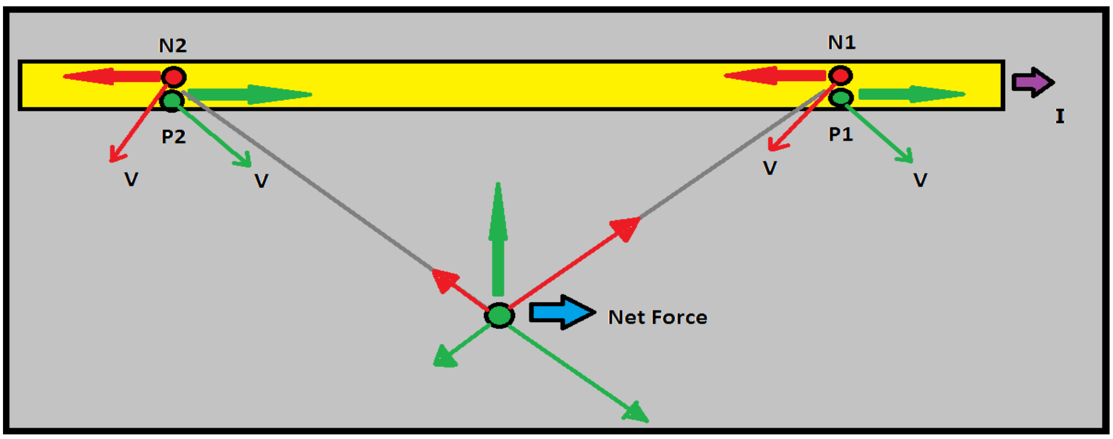
$$\boldsymbol{F} = \frac{\mu_0\, q\, v_1\, I}{2\, \pi\, r}$$

Where μ_0 is the permeability.

5.5.2.The Magnetic Force on a Test-charge Moving Perpendicular to the Current

As shown in the figure (1) the velocity of the test-charge perpendicular to the current , in the direction shown in the figure , will cause the angle between the relative velocity (v) and the direction of the distance between the two charges to decrease for the negative charge (N1) and the Positive charge (P2) and to increase for the positive charge (P1) and the negative charge (N2) and because of the dependence of the force on this angle as given by the formulation , a net force will act on the test-charge. The total force can be obtained by integration.

Fig (1): **the motion of the test charge perpendicular to the wire cause the angle between relative velocity of the positive charge carriers on the left and the negative charge carriers on the right and the distance of interaction to decrease and to increase for the positive charge carriers on the right and the negative charge carriers on the left and so a net force is obtained.**



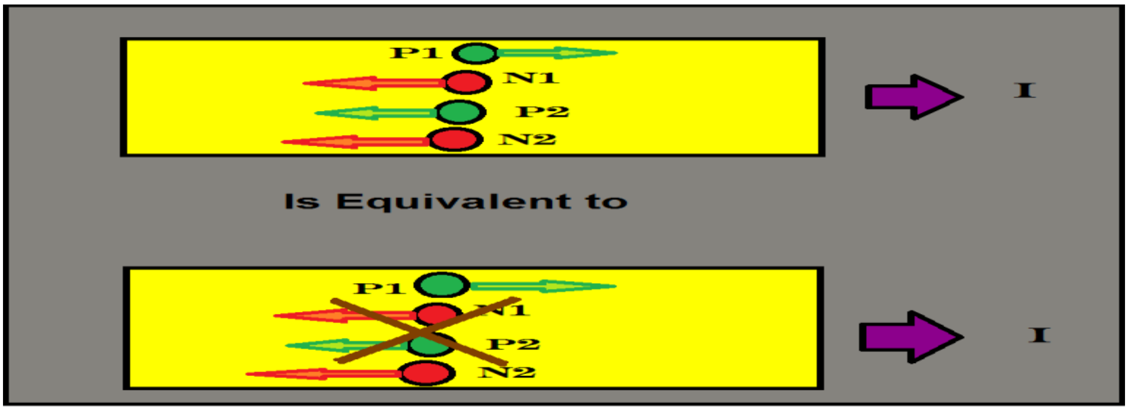
Using this formulation the magnetic force of an infinitely long straight wire acting on a test charge moving perpendicular to the wire for non-relativistic speeds can be shown to be:

$$\boldsymbol{F} = \frac{\mu_0 \, \boldsymbol{q} \, \boldsymbol{v} \, \boldsymbol{I}}{3 \, \boldsymbol{\pi} \, \boldsymbol{r}}$$

Carrying through the calculation using the formulation and integration one obtains the electromagnetic relation in each case .

This can also be applied in the practical case in which the charge carriers are the free electrons and the positively charged ions do not move from their position in the lattice but they are constantly vibrating . The electric current makes the positive metal ions vibrate more vigorously . If we apply the principles of statistics we can see that at any moment half of these ions move in the direction of the movement of the free electrons and the second half move in the opposite direction . As simplified in fig.(3) , the effect of the positive charges (P2) moving in the direction of the movement of the free electrons will cancel the effect of some of the free electrons (N1) and the result of this is that the net effect at any moment is equivalent to the effect of a conductor with negative (N2) and positive (P1) carriers . Because , the charged particles are distributed and moving in such a way that the applied electric field inside the wire is canceled , the instantaneous opposite currents of the positive and negative charge carriers are equal.

Fig (3) **: the effect of the positive charges (P2) instantaneously moving in the direction of the movement of the free electrons will cancel the effect of some of the free electrons (N1) .and the result of this is that the net effect at any moment is equivalent to the effect of a conductor with negative (N2) and positive (P1) carriers.**



5.5.3.Electromagnetic Radiation

According to this formulation and because of the dependence of the electric force on the velocity of the charged particle , accelerating a charged particle causes the electric field around the charge to change . If we apply the formulation to calculate the rate of change of energy of the electric field , we get good agreement with Larmor formula.

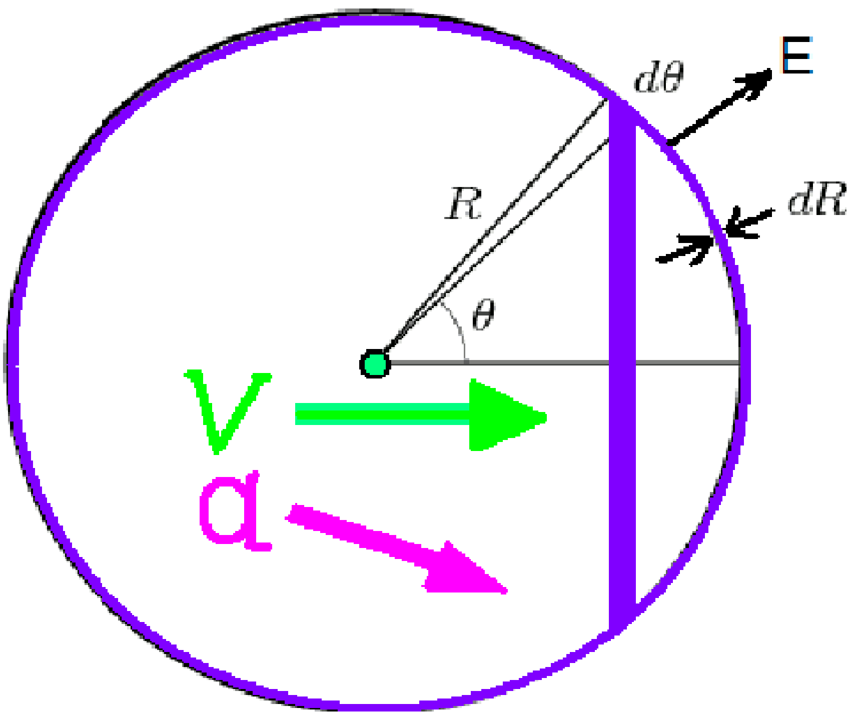
Let us consider the rate of change of energy of the electric field around a positive charge (**q**) moving with velocity (**V**) and acceleration (a) using the relation ;

$$u = \frac{1}{2}\varepsilon|\mathbf{E}|^2,$$

Where (***u***) is the energy density, (***ε***) is the permittivity of the medium and (**E**) is the electric field.

The energy density of the field at a distance (R) is :

$$U = \frac{q^2}{32\,\pi^2\,\varepsilon\,R^4}\;\frac{c^2 - v^2(\sin\theta)^2}{c^2 - v^2}$$



The energy in the element of volume is ;

$$\frac{q^2}{16\,\pi\,\varepsilon\,R^2}\;\frac{c^2 - v^2(\sin\theta)^2}{c^2 - v^2}\sin\theta\;d\theta\,dR$$

Now , let us calculate the total energy in the frame in which $(v = dv)$ which is given by integration for (θ) from zero to (π) and integration for (R) from $(c\ dt)$ to (∞) .this leads to expression :

$$\frac{q^2}{12\,\pi\,\varepsilon\,c\,dt}\;\frac{v^2 - (\frac{3}{2})c^2}{c^2 - v^2}$$

The rate of change in total energy in the frame in which $(v = dv)$ is obtained by differentiating with respect to time :

$$\frac{q^2}{6\,\pi\,\varepsilon\,c\,dt}\;\frac{c^2 dv\, a}{(c^2 - v^2)^2}$$

This leads to the following expression for the rate of change of the total energy:

$$\frac{q^2}{6\,\pi\,\varepsilon\,c}\;\frac{c^2 a^2}{(c^2 - (dv)^2)^2}$$

This agrees with Larmor formula :

$$\text{Power} = \frac{q^2 a^2}{6\,\pi\,\varepsilon\,c^3}$$

5.5.4.Electromagnetic Shielding

In this treatment based on Covariant Coulomb's law, the magnetic force on a charge moving near a wire carrying electric current is basically an electric force. Whereas electric forces can be shielded by a sheet of thin metal between wire and moving test charge, this is not possible for the magnetic force (v x B). This fact can be explained as follows :

Shielding is caused by the electric field which produces forces on the charge carriers in the metal sheet and causes displacement of charges that cancels the applied field inside the sheet and reduces the field outside.

But when the metal sheet is static with respect to the wire , the sheet is not affected by electric field according to the formulation so there is no shielding.

5.5.5.The Disappearing of Mansuripur's Paradox[2]

In a thought experiment of a magnetic dipole (a small neutral loop of current)fixed at a distance from a point charge , in the rest frame of the charge and the dipole , the dipole experience neither a force nor a torque from the point charge . However, for an observer who watches the charge and the dipole move with constant velocity , according to Lorentz transformation , a net torque acts on the pair of the dipole.

The appearance of this torque in one frame in the absence of a corresponding torque in another frame is known as Mansuripur's Paradox.

But when we consider this experiment using the Covariant Formulation , the paradox does not exist because as shown in figure (1) and figure (2) , the relative velocity between the charge and the neutral wire that carrying electric current is the necessary condition for the existence of interaction (magnetic force) between the wire and charge.

6.Conclusion

One can look on the continual development of our picture about the relation between electricity and magnetism as a process of evolution from the first picture which looks on magnetism as being essentially different to , and independent from electricity and to the next steps which are summarized by Maxwellian picture in which electricity and magnetism find their place in one theory as mutual interacting and symmetrical properties of nature, and then the step made by relativity in which electricity and magnetism are combined in one form which appears in different mixtures if we change our frame of reference . The covariant formulation of Coulomb's law is another role for relativity in this process of development , because it enables us to make another step in the same direction , showing that : *Electricity and Magnetism are Two Names of One Thing* .

References

[1] See for example: *The Feynman Lectures in Physics* (vol. 2 ch. 13-6), see also : Edward M. Purcell (1965,85) *Electricity and Magnetism: Berkeley Physics Course* Volume 2, published by Mc Graw-Hill .

[2] See : *Trouble with the Lorentz Law of Force , Incompatibility with Special Relativity and Momentum Conservation*. Masud Mansuripur, *arxiv.org/pdf/1205.0096*