Essentials of the Hilbert Book Model

By Ir J.A.J. van Leunen

Retired physicist & software researcher

Location: Asten, the Netherlands

Website: http://www.e-physics.eu/

Communicate your comments to info at that site.

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Abstract

In order to support clear discussions, a simple self- consistent model of fundamental physics is constructed. It is strictly based on the axioms of traditional quantum logic and uses the isomorphism of the set of propositions of this logic system with the set of closed subspaces of a quaternionic separable Hilbert space. This primitive model cannot represent dynamics and does not include a representation of fields. For that reason the model also uses the Gelfand triple of the Hilbert space and a set of links that connect the eigenvectors of a particle location operator in the Hilbert space with the continuum eigenspace of a corresponding location operator in the Gelfand triple. These links are quaternionic probability amplitude distributions (QPAD's) that will act as the quantum state functions of the particles. The combination of a quaternionic Hilbert space, its Gelfand triple and the set of linking QPAD's still can only represent a static status quo of the universe. However, a dynamic model may consist of an ordered sequence of such sandwiches. The subsequent sandwiches form the pages of a book that we will call the Hilbert Book Model.

The quaternionic derivative of a QPAD leads to a continuity equation that describes how the derivative is coupled to its source. When the strength of the coupling is made explicit, then the resemblance with linear equations of motion becomes apparent.

This interpretation extends conventional quantum physics with quantum fluid dynamics. It throws an unprecedented view on the undercrofts of fundamental physics.

The logic model

Lattice structure

In the beginning quantum physics was done by quantization of the equations of classical mechanics. Schrödinger and Heisenberg each found a different way to perform this act. Dirac was able to prove that these ways are in fact two different views of the same methodology. John von Neumann and Garret

Birkhoff were able to formulate the fundamental reason why quantum physics differs from classical physics. They found that physics cheats with classical logic and instead obeys a more weakly structured logic, which was called quantum logic. They also proved that the set of propositions of quantum logic is lattice isomorphic with the set of closed subspaces of a separable Hilbert space. Later Constantin Piron proved that the number system in which the inner product of the Hilbert space is specified must be taken from a division ring. Only three suitable division rings exist: the real numbers, the complex numbers and the quaternions. Selecting the quaternions leads to a generalized quaternionic separable Hilbert space. This is the choice that will be applied in the Hilbert Book Model. For historic information see [1], Piron's theorem and reference [15] in that paper

Extension

We now have a double model. Besides a pure logic model we have a lattice isomorphic model that uses a quaternionic Hilbert space, which is far more suitable for the mathematical formulation of the physical laws. However, these models are still very primitive. They do not support fields and they do not implement dynamics. The separable Hilbert space only supports countable operator eigenspaces and it does not support an operator that offers time as a continuous eigenvalue. With other words this primitive model must be extended.

First we add fields. We do this (in a unorthodox way) by adding the Gelfand triple of the Hilbert space and constructing a set of links between the eigenvectors of the particle location operator that resides in the separable Hilbert space and the continuum eigenspace of a corresponding location operator that resides in the Gelfand triple. The link has the form of a quaternionic probability amplitude distribution (QPAD). It will act as the quantum state function of the particle. It defines the probability of detecting the particle at the location that corresponds to the parameter of the QPAD.

Since we did not obstruct the lattice isomorphism between the Hilbert space and quantum logic, we can construct a similar extension to quantum logic. It adds fuzzy (=blurred) observations to the logic.

Dynamics

Still the sandwich consisting of Hilbert space, Gelfand triple and set of QPAD's can only represent a static status quo of the universe.

However, when we arrange the subsequent sandwiches in a sequence then the sandwiches become the pages in a book. This leads to the Hilbert Book Model. The HBM represents a simple dynamic model of physics that comprises its fields in the set of linking QPAD's.

It means that according to this model, nature moves with universe wide steps. The page number of the book acts as a progression parameter. The progression step is a model invariant.

Quaternionic probability amplitude distributions

QPAD's are quaternionic distributions. Quaternions are afflicted with eight different sign selections, which are combinations of four independent sign selections. One of them is the quaternionic conjugation. It changes the sign of the imaginary part. The three other independent sign selections are reflections, one for each imaginary base direction. In continuous quaternionic distributions the sign selections do not change. Thus, continuous distributions exist in eight different sign flavors. This applies to QPAD's and as a consequence it applies to quantum state functions.

A QPAD contains a scalar field in its real part and a vector field in its imaginary part. The scalar field can be interpreted as to represent a charge density distribution. The vector field can be interpreted as to represent a current density distribution.

Differentials, continuity equations and coupling equations

Derivative

The differential of a quaternionic distribution has a real and an imaginary part.

$$g(q) = g_0(q) + \boldsymbol{g}(q) = \nabla f(q) \tag{1}$$

$$= \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle$$

$$\pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right)$$

The blue colored $\mp$ and $\pm$ signs refer to the influence of conjugation of $f(q)$ on quaternionic multiplication. The red $\pm$sign refers to the influence of reflection of $f(q)$.

Continuity equation

If the function f(q) is a QPAD then an understanding reader might recognize the real part as a continuity equation (also called balance equation). Here f_0 plays the roles of a charge density and $\boldsymbol{f} = f_0\, \mathbf{v}$ plays the role of current density. Thus in quaternionic terms the full equation is also a continuity equation and the QPAD f combines a charge density distribution f_0 and a current density distribution $\boldsymbol{f}$.

Coupling equation

Now write the differential as a coupling equation

$$\nabla f(q) = m\, h(q) \tag{2}$$

where m is interpreted as a coupling strength. Here two QPAD's $\{f, h\}$ are coupled with strength m.

This equation resembles the Dirac equation and the Majorana equation when the gamma matrices are removed.

The Dirac equation for the electron runs

$$\nabla f(q) = m\, f^*(q) \tag{3}$$

Thus here the QPAD $h(q)$ is a sign flavor variant of $f(q)$.

The Majorana equation uses a different sign flavor variant.

Particles and waves

If we restrict to sign flavor variants then equation (2), apart from possible variants of m, offers 64 variants of $\{f, h\}$. The combinations $\{f, f\}$ will become zero couplings $(m = 0)$. Thus 56 different particle types result. The sign flavors determine the properties (electric charge, color charge and spin).

The free (= uncoupled) QPAD's must oscillate. They represent the waves. Eight different waves exist.

The result is a striking correspondence with the standard model, but also strange differences exist.

Inertia and space curvature

Quantum fluid dynamics

The continuity equation extends quantum physics with quantum fluid dynamics. Quantum fluid dynamics differs from conventional fluid dynamics in the nature of the moving objects. The moving objects are carriers of properties. In conventional fluid dynamics the moving objects are elements of a gas or a liquid. In quantum fluid dynamics the moving objects are tiny patches of the parameter space of the charge and current density distributions. The coupling of QPAD's affects the local density of the carriers. As a consequence it affects the curvature in the environment of this coupling. This explains the origin of gravitation.

Inertia

The QPAD that acts as a source in the coupling equation is coupled to the quantum state function of the particle. It is formed by the local superposition of the tails of the quantum state functions of distant particles. Apart from that they superpose, quantum state functions influence each other. This influence decreases with distance. However, the number of particles that contribute to the influence increases with distance. The result is a huge uniform background potential at every location in universe. Now field theory tells that the acceleration of a local particle goes together with the existence of an extra field that counteracts the acceleration. This effect is called inertia.

Palestra

All quantum state functions share their parameter space as affine spaces. Due to the fact that the coupling of QPAD's affect the parameter space, this shared space is curved. The curvature is not static. With other words patches in the parameter space move and densities in the distribution of these patches change. Due to the particular properties of this space it deserves a special name. it is called ***Palestra***.

Spacetime metric

The Palestra is defined with respect to a flat parameter space, which is spanned by the quaternions. The specification is performed by a continuous quaternionic distribution $\wp(x)$ that acts as a distance function. This distance function defines a quaternionic infinitesimal interval ds. On its turn this definition defines a metric tensor.

$$ds(x) \ = ds^{\nu}(x) e_{\nu} = \sum\nolimits_{\mu=0\ldots3} \frac{\partial\wp}{\partial x_{\mu}} dx_{\mu} = \ q^{\mu}(x)\, dx_{\mu} \tag{1}$$

The base e_{ν} and the coordinates x_{μ} are taken from the **flat** parameter space of $\wp(x)$. That parameter space is spanned by the quaternions.

ds is the infinitesimal quaternionic step that results from the combined real valued infinitesimal dx_{μ} steps that are taken along the e_{μ} base axes in the (flat) parameter space of $\wp(x)$.

$dx_0 = c\, d\tau$ plays the role of the infinitesimal space time interval ds_{st}[1]. It is a physical invariant. $d\tau$ plays the role of the proper time interval and it equals the infinitesimal progression interval. The progression step is an HBM invariant. Without curvature, dt in $\|ds\| \ = \ c\, dt$ plays the role of the infinitesimal coordinate time interval.

$$c^2\, dt^2 = \ ds\, ds^* = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \tag{2}$$

$$dx_0^2 = {ds_{st}}^2 = c^2\, dt^2 - \ dx_1^2 - dx_2^2 - dx_3^2 \tag{3}$$

dx_0^2 is used to define the local spacetime metric tensor. With that metric the Palestra is a pseudo-Riemannian manifold that has a Minkowski signature. The Palestra comprises the whole universe. It is the arena where everything happens.

The Palestra step

When nature steps with universe (Palestra) wide steps during a progression step Δx_0, then in the Palestra a quaternionic step $\Delta s_{\wp}$ will be taken that differs from the corresponding flat step Δs_f

$$\Delta s_f = \Delta x_0 + \boldsymbol{i}\, \Delta x_1 + \boldsymbol{j}\, \Delta x_2 + \boldsymbol{k}\, \Delta x_3$$

[1] Notice the difference between the quaternionic interval ds and the spacetime interval ds_{st}

$$\Delta s_{\wp} = q^0 \Delta x_0 + q^1\, \Delta x_1 + q^2\, \Delta x_2 + q^3\, \Delta x_3$$

The coefficients q^{μ} are quaternions. The Δx_{μ} are steps taken in the (flat) parameter space of the distance function .$\wp$(x)

Fields

The HBM considers several types of fields that are al derived from QPAD's.

First of all any continuous quaternionic distribution contains a scalar field in its real part and a vector field in its imaginary part. For quaternionic probability amplitude distributions (QPAD's) these fields become a special interpretation. The real scalar field can be interpreted as to correspond to a charge density distribution. The imaginary field can be interpreted as to correspond to a current density distribution. The charges are properties or ensembles of properties of their carriers. The carriers can be interpreted as tiny patches of the parameter space of the QPAD. With other words, the currents may affect the local density of the distribution of the carriers.

In the HBM elementary particles are constituted by the coupling of two QPAD's. This coupling can be characterized by a set of properties. These properties are the properties of the corresponding particle. These properties exert influence on the environment of the particle. This influence can be represented by a second type of fields. Each type of property corresponds to a type of secondary fields. Together with the objects that correspond to a zero coupling factor, these secondary fields are the physical fields that we are used to.

Complex and quaternionic quantum physics

Quaternionic probability amplitude distributions (QPAD's) extend the functionality of complex probability amplitude distributions (CPAD's). Their imaginary part is extended with two dimensions. On the other hand the product of two quaternionic distributions will in general not commute. This poses problems when differentiations of products of quaternionic distributions are treated. Then in general

$$\nabla(f\ g) \neq\ f\ \nabla\ g\ +\ (\nabla f)g$$

The switch from CPAD's to QPAD's does not affect physical reality. It only affects the view that we have at the undercrofts of fundamental physics.

The HBM supports complex quantum physics as well as quaternionic quantum physics. Complex quantum physics suits better for one-dimensional or one-parametric applications. Quaternionic quantum physics suits better for multidimensional applications, such as investigating the geometry of particles.

The Hilbert Book Model

The HBM is treated in greater detail in "Features of the Hilbert Book Model". The corresponding free accessible manuscript can be found at [2] and in the author's e-print archive [3].

HBM versus contemporary physics

The way that fields and dynamics are implemented in the HBM differs considerably from contemporary physics. In the HBM, nature steps with universe (Palestra) wide steps from one static status quo to the next. The corresponding progression step is considered a model invariant. In contemporary physics the infinitesimal spacetime interval is considered to be a physical invariant. Both models are Lorentz invariant.

Quantum electro dynamics and quantum chromo dynamics treat fields similarly as wave functions. The approach makes extensive use of canonical derivatives. The HBM uses QPAD's as quantum state functions and derives all fields directly or indirectly from these QPAD's.

The HBM is a quaternionic quantum physics that incorporates complex quantum physics as a specialization to one-dimensional or one-parametric applications. It extends complex quantum physics with quantum fluid dynamics. Conventional quantum physics sticks with complex numbers. It adapts to quaternionic behavior via spinors, matrices, Clifford algebras, Grassmann algebras, and Jordan algebras.

References

[1] Stanford Encyclopedia of Philosophy, *Quantum Logic and Probability Theory*, http://plato.stanford.edu/entries/qt-quantlog/

[2] J.A.J. van Leunen, *Features of the Hilbert Book Model*; http://www.e-physics.eu/FeaturesOfTheHBM.pdf

[3] J.A.J. van Leunen, e-print archive, http://vixra.org/author/j_a_j_van_leunen