Nonexistence of Nontrivial Cycles in the $3n + 1$ Problem

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Abstract

The Collatz conjecture is a famous problem in number theory. Given an integer, if it's odd, multiply it by three and add one, or, if it's even, divide it by two. The Collatz conjecture states that any trajectory of iterates of this Collatz transformation on the positive integers will reach one in a finite number of steps. This problem explores the behavior of a complicated discrete dynamical system that has eluded solution for over seventy years.

This paper addresses half the Collatz conjecture by altering the Collatz transformation into a friendlier format, which tells us what to do with an odd integer given its congruence modulo eight. We then describe how to find the numbers whose first few iterates follow a given pattern, which leads us to a directed graph that every trajectory must eventually enter. This directed graph then shows us that, in a finite number of steps, every iterate of a trajectory must either converge to one or strictly increase thereafter, proving the nonexistence of nontrivial cycles.

1 Introduction

The $3n + 1$ problem was first conjectured by Lothar Collatz in 1937. Known by many names – including the Syracuse problem, Hasse's algorithm, and, of course, the Collatz conjecture – this problem has drawn much attention because it is very easy to state but has been very difficult to prove. Since this problem is so known, a thorough background is not provided.

Definition 1.1. The *Collatz transformation* $T_c : \mathbb{Z}^+ \to \mathbb{Z}^+$ is

$$T_c(n) = \begin{cases} 3n + 1, & n \equiv 1 \pmod 2 \\ n/2, & n \equiv 0 \pmod 2 \end{cases}$$

Conjecture 1.2 (Collatz). *Let $a_0 \in \mathbb{Z}^+$, and, for all $i \geq 1$, let $a_i = T_c(a_{i-1})$. Then there is a $k \in \mathbb{Z}^+$ such that $a_k = 1$.*

2 Overview

Definition 3.1 and proposition 3.2 show that we can make a slight alteration to the Collatz transformation so that we map from odd integers to odd integers, the function has three pieces, and each piece has a fixed, closed-form transformation (i.e., we no longer have to divide by an unknown number 2^k, as in the odd-only Collatz transformation).

Definition 3.3 introduces notation to represent a finite start of a trajectory, called a *branching* of T. This allows us to consider, for example, all numbers that go "up, down a little, up, down a lot, up, down a little" using the algorithm presented in proposition 3.4.

Lemma 3.5 tells us how we can find the numbers that satisfy a particular branching, if there is such a number. In particular, for a given branching, it shows us that there's only one path we can follow to extend that branching if we don't want to raise the minimum number satisfying our extension. Since every number is bounded (by itself), every infinite trajectory must eventually follow the prescribed path.

Unfortunately, the forced congruences do not necessarily lead to the same congruences modulo 8 of the numbers a and b from the algorithm in proposition 3.4 every time. To overcome this pitfall, lemma 3.7 describes how we can use the quotients of a and b divided by 8 to determine which pair of congruences will come next. Lemma 3.7 can clearly be extended by induction to apply to nested quotients of quotients. For instance, $39 = 8 \cdot 5 - 1 = 8(4 \cdot 2 - 3) - 1 = 8(4(4 \cdot 1 - 2) - 3) - 1 = 8(4(4(4 \cdot 1 - 3) - 2) - 3) - 1 = \cdots$. Such a nesting will always reach a point where each inner term is $4 \cdot 1 - 3 = 1$ when it represents a finite number. Therefore, we find that, even though we have many choices of paths, there will eventually be a point where only one path becomes possible, which is presented in theorem 3.8. This path can be described by a disconnected directed graph, where each connected component settles into a cycle of length two. However, both elements in each cycle correspond to the same congruence modulo 8, which means that every trajectory eventually settles to a recurrence that is either increasing or decreasing to one.

3 Proof

Definition 3.1. For n odd, let

$$T(n) = \begin{cases} (3n+1)/2, & n \equiv 3 \pmod 4 \\ (3n+1)/4, & n \equiv 1 \pmod 8 \\ (n-1)/4, & n \equiv 5 \pmod 8 \end{cases}$$

and let $x_{i+1} = T(x_i)$ for a given $x_0 \in \mathbb{Z}^+$.

Proposition 3.2. *The Collatz conjecture holds if there exists a $k \in \mathbb{Z}^+$ for every $x_0 \in \mathbb{Z}^+$ such that $x_k = 1$.*

Proof. We show that $x_k = T^k(x_0) = 1$ implies there exists a k_c such that $a_{k_c} = T_c^{k_c}(x_0) = 1$ by induction on k. For $k = 1$, there are two cases. If $x_0 \equiv 1 \pmod 8$, then $x_0 = 1$, and $T(x_0) = 1 = T_c^3(x_0)$, so that $k_c = 3$. If $x_0 \equiv 5 \pmod 8$, then $x_0 = 5$, and $T(x_0) = 1 = T_c^5(x_0)$, so that $k_c = 5$. Suppose $T^k(x) = 1$ implies $T_c^{k'_c}(x) = 1$ for all $x \in \mathbb{Z}^+$. Then, $T^{k+1}(x_0) = 1 = T^k(T(x_0))$ implies $T_c^{k'_c}(T(x_0)) = 1$, by hypothesis. Now, there are three cases:

- If $x_0 \equiv 1 \pmod 8$, then $T(x_0) = T_c^3(x_0)$, so that $k_c = k'_c + 3$.
- If $x_0 \equiv 3 \pmod 4$, then $T(x_0) = T_c^2(x_0)$, so that $k_c = k'_c + 2$.
- If $x_0 \equiv 5 \pmod 8$, then $T_c(T(x_0)) = T_c^3(x_0)$, since $(3(4x+1)+1)/4 = 3x + 1$, so that $k_c = k'_c + 2$.

Thus, the proposition holds by induction and the fact that the Collatz conjecture holds for all positive integers if and only if it holds for odd positive integers. □

Definition 3.3. Let $S = s_0, s_1, \ldots, s_{k-1}$ be a sequence in $\{1, 3, 5\}$ of length k. S is called a *branching* of T. If $x_0 \in \mathbb{Z}^+$, and $x_i \equiv s_i \pmod 8$ or $x_i \equiv s_i \pmod 4$, if $s_i = 3$, then x_0 is said to *satisfy* S.

Proposition 3.4. *Let $S = s_0, s_1, \ldots, s_{k-1}$ be a branching of T. If there is an $x_0 \in \mathbb{Z}^+$ satisfying S, then we can find all numbers satisfying S with the following algorithm:*

1. *Assign the numbers a' and b' according to s_0:*
   (a) *$s_0 = 1$ implies $a' \leftarrow 8$ and $b' \leftarrow 7$.*
   (b) *$s_0 = 3$ implies $a' \leftarrow 4$ and $b' \leftarrow 1$.*
   (c) *$s_0 = 5$ implies $a' \leftarrow 8$ and $b' \leftarrow 3$.*
2. *Assign the following:*
   (a) *$i \leftarrow 0$.*
   (b) *$c \leftarrow 1$.*
   (c) *$d \leftarrow 0$.*
   (d) *$a \leftarrow 1$.*
   (e) *$b \leftarrow 0$.*
3. *Go to step 5.*
4. *Assign $a', b' \in \mathbb{Z}$, $a' > 0, b' \geq 0$, the smallest integers satisfying a congruence according to s_i:*
   (a) *$s_i = 1$ implies $a(a'x - b') - b \equiv 1 \pmod 8$*
   (b) *$s_i = 3$ implies $a(a'x - b') - b \equiv 3 \pmod 4$*
   (c) *$s_i = 5$ implies $a(a'x - b') - b \equiv 5 \pmod 8$.*

5. *Assign* $c, d \in \mathbb{Z}^+$ *from* c, d, a', b'*:*
   (a) $d \leftarrow (d + cb')$.
   (b) $c \leftarrow ca'$.
6. *Assign* b, a *from* b', a' *according to* s_i*:*
   (a) $s_i = 1$ *implies* $b \leftarrow (3(b + ab') - 1)/4$ *and then* $a \leftarrow 3aa'/4$.
   (b) $s_i = 3$ *implies* $b \leftarrow (3(b + ab') - 1)/2$ *and then* $a \leftarrow 3aa'/2$.
   (c) $s_i = 5$ *implies* $b \leftarrow (b + ab' + 1)/4$ *and then* $a \leftarrow aa'/4$.
7. *Assign* $i \leftarrow i + 1$.
8. *If* $i < k$*, go to step* 4*. Otherwise, done.*

When the algorithm terminates, for all $x \in \mathbb{Z}^+$*,* $x_0 = cx - d$ *satisfies* S *and* $x_{k+1} = ax - b$*. In particular, the smallest positive integer satisfying* S *is* $c - d$*.*

Proof. By computation of the three possibilities of $S = s_0$, the theorem holds for any branching of length 1. For $S_{k+1} = s_0, s_1, \ldots, s_k$ a branching of length $k + 1$, apply the algorithm to find $a, b, c, d \in \mathbb{Z}^+$ such that $cx - d$ satisfies the branching $S_k = s_0, s_1, \ldots, s_{k-1}$ for all $x \in \mathbb{Z}^+$. Clearly, if x_0 satisfies S_{k+1}, then x_0 satisfies S_k, so any $x_0 = cx - d$, for some $x \in \mathbb{Z}^+$, satisfies S_{k+1} if and only if one of the following holds

1. $s_k = 1$ and $x_k \equiv 1 \pmod 8$
2. $s_k = 3$ and $x_k \equiv 3 \pmod 4$
3. $s_k = 5$ and $x_k \equiv 5 \pmod 8$

However, if $x_0 = cx - d$, then $x_k = ax - b$, since step 6 assigns a, b by computing $x_k = T(a(a'x - b') - b)$, where a, a', b, b' here are from the previous iteration. Thus, step 4, when $i = k$, finds a', b' such that the "new" x_k, the value $a(a'x - b') - b$, is exactly those numbers satisfying the required conditions. □

Lemma 3.5. *Let* $a, b, c, d, i \in \mathbb{Z}^+$ *be the results of an iteration where* $1 \le i < k$ *of the algorithm in proposition 3.4 applied to* $S = s_0, s_1, \ldots, s_{k-1}$*, and let* $\overline{a}, \overline{b}$ *be such that* $\overline{a} \equiv a \pmod 8$*,* $\overline{b} \equiv b \pmod 8$*, and* $0 \le \overline{a}, \overline{b} < 8$*. Then, the minimum* x_0 *satisfying the branching* $S_i = s_0, s_1, \ldots, s_{i-1}$ *is less than or equal to the minimum* x_0 *satisfying* $S_{i+1} = s_0, s_1, \ldots s_i$*, and these minima are equal if and only if* s_i *is equal to the value in the label of the column containing the bold entry in the row corresponding to* $(\overline{a}, \overline{b})$ *in the following table:*

$(\overline{a},\overline{b})$	$s_i=3$	$s_i=5$	$s_i=1$
$(2,1)$	$2x$	$4x-1$	$\mathbf{4x-3}$
$(2,3)$	$\mathbf{2x-1}$	$4x$	$4x-2$
$(2,5)$	$2x$	$\mathbf{4x-3}$	$4x-1$
$(2,7)$	$\mathbf{2x-1}$	$4x-2$	$4x$
$(4,1)$	$\mathbf{x}$		
$(4,3)$		$2x$	$\mathbf{2x-1}$
$(4,5)$	$\mathbf{x}$		
$(4,7)$		$\mathbf{2x-1}$	$2x$
$(6,1)$	$2x$	$\mathbf{4x-3}$	$4x-1$
$(6,3)$	$\mathbf{2x-1}$	$4x$	$4x-2$
$(6,5)$	$2x$	$4x-1$	$\mathbf{4x-3}$
$(6,7)$	$\mathbf{2x-1}$	$4x-2$	$4x$

Proof. That the minimum x_0 satisfying the branching S_i is less than or equal to the minimum x_0 satisfying S_{i+1} is clear. If $a,b,c,d,i \in \mathbb{Z}^+$ are the result of an iteration with $1 \leq i < k$, then, by the algorithm in proposition 3.4, the values of the variables a' and b' in step 4 of the subsequent iteration are given in the table by $a'x-b'$ being the entry in row $(\overline{a},\overline{b})$ and column s_i, and it follows that the minimum x_0 satisfying S_{i+1} is unchanged from the minimum x_0 satisfying S_i if and only if $1=a'x-b'$ for some $x \in \mathbb{Z}^+$, which is only true for the bold entries. □

Corollary 3.6. *If we write $a=8k_1-(8-\overline{a})$ and $b=8k_2-(8-\overline{b})$, then the transformations $T(ay-b)$, where y is the bold entry in the corresponding row from lemma 3.5, are given by the following table:*

$(\overline{a},\overline{b})$	$T(ay-b)$
$(2,1)$	$[8(3k_1-2)-2]x-[6(3k_1+k_2)-19]$
$(2,3)$	$[8(3k_1-2)-2]x-[12(k_1+k_2)-17]$
$(2,5)$	$(8k_1-6)x-[4(3k_1+k_2)-5]$
$(2,7)$	$[8(3k_1-2)-2]x-[12(k_1+k_2)-11]$
$(4,1)$	$(12k_1-6)x-(12k_2-11)$
$(4,3)$	$(12k_1-6)x-[6(k_1+k_2)-7]$
$(4,5)$	$(12k_1-6)x-(12k_2-5)$
$(4,7)$	$(4k_1-2)x-[2(k_1+k_2)-1]$
$(6,1)$	$(8k_1-2)x-[2(3k_1+k_2)-3]$
$(6,3)$	$[8(3k_1)-6]x-[12(k_1+k_2)-11]$
$(6,5)$	$[8(3k_1)-6]x-[6(3k_1+k_2)-7]$
$(6,7)$	$[8(3k_1)-6]x-[12(k_1+k_2)-5]$

The corresponding $(\overline{a},\overline{b})$ of a subsequent iteration of the algorithm in 3.4 is then determined by the congruences of k_1 and k_2 modulo 4, as shown in the following table:

$(\overline{k_1}, \overline{k_2})$	(2, 1)	(2, 3)	(2, 5)	(2, 7)	(4, 1)	(4, 3)	(4, 5)	(4, 7)	(6, 1)	(6, 3)	(6, 5)	(6, 7)
(0, 0)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 1)	(2, 3)	(6, 7)	(6, 5)	(2, 5)	(2, 1)	(2, 3)
(0, 1)	(6, 3)	(6, 3)	(2, 7)	(6, 1)	(2, 1)	(2, 7)	(2, 7)	(6, 1)	(6, 7)	(2, 1)	(2, 7)	(2, 7)
(0, 2)	(6, 1)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 5)	(2, 3)	(6, 3)	(6, 1)	(2, 5)	(2, 5)	(2, 3)
(0, 3)	(6, 7)	(6, 3)	(2, 7)	(6, 1)	(2, 1)	(2, 3)	(2, 7)	(6, 5)	(6, 3)	(2, 1)	(2, 3)	(2, 7)
(1, 0)	(6, 7)	(6, 3)	(2, 7)	(6, 1)	(6, 5)	(6, 7)	(6, 3)	(2, 1)	(6, 3)	(2, 1)	(2, 3)	(2, 7)
(1, 1)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(6, 1)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 1)	(2, 3)
(1, 2)	(6, 3)	(6, 3)	(2, 7)	(6, 1)	(6, 5)	(6, 3)	(6, 3)	(2, 5)	(6, 7)	(2, 1)	(2, 7)	(2, 7)
(1, 3)	(6, 1)	(6, 7)	(2, 3)	(6, 5)	(6, 1)	(6, 1)	(6, 7)	(2, 7)	(6, 1)	(2, 5)	(2, 5)	(2, 3)
(2, 0)	(6, 1)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 5)	(2, 3)	(6, 3)	(6, 1)	(2, 5)	(2, 5)	(2, 3)
(2, 1)	(6, 7)	(6, 3)	(2, 7)	(6, 1)	(2, 1)	(2, 3)	(2, 7)	(6, 5)	(6, 3)	(2, 1)	(2, 3)	(2, 7)
(2, 2)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 1)	(2, 3)	(6, 7)	(6, 5)	(2, 5)	(2, 1)	(2, 3)
(2, 3)	(6, 3)	(6, 3)	(2, 7)	(6, 1)	(2, 1)	(2, 7)	(2, 7)	(6, 1)	(6, 7)	(2, 1)	(2, 7)	(2, 7)
(3, 0)	(6, 3)	(6, 3)	(2, 7)	(6, 1)	(6, 5)	(6, 3)	(6, 3)	(2, 5)	(6, 7)	(2, 1)	(2, 7)	(2, 7)
(3, 1)	(6, 1)	(6, 7)	(2, 3)	(6, 5)	(6, 1)	(6, 1)	(6, 7)	(2, 7)	(6, 1)	(2, 5)	(2, 5)	(2, 3)
(3, 2)	(6, 7)	(6, 3)	(2, 7)	(6, 1)	(6, 5)	(6, 7)	(6, 3)	(2, 1)	(6, 3)	(2, 1)	(2, 3)	(2, 7)
(3, 3)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(6, 1)	(6, 5)	(6, 7)	(2, 3)	(6, 5)	(2, 5)	(2, 1)	(2, 3)

Proof. For $(\overline{a}, \overline{b}) = (2, 1)$, we have $y = 4x - 3$, so that

$$\begin{aligned} T(ay - b) &= T((8k_1 - 6)(4x - 3) - (8k_2 - 7)) \\ &= (3(4(8k_1 - 6)x - (3(8k_1 - 6) + 8k_2 - 7)) + 1))/4 \\ &= (8(3k_1 - 2) - 2)x - (6(3k_1 + k_2) - 19) \end{aligned}$$

We complete the first table by continuing in this fashion. For the second table, since we are calculating congruence modulo 8, and every coefficient of k_1 or k_2 is even, it suffices to know congruences modulo 4. The completion of the table is straightforward by substituting $k_1 = \overline{k_1}$, $k_2 = \overline{k_2}$ and computing congruence modulo 8. □

Lemma 3.7. *For a given $(\overline{a}, \overline{b})$, where $a = 8k_1 - (8 - \overline{a})$, $b = 8k_2 - (8 - \overline{b})$, let $k_1 = \alpha k_1' - \beta$ and $k_2 = \alpha k_2' - \beta'$, where $\alpha = 2$ if every coefficient of k_1, k_2 in the corresponding entry for $(\overline{a}, \overline{b})$ in the first table in corollary 3.6 is divisible by 4 and $\alpha = 4$ otherwise, and β, β' are integers in $[1, \alpha)$. Then the congruences of k_1' and k_2' modulo 4 determine the subsequent path, according to the second table in corollary 3.6, in the same fashion as k_1 and k_2, i.e., for the paths such that it suffices to know the congruences of k_1, k_2 modulo 2, it also suffices to know the congruences of k_1', k_2' modulo 2.*

Proof. For $(\overline{a}, \overline{b})$, we have $\alpha = 4$. Let us chose $(\overline{k_1}, \overline{k_2}) = (1, 3)$, so that we are selecting the path $(6, 1)$ from the second table in corollary 3.6, and $\beta = 3$, $\beta' = 1$. Then, we have

$$\begin{aligned} (8(3k_1 - 2) - 2)x - (6(3k_1 + k_2) - 19) &= \\ (8(3k_1 - 2) - 2)x - (6(3k_1 + k_2) - 19) &= \\ (8(3(4k_1' - 3) - 2) - 2)x - (6(3(4k_1' - 3) + (4k_2' - 1)) - 19) &= \\ (8(12k_1' - 9) - 2) - 2)x - (8(9k_1' + 3k_2' - 9) - 7) & \end{aligned}$$

Thus, we are forced into the path $(6, 1)$, and the congruences modulo 4 (or 2) of the "new" k_1 and k_2, which are respectively $12k_1' - 9$ and $9k_1' + 3k_2' - 9$, are determined by the congruences of k_1' and k_2' modulo 4 (or 2). This follows from the appropriate selection of α to produce odd coefficients for k_1' and k_2'. □

Theorem 3.8. *For any $x_0 \in \mathbb{Z}^+$, there is a $k \in \mathbb{Z}^+$ such that either $x_n \equiv 1 \pmod 8$ for all $n \geq k$ or $x_n \equiv 3 \pmod 8$ for all $n \geq k$.*

Proof. If S_i is the branching of T of length $i \in \mathbb{Z}^+$ that a given $x_0 \in \mathbb{Z}^+$ satisfies, then the minimum positive integer satisfying S_i must be less than or equal to x_0 for all i, so that there is a $k \in \mathbb{Z}^+$ such that each iteration from s_{k+n-2} to s_{k+n-1}, $n \in \mathbb{Z}^+$, follows according to the bold entries in lemma 3.5.

Now, let a, b result from the first iteration such that all subsequent iterations are from the bold entries in lemma 3.5. Let $k_1^{(j)}$, $k_2^{(j)}$ be sequences of positive integers such that $a = 8k_1^{(0)} - (8 - \overline{a})$, $b = 8k_2^{(0)} - (8 - \overline{b})$, $k_1^{(j)} = \alpha_j k_1^{(j-1)} - \beta_j$, and $k_2^{(j)} = \alpha_j k_2^{(j-1)} - \beta_j'$, where $\alpha_j, \beta_j, \beta_j'$ are positive integers, $\alpha_j \in \{2, 4\}$ and $\beta_j, \beta_j' \in [1, \alpha_j)$. By lemma 3.7 and induction, these sequences describe subsequent iterations. However, since a, b are finite, there must be a $J \in \mathbb{N}$ such that $k_1^{(j)} = k_2^{(j)} = 1$ for all $j \geq J$. In other words, we eventually become restricted to the paths in the row where $(\overline{k_1}, \overline{k_2}) = (1, 1)$ in the second table of corollary 3.6, which can be expressed in the following form:

$(\overline{a}, \overline{b})$	$s_i = 3$	$s_i = 5$	$s_i = 1$
$(2, 1)$			$(6, 5)$
$(2, 3)$	$(6, 7)$		
$(2, 5)$		$(2, 3)$	
$(2, 7)$	$(6, 5)$		
$(4, 1)$	$(6, 1)$		
$(4, 3)$			$(6, 5)$
$(4, 5)$	$(6, 7)$		
$(4, 7)$		$(2, 3)$	
$(6, 1)$		$(6, 5)$	
$(6, 3)$	$(2, 5)$		
$(6, 5)$			$(2, 1)$
$(6, 7)$	$(2, 3)$		

Thus, every $(\overline{a}, \overline{b})$ eventually finds itself either in the loop $(6, 5) \to (2, 1) \to (6, 5) \to \cdots$ or in the loop $(6, 7) \to (2, 3) \to (6, 7) \to \cdots$. Since $(6, 5)$ and $(2, 1)$ both correspond to $a(a'x - b') - b \equiv 1 \pmod 8$ and since $(6, 7)$ and $(2, 3)$ both correspond to $a(a'x - b') - b \equiv 3 \pmod 8$, it follows that every S_i that a fixed x_0 satisfies, for i sufficiently large ($i > k$), has either $s_k, s_{k+1}, \ldots, s_{i-1} = 1$ or $s_k, s_{k+1}, \ldots, s_{i-1} = 3$. □

Corollary 3.9. *There are no non-trivial cycles of iterates of the Collatz transformation over the positive integers.*

Proof. Since $(3x + 1)/4 < x$ and $(3x + 1)/2 > x$, for all $x \in \mathbb{Z}^+$, $x > 1$, every trajectory either converges to 1 or diverges. □

4 Future Work

The trick that allowed this result may lead to a proof of convergence to 1 and also can be applied to other specific instances of a more general Collatz-type problem. For instance, in the $5n+1$ problem, we have $(5(16x+3)+1)/16 = 5x+1$, which allows us to simplify this transformation just like we did in definition 3.1, yielding the following function:

$$T_{5,1}(n) = \begin{cases} (5n+1)/2, & n \equiv 1 \pmod 4 \\ (5n+1)/4, & n \equiv 7 \pmod 8 \\ (5n+1)/8, & n \equiv 11 \pmod{16} \\ (5n+1)/16, & n \equiv 3 \pmod{32} \\ (n-3)/16, & n \equiv 19 \pmod{32} \end{cases}$$

This function definition may make a description of the trajectories of the $5n+1$ problem possible.