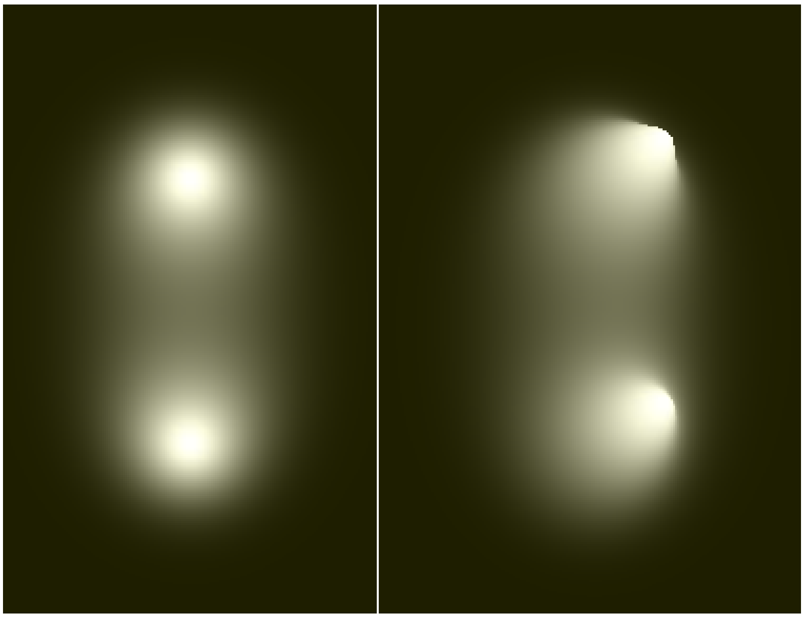

Figure 1: Left panel is the planar graph of double-breast structure, and right panel is its corresponding 3-dimensional demonstration. The graph and demonstration are based on Galaxy Anatomy graphic software.

Heaven Breasts and Heaven Calculus

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Abstract Since the birth of mankind, human beings have been looking for the origin of life. The fact that human history is the history of warfare and cannibalism proves that humans have not identified their origin. Humanity is still in the dark phase of lower animals. Humans can see the phenomenon of life only on Earth, and humans' vision does not exceed the one of lower animals. However, it is a fact that human beings have inherited the most advanced gene of life. Humans should be able to answer the following questions: Is the Universe hierarchical? What is Heaven? Is Heaven the origin of life? Is Heaven a higher order of life? For more than a decade, I have done an in-depth study on barred galaxy structure. Today (September 17, 2012) I suddenly discovered that the characteristic structure of barred spiral galaxies resembles the breasts of human female essentially. If the rational structure conjecture presented in the article is proved then Sun must be a mirror of the universe, and mankind is exactly the image on earth of the Heaven.

keywords: Double-humped Structure, Human Breasts, Galaxy Structure, Calculus

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1 Heaven Breasts

For the last seven years, I have propagated the result of my study on galaxy structure. Spiral galaxies are flat-shaped. As I said before, the shorter-wavelength images of spiral galaxies show the phenomenon of living things such as flying clouds of dust and gas, spectacular birth of stars. Longer-wavelength images of spiral galaxies demonstrate the smooth structure of overall stellar distribution. Galaxy structure is natural one, and a natural structure is an uneven distribution of matter. Galaxy structure is the pattern of stellar density varying with spatial position. How to describe the spatial change of a matter distribution? Fortunately, Newton and Leibniz discovered Calculus over three hundred years ago. Calculus is essentially the mathematical description of matter change. Because Newton and Leibniz's calculus is applicable to the description of stellar density change in galaxy structure, I call Newton and Leibniz's calculus the Heaven Calculus.

For more than seven years, I have tried hard to tell the public world that galaxy structure is harmonic and rational. My only assumption is that galaxy structure is constructed from a vascular network. That is, the stellar densities at both sides of any blood vessel are proportional along the vessel. This vascular network is composed of two clusters of parallel curves. The curves from the two clusters are mutually orthogonal.

The spiral galaxies which have no bars are called the ordinary spirals. The basic structure of ordinary spirals is an axisymmetric disk whose stellar density decreases exponentially outwards from the galaxy center. Exponential disks are rational structure. Their vascular network consists of all golden spirals. Coincidentally, astronomical observations show that the spiral arms of ordinary spiral galaxies are all golden spirals.

For more than seven years, I have tried hard to tell the public world that the bars of barred spiral galaxies are rational too. The bar of a barred galaxy is composed of two or three pairs of double-humped structures. Today (September 17, 2012), I suddenly realized that the double-humped structure resembles the breasts of human females very much. From now on, I call the double-humped structure the double-breast structure, or the Heaven Breasts. The vascular network of Heaven Breasts consists of all confocal ellipses and confocal hyperbolas. Here, I present two conjectures to the public world, and hope that the conjectures will be proved or disproved in some day.

Ellipse-Hyperbola Conjecture: Elliptical and hyperbolic curves are the basic properties of all natural things.

Rational Structure Conjecture: Heaven Breasts is the only non-axisymmetric rational structure. In other words, any non-axisymmetric rational structure is either a Heaven Breasts itself or a superimposed structure of a number of Heaven Breasts and a special axisymmetric rational structure.

If Rational Structure Conjecture is proved, then humans must admit that the Universe is hierarchical, galaxies are the real life, Heaven does exist, and Heaven is the life of superior level. Mankind must also recognize that the Milky Way galaxy is the mother of human beings both materially and spiritually.

The left panel of Figure 1 is the planar graph of double-breast structure, and the right panel is its corresponding 3-dimensional demonstration. The graph and demonstration are based on the Galaxy Anatomy graphic software [1] (http://www.galaxyanatomy.com/, currently people can purchase the software via Paypal). Note that Figure 1 is the direct

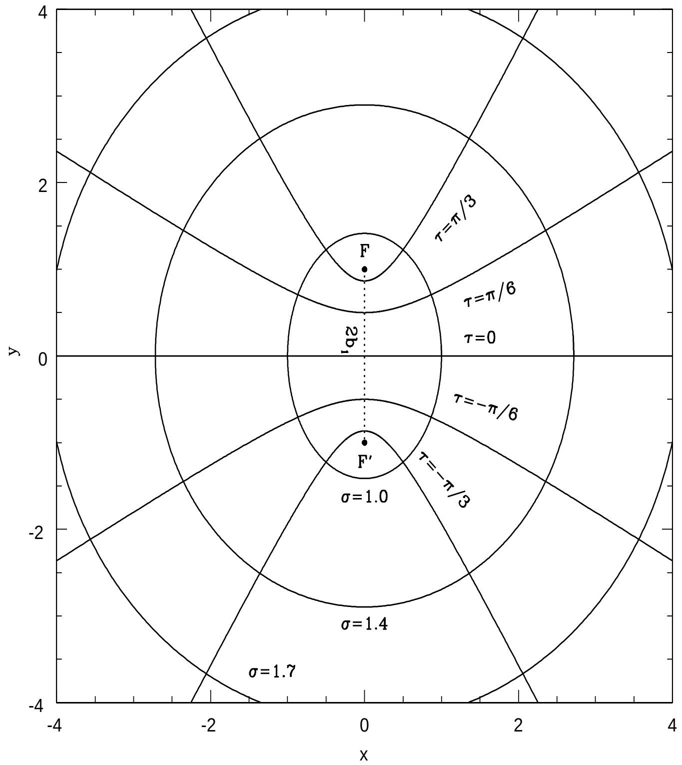

Figure 2: The orthogonal curves of confocal ellipses and hyperbolas. The distance between the two foci, F and F', is $2b_1$ which measures the distance between the two humps.

density demonstration of Heaven Breasts. However, the demonstration in the software is not directly the density. It is the squared stellar density. This is because squared density helps make a better display of galaxy structure. Figure 2 is the demonstration of the vascular network in Heaven Breasts. They are mutually orthogonal confocal-ellipses and hyperbolas. In the following Sections, I prove that the rational structure which corresponds to confocal ellipses and hyperbolas must be the Heaven Breasts. In other words, the stellar densities at both sides of any ellipse or any hyperbola are proportional along the curve.

2 Heaven Calculus

This Section is taken from the reference [2].

1. Logarithmic Density of Galaxy Stellar Structure. A longer-wavelength galaxy image is the distribution of stellar densities. Therefore, we use a mathematical function to describe a distribution of densities. Because spiral galaxies are planar, we use a function of two variables, x, y, to describe the stellar distribution of a face-on spiral galaxy:

$$\rho(x, y) \tag{1}$$

where x, y is the rectangular Cartesian coordinates on the spiral galaxy plane. The coordinate origin is the galaxy center. Therefore, $\rho(0, 0)$ is the stellar density at the galaxy center. We want to study the ratio of the density ρ_2 to the density ρ_1 at two positions 2 and 1 respectively: ρ_2/ρ_1. In fact, the logarithm of the ratio divided by the distance s between the two positions is approximately the directional derivative of the logarithmic density $f(x, y)$ $(= \ln \rho(x, y))$ along the direction of the two positions: $(\ln(\rho_2/\rho_1))/s \approx \partial f/\partial s$. There is no systematic mathematical theory on ratios. Therefore, we from now on, study the logarithmic function $f(x, y)$ instead of the density function $\rho(x, y)$:

$$f(x, y) = \ln \rho(x, y). \tag{2}$$

2. Description of a Net of Orthogonal Curves. The following equations

$$x = x(\lambda, \mu), \ y = y(\lambda, \mu) \tag{3}$$

tell us how to describe a net of curves by employing coordinate equations. Given two functions, $x(\lambda, \mu)$, $y(\lambda, \mu)$, you have the transformation between the curvilinear coordinates (λ, μ) and the rectangular Cartesian coordinates (x, y). It describe a net of curves. Letting the second parameter μ be a constant, you have a curve (called a row curve, i.e., the proportion row defined in the following). That is, the above formula is a curve with its parameter being λ. For the different values of the constant μ, you have a set of "parallel" rows. Similarly, you have a set of "parallel" columns of parameter μ. However, The row curves and the column curves are not necessarily orthogonal to each other. The following equation is the necessary and sufficient condition for the net of curves to be orthogonal:

$$\frac{\partial x}{\partial \lambda}\frac{\partial x}{\partial \mu} + \frac{\partial y}{\partial \lambda}\frac{\partial y}{\partial \mu} \equiv 0. \tag{4}$$

To study the rational structures described in the following, we need more knowledge of the description of row and column curves. The arc length of the row curve is s whose differential is $ds = \sqrt{x_\lambda'^2 + y_\lambda'^2} d\lambda = P d\lambda$ where P is the arc derivative of the row curve. The arc length of the column curve is t, and Q is its arc derivative:

$$P = s_\lambda' = \sqrt{x_\lambda'^2 + y_\lambda'^2}, \ Q = t_\mu' = \sqrt{x_\mu'^2 + y_\mu'^2}. \tag{5}$$

3. The Condition of Rational Structure. The formulas (3) and (4) are the general description of a net of orthogonal curves, and the formula (1) is the general description of a distribution of densities (a structure). In this paper we discuss on rational structure. A distribution of densities is called the rational structure if the density varies proportionally along some particular net of orthogonal curves. That is, you walk along a curve from the net and the ratio of the density on your left side to the immediate density on your right is constant along the curve (see the figures in the reference [3]). However, the constant ratio of this curve is generally different from the constant ratios of the other curves.

We have shown that a logarithmic ratio of two densities divided by the distance between the two positions is approximately the directional derivative of the logarithmic density along the direction of the distance. Therefore, we always study the logarithmic function $f(x, y)$. If we know the two partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ then the structure $f(x, y)$ is found. The partial derivatives of $f(x, y)$ are the directional derivatives along the straight directions of the rectangular Cartesian coordinate lines. However, we are interested in the net of orthogonal curves and what we look for is the directional derivatives along the tangent directions of the curvilinear rows and columns. These are denoted $u(\lambda,\mu)$ and $v(\lambda,\mu)$ respectively. The condition of rational structure is that u depends only on λ while v depends only on μ:

$$u = u(\lambda),\ v = v(\mu). \tag{6}$$

What a simple condition for the solution of rational structure.

Now we prove the condition. Assume you walk along a row curve. The logarithmic ratio of the density on your left side to the immediate density on your right side is approximately the directional derivative of $f(x,y)$ along the column direction. That is, the logarithmic ratio is approximately the directional derivative $v(\lambda,\mu)$. Because $v(\lambda,\mu)$ is constant along the row curve (rational), $v(\lambda,\mu)$ is independent of λ: $v = v(\mu)$. Similarly, we can prove that $u(\lambda,\mu) = u(\lambda)$.

4. The Equation of Rational Structure. It is known that, given an arbitrary function, we can have its two partial derivatives. However, given two functions, we may not find the third function whose partial derivatives are the given functions. For the given two functions to be some derivatives, a condition must be satisfied. The condition is the Green's theorem. In the case of derivatives along orthogonal curves, the Green's theorem is the following:

$$\frac{\partial}{\partial\mu}(u(\lambda,\mu)P) - \frac{\partial}{\partial\lambda}(v(\lambda,\mu)Q) = 0 \tag{7}$$

In the case of rational structure, directional derivatives are the functions of the single variables, λ and μ respectively. Therefore, the Green's theorem turns out to be the following which is called the rational structure equation:

$$u(\lambda)P'_\mu - v(\mu)Q'_\lambda = 0 \tag{8}$$

This equation determines rational structure. To find a rational structure, generally we are, first of all, given a net of orthogonal curves. Accordingly the arc derivatives of both the rows and columns, $P(\lambda,\mu)$ and $Q(\lambda,\mu)$, are known. Therefore, the remaining functions, $u(\lambda)$ and $v(\mu)$, are the only unknowns. Because the rational structure equation involves no derivative of the unknowns, the equation is not a differential equation at all. It is an algebraic equation and what we need to do is to add two factors (the two unknowns) to the derivatives of P and Q so that the rational structure equation holds: factorization. If the factorization fails then the net of orthogonal curves does not correspond to a rational structure. In fact, most orthogonal curves do not correspond to rational structure. That is why I proposed the Rational Structure Conjecture in Section 1: Heaven Breasts is the only non-axisymmetric rational structure. How simple is rational structure.

6. An Example: Double-humped Structure (i.e., Heaven Breasts). Now we study galactic bar model. A bar pattern is composed of two or three double-humped structures which are generally aligned with each other (spiral galaxy NGC 1365 is an exception). The double-humped structure is determined by the following orthogonal curves of confocal ellipses and hyperbolas:

$$\begin{cases} x = e^\sigma \cos\tau,\ y = \sqrt{e^{2\sigma} + b_1^2}\sin\tau, \\ -\infty < \sigma < +\infty,\ 0 \le \tau < 2\pi. \end{cases} \tag{9}$$

where $b_1(>0)$ is a constant. The orthogonal coordinate system is a generalization of the polar coordinate system. The coordinate lines are confocal ellipses and hyperbolas (Figure 2). The distance between the two foci is $2b_1$ which measures the distance between the two humps. The eccentric anomaly of the ellipses is τ. The inverse coordinate transformation of the formulas is easily found,

$$
\begin{aligned}
&p(x,y) = e^{\sigma} = \sqrt{\left(r^2 - b_1^2 + \sqrt{(r^2-b_1^2)^2 + 4b_1^2x^2}\right)/2}, \\
&\cos\tau = xe^{-\sigma} = x/p(x,y)
\end{aligned}
\tag{10}
$$

where $r^2 = x^2 + y^2$. We find the density of the double-humped structure,

$$
\begin{aligned}
&\rho(x,y) = b_0 \exp(f(x,y)), \\
&f(x,y) = (b_2/3)(p^2(x,y) + b_1^2x^2/p^2(x,y))^{3/2}
\end{aligned}
\tag{11}
$$

where b_0 is the dual handle density at the galaxy center. We need to choose $b_2 < 0$ so that $f < 0$ and $\rho \to 0$ when $r \to +\infty$. We can see that b_0 is proportional to the central strength and b_1 corresponds to the distance between the two humps while b_2 measures the density slope off the double humps. If we display the double-humped structure as a curved surface in 3-dimensional space then we can see that the surface resembles the breasts of human female essentially. Now I call double-humped structure the Heaven Breasts. You can display the structure with Galaxy Anatomy graphic software. If you do not have the software, you may use Matlab or Maple software to display the structure and see if it really resembles human female's breasts.

The Heaven Breasts (11) is expressed with the Cartesian coordinates x, y, not the parameters σ, τ. Further more, it is independent of the choice of coordinate scale. For example, let us replace the coordinates x, y with the coordinates xs, ys where s is an arbitrary scale. We should prove that new structure is identical to the old one

$$
\rho(x,y) \equiv \rho(xs, ys) \tag{12}
$$

if we choose some different values of the constants b_0, b_1, b_2. That is,

$$
\rho_{b_0,\,b_1,\,b_2}(x,y) \equiv \rho_{b_0,\,b_1 s,\,b_2 s^{-3}}(xs, ys). \tag{13}
$$

The above formula is left for readers' exercise.

In fact, the orthogonal net of ellipses and hyperbolas in Figure 2 can be expressed with some coordinate formulas other than (9). The following Section presents an example. However, galaxy structure depends only on the geometric curves, not the choice of coordinate formulas. The general proof is very simple, and is left for readers' exercise. The following Section serves as a special verification.

3 Further Proof of Heaven Breasts

The confocal ellipses and hyperbolas can be expressed with a more beautiful and symmetric formula

$$
\begin{cases}
x = a_1 \sinh\lambda\, \sin\mu,\ y = a_1 \cosh\lambda\, \cos\mu, \\
0 \le \lambda < +\infty,\ 0 \le \mu < 2\pi
\end{cases}
\tag{14}
$$

where a_1 is a constant. The curves, $\lambda =$ constant, are all confocal ellipses while the curves, $\mu =$ constant, are all confocal hyperbolas. The positive x-axis is the curve $\mu = \pi/2$.

The corresponding arc-length derivatives P, Q are

$$\begin{aligned} P(\lambda,\mu) &= s'_\lambda = a_1\sqrt{\sinh^2\lambda\,\cos^2\mu + \cosh^2\lambda\,\sin^2\mu}, \\ Q(\lambda,\mu) &= t'_\mu \equiv P(\lambda,\mu). \end{aligned} \tag{15}$$

Their partial derivatives are

$$\begin{aligned} P'_\mu &= (a_1/2)\sin 2\mu \,/\sqrt{\sinh^2\lambda\,\cos^2\mu + \cosh^2\lambda\,\sin^2\mu}, \\ Q'_\lambda &= (a_1/2)\sinh 2\lambda \,/\sqrt{\sinh^2\lambda\,\cos^2\mu + \cosh^2\lambda\,\sin^2\mu}. \end{aligned} \tag{16}$$

The rational structure equation (8) does help factor out the required directional derivatives for our orthogonal net of curves. That is, Heaven Breasts does exist:

$$u(\lambda) = a_2\sinh 2\lambda,\ v(\mu) = a_2\sin 2\mu \tag{17}$$

where a_2 is another constant.

Before we find the Heaven Breasts (11), we look for the inverse equation of formulas (14). Squaring both sides of formulas (14) and performing a little trick, we have

$$x^2 + (\sinh\lambda/\cosh\lambda)^2 y^2 = a_1^2\sinh^2\lambda, \tag{18}$$

$$(\cosh\lambda/\sinh\lambda)^2 x^2 + y^2 = a_1^2\cosh^2\lambda. \tag{19}$$

Subtraction of (18) from (19) leads to

$$y^2\tanh^4\lambda + (x^2 - y^2 + a_1^2)\tanh^2\lambda - x^2 = 0. \tag{20}$$

Therefore,

$$\tanh^2\lambda = \left(-(x^2 - y^2 + a_1^2) + \sqrt{(x^2 - y^2 + a_1^2)^2 + 4x^2y^2}\right)/(2y^2). \tag{21}$$

Recall the formula, $r^2 = x^2 + y^2$, and the formula $p(x,y)$ in (10). We have

$$\tanh^2\lambda = (p^2(x,y) - x^2)/y^2 \tag{22}$$

if we choose a_1 to be equal to b_1. Once again we square both sides of formulas (14). We have

$$\tan^2\mu = x^2/y^2\tanh^2\lambda = x^2/(p^2(x,y) - x^2). \tag{23}$$

Finally the formulas (22) and (23) are the inverse equations of formulas (14).

It is easy to prove that

$$\sin^2\mu = x^2/p^2(x,y). \tag{24}$$

Formula (14) indicates that $\sinh\lambda = x/(a_1\sin\mu)$. Therefore,

$$\sinh^2\lambda = p^2(x,y)/a_1^2. \tag{25}$$

Now we are prepared to calculate the Heaven Breasts structure $f(x,y)$. To find the logarithmic density f at the point (x,y) whose corresponding parameter values are (λ,μ), we perform path integration of the directional derivative $u(\lambda)$ or $v(\mu)$ along some coordinate curves which connect the concerned point (x,y) to the center point $(x=0,y=0)$, so that we obtain the incremental value Δf between the two points. The parameter values of the center point can be chosen to be $\lambda=0$ and $\mu=\pi/2$. Therefore, the first part of the path integration is chosen to be along the x-axis ($\mu=\pi/2$):

$$(\Delta f)_1 = \int_{(\lambda=0,\mu=\pi/2)}^{(\lambda=\lambda,\mu=\pi/2)} u(\lambda)P(\lambda,\mu)d\lambda. \tag{26}$$

The second part of the path integration is along the elliptical coordinate curve ($\lambda =$ constant):

$$(\Delta f)_2 = \int_{(\lambda=\lambda,\mu=\pi/2)}^{(\lambda=\lambda,\mu=\mu)} v(\mu)Q(\lambda,\mu)d\mu. \tag{27}$$

Finally, $\Delta f = (\Delta f)_1 + (\Delta f)_2$. Straightforwardly we have

$$\begin{aligned} (\Delta f)_1 &= \textstyle\int_0^\lambda a_2 \sinh 2\lambda \; a_1\sqrt{\sinh^2\lambda\,\cos^2\mu + \cosh^2\lambda\,\sin^2\mu}\,d\lambda \\ &= a_1a_2 \textstyle\int_0^\lambda \sinh 2\lambda\,\cosh\lambda\,d\lambda \\ &= (2a_1a_2/3)(\cosh^3\lambda - 1). \end{aligned} \tag{28}$$

$$\begin{aligned} (\Delta f)_2 &= a_1a_2 \textstyle\int_{\pi/2}^{\mu} \sin 2\mu\,\sqrt{\sinh^2\lambda\,\cos^2\mu + \cosh^2\lambda\,\sin^2\mu}\,d\mu \\ &= a_1a_2 \textstyle\int_{\pi/2}^{\mu} \sin 2\mu\,\sqrt{\sinh^2\lambda + \sin^2\mu}\,d\mu \\ &= a_1a_2 \textstyle\int_{\pi/2}^{\mu} \sqrt{\sinh^2\lambda + \sin^2\mu}\,d\sin^2\mu \\ &= (2a_1a_2/3)\left((\sinh^2\lambda + \sin^2\mu)^{3/2} - (\sinh^2\lambda + 1)^{3/2}\right) \\ &= (2a_1a_2/3)\left((\sinh^2\lambda + \sin^2\mu)^{3/2} - \cosh^3\lambda\right). \end{aligned} \tag{29}$$

Finally, we have, by ignoring a constant,

$$\Delta f = (2a_1a_2/3)(\sinh^2\lambda + \sin^2\mu)^{3/2}. \tag{30}$$

Now we need the inverse coordinate equations (24) and (25),

$$\Delta f = (2a_2/3a_1^2)\left(p^2(x,y) + a_1^2x^2/p^2(x,y)\right)^{3/2}. \tag{31}$$

Recalling that $a_1 = b_1$, we finally recover the expression of Heaven Breasts if we choose $a_2 = b_2b_1^2/2$.

In fact, galaxy structures depend only on the geometric curves, not the choice of coordinate parameters. Therefore, the following is the general expression for confocal ellipses and hyperbolas:

$$\begin{cases} x = a_1 \sinh f(\lambda)\,\sin g(\mu), \\ y = a_1 \cosh f(\lambda)\,\cos g(\mu) \end{cases} \tag{32}$$

where $f(\lambda), g(\mu)$ are arbitrary functions. All these expressions give the same confocal ellipses and hyperbolas and generate the same Heaven Breasts, independent of the choice of coordinate parameters.

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