Relativity and the Luminal Structure of Matter

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Abstract

Lorentz Invariance implies definite structural constraints on the massive particles. It is shown, from the basic physics of luminal waves of any kind, that multi-component wave systems conform to the usual relativistic mechanics for matter. This motivates further consideration of luminal wave models of matter. The usual length contraction and time dilation phenomena are shown in such models, leading to a paradox-free wave interpretation of Lorentz Invariance and the conclusion that internal movements referred to the co-moving frame will be luminal in any Lorentz invariant particle model.
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1 Introduction

> *"But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created."* - Einstein & Infeld [1].

Relativistic wave equations, especially the d'Alembert, Helmholtz and Dirac [2] equations, are indispensable to Modern Physics. For example, the nonrelativistic Schroedinger wave equation is contained in the Dirac Equation as the low velocity, no spin limit. These relativistic equations either feature propagation at the characteristic velocity, c, or, in the language of the operator formalism, a velocity operator of constant modulus equal to c [3]. There are also many Lorentz invariant classical field theories in the literature, including nonlinear theories with subluminal soliton solutions that serve as candidate models for the fermions. [4] - [10] are just a few to illustrate the diverse range of approaches.

This Article considers the basic mechanics of luminal wave systems, *i.e.* systems of waves that propagate at c. We shall adapt the Newtonian momentum equation, $p = mv$, for use with constant speed luminal waves and then apply universally accepted basic principles of mechanics to luminal waves. This leads to a general structural analysis of luminal wave systems that is inherently relativistic without asserting any principle of relativity. The usual relativistic mechanics of matter can therefore be interpreted as the basic mechanics of subluminally moving systems constructed entirely from luminal waves. It is also shown that this interpretation is uniquely simple and free of all paradoxes.

The first, necessary step towards achieving Einstein's goal for a pure field physics is therefore to recognise that, whether it appears as radiation or as matter, energy is a propagative phenomenon. The proposed luminal wave ontology also provides new perspectives on many issues including the Dirac velocity operator, angular momentum quantisation, the structure of Electromagnetics [11], gravity [12], the existence of nonlocal relations between observables, and interference phenomena in matter beams.

Section 2 defines the basic principles of mechanics that are regarded as universally accepted and identifies the simple general relationship that governs the connection between inertial frames for systems of luminal wave momenta. Section 3 shows that the usual relativistic momentum equation for particles applies to systems of luminal wave momenta. Section 4 derives the (forward) relativistic transformation of wave momenta in a form that is useful for analysing wave systems as a whole. Section 5 extends the results to any kind of luminal wave system, provided a wave vector in the direction of propagation can be defined, linear momentum is locally conserved, and propagation is luminal. In particular, linear superposition of field variables is not required so the method is applicable to nonlinear wave systems with subluminal soliton solutions. For luminal waves the speed of propagation is, by definition, fixed and any luminal wave model of a subluminal massive particle is immediately subject to the kinematic constraint that, when the speed of the particle changes, the speed of its constituent wave components does not. Sections 6 and 7 show that length contraction and time dilation are the consequences of this kinematic constraint so luminal systems display all the usual relativistic phenomena.

Section 8 addresses the question how the physical phenomena of length contraction and time dilation constrain the coordinate transformations. Selleri has analysed this in detail [13, 14]. He showed that, subject mainly to the use of Einstein clock synchronisation, Lorentz Transformations follow directly from length contraction and time dilation, which are derived here from the basic principles of mechanics without making any further assumptions. As discussed in Subsection 8.2, the proposed wave interpretation is also equipped with a readily observable preferred frame, eliminating the paradoxes usually associated with the relativist interpretation of Lorentz Invariance. It remains only to point out, in Section 9, that any form of non-luminal structure for the massive particles is implausible, hence the conclusion that the relativistic phenomena imply the luminal structure. Section 10 then outlines the reasons why Lorentz Invariance does not preclude nonlocal relations between observables in this pure field context.

2 Basic Physics of Luminal Waves

2.1 The Basic Principles of Mechanics

The usual classical field approach to mechanics in wave systems begins by choosing wave or field equations. Any analysis is immediately limited to the mechanics of one particular kind of wave system. We would identify various solutions to the chosen equations, which are in general expressed as spatial distributions of some field variables. Field energy and momentum densities must be induced from these field variables. After evaluating the spatial integrals of the energy and momentum densities we would arrive at expressions for the momenta and energies of the wave solutions and we could begin the mechanics.

Unfortunately, in many circumstances we do not know what equations to use, much less their solutions. Moreover, the great variety of Lorentz covariant wave equations suggests that relativistic mechanics is a general feature that many wave systems have in common. What kind of wave systems? As mentioned in the introduction, the leading relativistic wave equations feature the characteristic velocity, suggesting that, when the field energy-momentum in a wave system is constrained to propagate at c (i.e. luminally), then the system displays the usual relativistic mechanics.

Therefore, instead of taking the usual fields approach to mechanics let us take a *mechanics* approach to *fields*, applying the basic principles of mechanics directly to a field energy-momentum density that propagates at c. The universally accepted principles to rely upon can be stated as follows:

1. The momentum of an object is defined as the product of its inertia times its velocity. Similarly, field momentum density is the product of inertia density and velocity.

2. Momentum is conserved. Field momentum is locally conserved.

3. The principle of local action means that wave objects, as defined below, may interact with each other only in regions of space where they overlap.

4. The force acting on an object is equal to its rate of change of momentum.

5. The resulting change in the energy of the object is given by the work integral.

6. Energy is conserved. Field energy is locally conserved.

Here 'wave object' means: some set of functions on a 3-space[1], which together induce a field momentum density, $\vec{\rho_p}(x, y, z, t)$, that a) propagates luminally according to a unique unit wave vector, $\hat{\mathbf{k}}(x, y, z, t)$ and b) whose spatial integral, $\int\int\int_{-\infty}^{+\infty} \vec{\rho_p}(x, y, z, t)dxdydz$, is finite.[2,3,4]

We are interested in the mechanics of systems that comprise multiple wave objects. This begins with non-interacting systems where the wave objects are not presently interacting with each other. The next Subsection focusses on the case where each object's unit wave vector, $\hat{\mathbf{k}}(x, y, z, t)$, is a constant vector, independent of x, y, z and t. The momentum density distribution of such wave objects moves through space in a self similar form at c. We shall refer to this special kind of wave object as a light flash.

2.2 Application to Light

Consider a source that simultaneously emits a set of N light flashes in various directions. The development here can be applied to any kind of light flashes, including individual photons, short segments of laser beams, or collimated beams in general, monochromatic or not. We require only that each flash propagates at c, carrying a finite linear momentum in a well-defined direction in space.

Let the i^{th} light flash carry linear momentum $\mathbf{p}_i$. According to the first basic principle, momentum equals the product of inertia and velocity and the wave inertia of the i^{th} light flash is therefore defined as $m_i = p_i/c$, where $p_i = |\mathbf{p}_i|$ is the magnitude of the momentum of the i^{th} light flash, also called the 'scalar momentum':

$$p_i = m_i c \tag{1}$$

This Article is essentially a consistent application of the basic mechanics principles, using (1) in place of the familiar $p = mv$, where the speed v is a variable. Note that, *prima facie*, the inertia, m_i, of a wave propagating in a well-defined direction in space has nothing to do with the mass of a particle. However we use the symbol m_i because, unless they ALL propagate in the same direction, the total inertia of a set of N waves will be found to correspond to the usual (relativistic) particle mass. The time differential of (1) is:

$$\frac{dp_i}{dt} = c\frac{dm_i}{dt} \tag{2}$$

Having fixed the propagation speed, c, changes of the scalar momentum are thus associated with changes of the wave inertia. It will become clear in Sect. 8 that the inertia changes we will be discussing throughout this Article are in fact frequency changes. Such changes may be due to a change of observer or they may be physical changes due to any forces that are acting on the wavefield.

[1] That is, spatial distributions of field variables.

[2] In addition to inducing the field momentum density, the space functions that define wave objects in a nonlinear field theory may also act as sufficient causes for any interactions that there may be.

[3] Note that infinite plane waves are not wave objects.

[4] Neither the propagation of the space functions nor their relation to the linear momentum density are specified here. This allows for wave objects with intrinsic field angular momentum and, more generally, the definition accommodates two kinds of internal evolution, via the internal movements of an otherwise invariant set of functions and via their individual time evolutions.

In general, if a force acts on a light flash then, since (2) is the force component parallel to the light flash's motion, the work integral is:

$$W = \int_{p_\mathrm{s}}^{p_\mathrm{f}} \frac{d\mathbf{p}}{dt}.d\mathbf{s} = \int_{m_\mathrm{s}}^{m_\mathrm{f}} c\frac{dm}{dt}\,cdt = (m_\mathrm{f} - m_\mathrm{s})c^2 \qquad (3)$$

Where subscripts s and f refer to the words 'start' and 'finish'. The radiation reaction force that acts on a light flash reflected by a moving mirror is an example that highlights the role of the work integral in a basic mechanics calculation[5]. According to the fifth basic principle, the work done equals the energy change, and we may assume that a light flash that has zero momentum requires zero energy, so the energy of the i^{th} flash is:

$$E_i = m_i c^2 = cp_i \qquad (4)$$

According to the second basic principle, momentum is conserved so the total momentum of a set of N wave objects is given by the vector sum over their momenta:

$$\mathbf{P} = \sum_{i=1}^{N} \mathbf{p}_i \qquad (5)$$

Suppressing the summation range henceforth, we write the total inertia as $m_e = \sum_i m_i$. The total energy of the set is then:

$$E = \sum_i cp_i = m_e c^2 \qquad (6)$$

According to the (first and second) basic principles, the velocity of the centre of inertia of a system of objects is the inertia weighted average velocity, $\mathbf{V} = \sum_i m_i \mathbf{v}_i / \sum_i m_i$, so that:

$$\mathbf{V} = \frac{\sum_i \mathbf{p}_i}{m_e} \Rightarrow \mathbf{P} = m_e \mathbf{V} \qquad (7)$$

For a relativistic analysis, these basic Equations (1) - (7) must of course be good for any observer, however, since we intend *inter alia* to show it, no principle of relativity is asserted *a priori*.

Consider an incremental change that affects the system of light flashes as a whole. For example, an incremental change in the condition of motion of the observer would at once alter all his observations of the $\mathbf{p}_i$. Similarly, a single observer considering light flashes emitted by otherwise identical sources that are in different conditions of motion will find different values for the $\mathbf{p}_i$. Since these two cases are not *a priori* assumed equivalent, consider the latter one, and consider, specifically, two otherwise identical sources moving at velocities $\mathbf{v}$ and $\mathbf{v} + d\mathbf{v}$ in the inertial frame of a single inertial observer.

This scenario closely corresponds to applying a Lorentz boost to a system of wave momenta. We may write the momenta of the light flashes as $\mathbf{p}_i$ and $\mathbf{p}_i + d\mathbf{p}_i$ respectively and their totals as $\mathbf{P}$ and $\mathbf{P} + d\mathbf{P}$. We are interested in how the $d\mathbf{p}_i$ are related to $d\mathbf{P}$. As shown in Appendix 2, this is determined by the relevant known facts, the relativistic Doppler shift and aberration phenomena, which together imply:

$$d\mathbf{p}_i = \frac{p_i}{m_e c}\, d\mathbf{P} \qquad (8)$$

We are assuming neither special relativity nor the relativity principle by referring to these phenomena. Indeed, while Lorentz Transformations correctly imply each of them, there exist other coordinate transformations [13] that also correctly predict these observables [15]. Because (8) is a direct consequence of the phenomena themselves[6], it necessarily applies to any theory that correctly accounts for them.

It is nonetheless relevant to consider what, if anything, the facts here are introducing over and above the basic principles stated above. If our coordinate transformations are to be linear and homogeneous, as is usually assumed, then $d\mathbf{p}_i$ will be linear in p_i. Similarly, when considering the case of a single light flash, (the case $N = 1$), $d\mathbf{p}_i$ should be linear in, and parallel to, any incremental change of momentum of the light source, $d\mathbf{P}_{LS}$, prior to emission. Since the same applies to each of the light flashes in our system it follows that $d\mathbf{p}_i \propto d\mathbf{P}$ and we can safely assume that: $d\mathbf{p}_i = \alpha_i p_i d\mathbf{P}$.

Eq. (8) means that all the weights, α_i, are the same, $\alpha_i = \alpha$. Summing over i gives $\alpha = 1/m_e c$ (since $\sum_i d\mathbf{p}_i = d\mathbf{P}$). In particular, under an incremental momentum boost of the whole system, the Doppler shift and aberration results require that the momentum shifts, $d\mathbf{p}_i$, applied to the various wave momenta, $\mathbf{p}_i$, depend linearly on their energies but not on their directions of propagation, $\hat{\mathbf{k}}_i$

The next two sections show how the incremental momentum boost, (8), governs the connection between inertial frames for systems of luminal wave momenta. In order to avoid asserting the relativity principle, the boost will not presently be associated with a change of observer. It will turn out to work relativistically, but for the present purposes (8) has only the restricted meaning of an incremental change $d\mathbf{v}$ in the velocity of a light source, the result of which is to add $d\mathbf{P}$ to the total wave momentum by adding wave momentum $d\mathbf{p}_i$ to each of the N constituent light flashes[7].

3 The Relativistic Momentum

This section shows that systems of luminal wave momenta that are connected by incremental momentum boosts obey the usual relativistic momentum equation for particles.

In subsection 2.2, the incremental change in the scalar momentum of the i^{th} light flash, dp_i, is given by the component of $\mathbf{dp}_i$ parallel to $\mathbf{p}_i$, namely:

$$dp_i = d\mathbf{p}_i \cdot \frac{\mathbf{p}_i}{p_i}$$

Substituting (8) in this gives $m_e c\, dp_i = \mathbf{p}_i \cdot d\mathbf{P}$. Noting that $\sum dp_i = c\, dm_e$, summing over i gives:

$$c^2 m_e dm_e = \mathbf{P} \cdot d\mathbf{P}$$

[5] See Appendix 1, which shows that the ratio of reflected and incident momenta is the square of the relativistic doppler shift.

[6] When Appendix 1 is generalised to the case of non-normal incidence, the result is the product of the two relativistic Doppler shift and aberration operations involved. The basic principles are thus arguably sufficient to derive (8) by themselves, however the analysis is tedious.

[7] Note that we do not need to assume that $d\mathbf{V} = d\mathbf{v}$

Integrating this we obtain the common expression for the invariance of the 4-momentum:

$$m_e^2 c^2 = P^2 + m_0^2 c^2 \tag{9}$$

Where m_0 is the value of m_e for $P = 0$. Let $\beta = V/c$ as usual so β is a +ve real number in the interval $[0, 1]$. The basic equations of relativistic mechanics, $\mathbf{P} = \gamma m_0 \mathbf{V}$ and $m_e = \gamma m_0$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, follow upon substituting (7) into (9).

4 Wave System Transformations in Momentum Space

In this section we show how the momenta of individual wave objects in a multi-object wave system transform under the action of (8).

By analogy to the usual comoving frame for massive particles, let us define the rest frame of a multi-object wave system as the (unique) inertial frame for which the right hand side of (5) vanishes. This definition is convenient, but not essential. Given the definition, let us now adopt the perspective of a single inertial observer who compares systems of light flashes emitted by two otherwise identical sources in different conditions of motion such that he considers one system's centre of inertia to be at rest, i.e. $\mathbf{P} = 0$ in (5) and $\mathbf{V} = 0$ in (7), and the other's to be moving at speed V in the x-direction, so that, from Section 3, $P = \gamma m_0 V$.

Let us refer to these two systems of light flashes as the 'rest system' and the 'moving system' respectively. We shall use a 0 subscript to refer to rest system momenta, so $\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$. The analysis is expressed in momentum coordinates and it does not involve anything about spatial relations between the waves until Section 6.

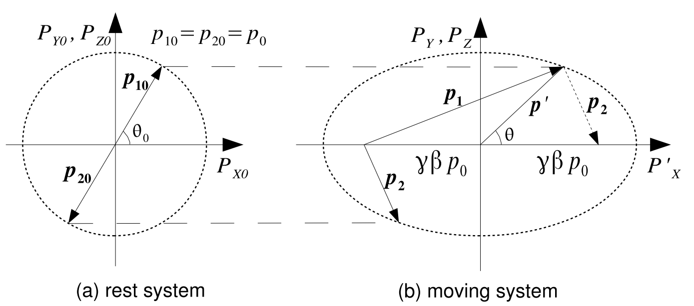

Figure 1: Binary light flash systems whose centers of inertia are (a) at rest (b) moving at speed $V = \beta c$.

The simplest case of a compound wave system where $\mathbf{P}_0 = 0$ consists of 2 light flashes of equal scalar momentum, $p_{10} = p_{20} = p_0$, propagating in opposite directions, as shown in Fig. 1a. The moving system is shown in Fig. 1b, where the x-components of the wave momenta, $\mathbf{p}_{10}$ and $\mathbf{p}_{20}$, have been modified in accordance with (8) so that the centre of inertia moves at speed V in the x-direction.

In Fig. 1a, $m_0 = (p_{10} + p_{20})/c = 2p_0/c$. Recalling from Section 3 that $m_e = \gamma m_0$, the sum of scalar momenta in the moving system of Fig. 1b is:

$$p_1 + p_2 = m_e c = 2\gamma p_0 \tag{10}$$

Whilst the total momentum, $\mathbf{P} = m_e \mathbf{V}$ is the vector sum of momenta:

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P} = \frac{2\gamma p_0}{c}\mathbf{V} = 2\gamma\beta p_0\, \hat{\mathbf{i}}$$

Consider the vector $\mathbf{p}'$ in Figure 1b, where $\mathbf{p}_1 = \mathbf{P}/2 + \mathbf{p}'$ and $\mathbf{p}_2 = \mathbf{P}/2 - \mathbf{p}'$. Using the law of cosines, its magnitude, p', is such that:

$$p_1^2 = p'^2 + (\gamma\beta p_0)^2 + 2\gamma\beta p_0 p' \cos\theta \tag{11}$$

$$p_2^2 = p'^2 + (\gamma\beta p_0)^2 - 2\gamma\beta p_0 p' \cos\theta \tag{12}$$

Where θ is the angle $\mathbf{p}'$ makes with the x-axis. Upon eliminating p_1 and p_2 from (10)-(12) we find that $p' = p'(\theta)$ is the ellipsoid:

$$p'(\theta) = \frac{p_0}{\sqrt{1 - \beta^2 \cos^2\theta}} \tag{13}$$

Let us write the momenta in component form as $(p_{ix}, p_{iy}, p_{iz})_{i=1,2}$. In Cartesian coordinates (13) is then the ellipsoid:

$$(p'_{ix}/\gamma)^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2$$

where $p'_{ix} = p_{ix} - \gamma\beta p_{i0}$, so that the moving system momenta satisfy the following equation:

$$(\frac{p_{ix} - \gamma\beta p_{i0}}{\gamma})^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2 \tag{14}$$

Eq. (14) is here derived only for the case N=2, however this equation also covers the general case, as we shall now show. Consider as initial condition an arbitrary system of light flashes, comprising a number $N \geq 2$ of wave momenta of scalar momentum, p_{i0}, whose directions of propagation are distributed in space such that $\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$ and $\sum_i p_{i0} = m_0\, c$. In the rest system components are such that:

$$p_{ix0}^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2 \tag{15}$$

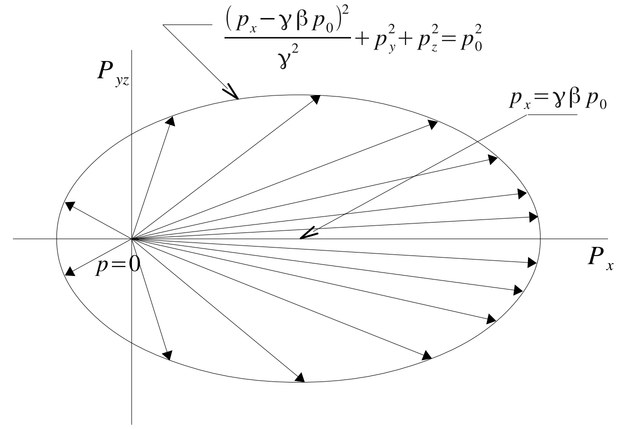

Figure 2: Individual momenta in an isotropic wave system modified such that $V = \beta c$.

The example for $N = 2$ above suggests that after (8) acts on the set, bringing the total momentum to $\mathbf{P} = \gamma m_0 V\, \hat{\mathbf{i}}$, then (14) applies to the moving system momenta. Fig. 2 shows the moving system momenta when all the rest system scalar momenta are the same, i.e. $p_{i0} = p_0$ for all i. Differentiating (14) with respect to β using $d\gamma/d\beta = \gamma^3\beta$ and $d\gamma^{-1}/d\beta = -\gamma\beta$ leads to:

$$\frac{dp_{ix}}{d\beta} = \gamma(p_{i0} + \gamma\beta p_{ix}) \tag{16}$$

Expanding the first term in (14) and using $\gamma^2\beta^2 = \gamma^2 - 1$ (twice) gives:

$$p_i = \frac{p_{i0} + \gamma\beta p_{ix}}{\gamma} \tag{17}$$

As $P_x = \gamma m_0 V$, we also have $dP_x = \gamma^3 m_0\, dV$, so that:

$$dp_{ix} = \frac{dp_{ix}}{dV}dV = \frac{dp_{ix}}{d\beta}\frac{dP_x}{\gamma^3 m_0 c} \tag{18}$$

Finally, substituting (16) and (17) in (18):

$$dp_{ix} = \frac{p_i}{\gamma m_0 c}dP_x$$

This is the x-component of (8). Due to the choice of coordinates, the y and z components of momentum were unaffected, so the ellipsoidally modified distribution (14) is generated by the action of (8) on our arbitrary initial condition as expected. Comparing (14) and (15), the components of the moving system wave momenta are:

$$p_{ix} = \gamma(p_{ix0} + \beta p_{i0}) \;\;,\;\; p_{iy} = p_{iy0} \;\;,\;\; p_{iz} = p_{iz0} \tag{19}$$

Note that these physical transformations due to changes in the condition of motion of a light source are identical to Lorentz Transformations of wave momenta between different reference frames in standard configuration. However, as we are not asserting the Principle of Relativity there is no guarantee (so far) that our analysis works relativistically, and (19) corresponds only to the forward transformations of wave momenta in relativity theory.

We can now calculate the relative velocity of the i^{th} light flash, which is to say its velocity relative to the centre of inertia of the system, which our observer considers to be moving at V in the x-direction. The total velocity of the i^{th} flash has components $v_{ix} = cp_{ix}/p_i$, $v_{iy} = cp_{iy0}/p_i$, and $v_{iz} = cp_{iz0}/p_i$. Using $\gamma^2\beta^2 = \gamma^2 - 1$ with (17) and (19), it is readily shown that the relative velocity, $\mathbf{v}_{\mathrm{r}i}$, has components[8]:

$$v_{\mathrm{r}ix} = v_{ix} - V = \frac{cp_{ix0}}{\gamma p_i}$$

$$v_{\mathrm{r}iy} = v_{iy} = \frac{cp_{iy0}}{p_i} \qquad ; \qquad v_{\mathrm{r}iz} = v_{iz} = \frac{cp_{iz0}}{p_i}$$

Finally, if $\mathbf{v}_{\mathrm{r}i}$ makes the angle ϑ_i with the x-axis, then:

$$\tan\vartheta_i = \sqrt{v_{\mathrm{r}iy}^2 + v_{\mathrm{r}iz}^2}/v_{\mathrm{r}ix} = \gamma\tan\vartheta_{i0} \tag{20}$$

where ϑ_{i0} is the corresponding angle in the rest system. Sect. 6 shows how this basic kinematic relationship leads to length contraction in 'pure field' models of the massive particles where all the field energy propagates luminally. Such models are discussed in the next section.

5 Luminal Wave Models of Matter

Up to this point the analysis has dealt with the linear momentum of systems of light flashes emitted by identical sources in different conditions of motion. No functional description of the light flashes was required, neither as photons nor as solutions to any particular wave equation. The fact that these systems obey the usual relativistic momentum equation for particles strongly suggests that the massive particles should also be thought of as luminally propagating field systems. This Section discusses how the basic mechanics principles can be applied quite generally to compound, interacting systems of wave objects that are commensurate with modelling subluminally moving systems.

5.1 Compound Wave Systems

At any point in a system of disjoint light flashes (*i.e.* whose momentum densities do not overlap), there is a single field momentum density associated to a well defined unit wave vector. This field momentum density might, at least in principle, be induced from a set of space functions in accordance with the definition of a wave object, so the entire system can be thought of as a single wave object. However, there are also wave systems that cannot be represented as single wave objects.

Consider instead a system of N light flashes that propagate towards each other. When the field momentum densities of the various light flashes meet and overlap, the physical situation is inevitably such that there are multiple waves coexisting at the same place, propagating in different directions[9]. Since the set of space functions that comprises a wave object induces only a single momentum density at each point, when wave objects collide a luminal wave description of the resulting system inherently requires us to consider multiple wave objects coexisting at the same place and time.

We shall now see that interactions between these distinct entities are required in order to construct luminal wave models of subluminally moving matter.

5.2 Forces, Field Variables and Superposition

The force operating on a wave object is, by definition, equal to its rate of change of momentum, which is to say the space integral of the rate of change of its momentum density[10]. Momentum is locally conserved, so forces necessarily manifest as reciprocal local exchanges of momentum between the momentum densities of the participating wave objects. These exchanges necessarily sum to zero locally as well as globally, so 'local action' can only mean that the objects' momentum densities must overlap.

[8] Since $\mathbf{V}$, $\mathbf{v}_i$ and $\mathbf{v}_{\mathrm{r}i}$ are all referred to the same observer

[9] Note that the vector addition of two non-collinear luminal wave vectors is not a luminal wave vector because there is no wave actually propagating at c in the direction of the resultant vector.

[10] There is also generally a rate of change of a wave object's momentum density at every fixed point due to the movement of the object, but the space integral of such changes obviously vanishes.

When the momentum density distribution of a wave object changes then so must the field variables that induce it, so the essential nature of forces in a wave theory is to modify wave objects.

In a compound wave system formed by intersecting light flashes, if there were no forces between wave objects, then the momentum distributions pertaining to each object would not change as they move through each other, the same space functions could be retained for each wave object throughout the encounter and it is reasonable to think of each object's field variables as being the same as if it were by itself. A linear field theory is then appropriate. In Electromagnetics, for example, the wavefields interact with charges but not with each other. The chosen field variables, $\mathbf{E}$ and $\mathbf{H}$, are force fields defined by the force that the wavefield exerts on a standard reference system, a 1 coulomb point charge. The global values of these field variables are given as linear superpositions of the disjoint values pertaining to individual wavefields.

In linear field theory, wave components evolve independently of each other, there are no interactions amongst the waves and any superposition must dissipate unless all the wave vectors are parallel, in which case the motion of the centre of inertia of the wave group is $V = c$. Electromagnetic field models of the massive particles are thus excluded. The idea that a finite subluminal image can be formed as an interference pattern can also be excluded as it requires infinite wave trains which requires infinite energy. Therefore, the construction of luminal wave models for the massive particles requires multiple distinct wavefields that share the same space and interact with each other to form bounded systems, which is to say they form wave solitons.

When the wavefields in a model do interact with each other, the forces that are actually operating on a given wave object still superpose (by definition). However, as discussed above, the definition of force also implies that wave objects are distorted under interaction. If the wave object is defined by force field variables, as in Electromagnetics, then its force fields (which are propensities to exchange momentum as opposed to actual forces) are not the same under interaction as would be the case if it had been disjoint. Furthermore, if a wave object in an interacting system persists in a self-similar form then that form depends in an essential way on the forces that are operating on it. It is obviously counterfactual to consider such a wave object as if it were disjoint from the other wave objects that are actually present. If they were not present, it would be a different object.

Overall, once we include interactions between wave objects, the global values of field variables cannot be expressed as a linear superposition of disjoint values so a nonlinear theory is required. If the chosen field variables are force fields, then global values are by definition still given as a linear superposition, but this is a linear superposition of conjoint values that correspond to actual transfers of wave momentum from one object to another.

Of course one might choose other field variables besides force fields. With water waves for example the vertical displacement of the water surface is commonly used as a field variable. Such alternatives also do not generally superpose linearly. However, whatever field variables we may choose and however they may induce it, the field momentum density is locally conserved. As we shall see in the next two sections, the field momentum density is also the physical basis for any mechanical quantities that we may observe including not just momenta but also lengths and times.

5.3 Wave Trajectories

Whereas a field variables description immediately confronts us with some unknown nonlinearity, we can focus directly on the inherently linear field momentum density by considering a wave trajectories description. This kind of description is often useful in Electromagnetics, where it arises from the field variables description as follows. Electromagnetic waves in a vacuum obey the well known d'Alembert wave equation:

$$\{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\}\psi = 0 \tag{21}$$

Where $\psi(x, y, z, t)$ may be any component of either the Electric field $\mathbf{E}$ or the Magnetic field $\mathbf{H}$. The individual field components are not linear momenta, but nor do they exist in isolation. Electromagnetic waves involve both Electric and Magnetic fields and there is a linear momentum density, $\overrightarrow{\rho}_p = \mathbf{S}/c^2$, where the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ is aligned with the wave vector, $\mathbf{k}$ (which by definition points in the direction of propagation). The field lines of the wave vector trace out well defined trajectories at the ray velocity $v_{ray} = c$ (in vacuo) [16, 17], and the linear momentum carried by the Electromagnetic wave propagates along these trajectories at the characteristic velocity.

Any luminal wave theory, linear or nonlinear, has a wave vector pointing in the direction of propagation, and once we have a wave vector, the wave trajectories description works as in Electromagnetics.

5.4 Closed Wave Systems

Whether we consider a subatomic particle or some macroscopic object, it is a basic premise that the energy that constitutes a persistent subluminally moving system must remain in the same general vicinity as the object. From the perspective of a luminal wave model where the energy is moving at c, any trajectory of the wave vector will remain bound to the system because any wave trajectory that leaves the system bleeds energy from it. Therefore, when considering luminal wave models for matter, we can restrict our attention to closed trajectory systems. The trajectories may or may not form closed loops, but any given trajectory will remain within some finite distance of the centre of inertia of the system.

5.5 Towards Coordinate Transformations

In order for wave trajectories to remain bound to a subluminally moving centre of inertia they must be curved. Therefore, the unit wave vector for any given wave object in a closed system must be position dependent and may in general also be time dependent. Consequently, space functions that describe light flashes, where the unit wave vector is constant (see for example [18]-[20]), will generally be unsuitable for describing closed systems, so we cannot think that the massive particles are constructed of light flashes. Therefore, we now require the incremental momentum boosts to operate directly on the momentum densities.

Eqs. (1) - (7) can be rewritten in terms of momentum densities, however it is more convenient to preserve the notation by converting momentum densities into momenta as follows. Let the entire space be divided into small regions of dimension $\delta x = \delta y = \delta z = \delta l$, where δl is sufficiently small that any of the momentum densities, $\overrightarrow{\rho}_{pi}(x, y, z, t)$, can be considered constant within each region so that $\overrightarrow{\rho}_{pi}(x, y, z, t)\, \delta l^3$ is a linear momentum propagating at c in a definite direction in space. Introducing a new subscript, k, to label the regions, we write the linear momentum of the i^{th} field in the k^{th} region as $\mathbf{p}_{ik}(t) = \overrightarrow{\rho}_{pi}(\mathbf{r}_k, t)\, \delta l^3$, where $\mathbf{r}_k$ is the position vector to the centre of the k^{th} region. Since the space integral of the momentum boost must recover (8) for all possible light flashes, the incremental momentum boost operating on the $\mathbf{p}_{ik}$ can only be:

$$d\mathbf{p}_{ik} = \frac{p_{ik}}{m_e c} d\mathbf{P} \tag{22}$$

Where:

$$\mathbf{P} = \sum_k \sum_i \mathbf{p}_{ik}$$

$$m_e = \sum_k \sum_i m_{ik} = \frac{1}{c} \sum_k \sum_i |\overrightarrow{\rho}_{pi}(\mathbf{r}_k, t)|\, \delta l^3$$

and the rest goes through as before.

The rest system in Sect. 4 could be a particle or any macroscopic system that is comoving with the observer. The moving system's internal momenta, $\mathbf{p}_{ik}$, are related to the $\mathbf{p}_{ik0}$ by (19), with an additional k subscript inserted. The system's momentum is $\mathbf{P} = \gamma m_0 \mathbf{V}$, where the velocity of the centre of inertia of the wavegroup, $\mathbf{V}$, is simply the observed velocity of the system. The relative velocity we developed at the end of the last section, $\mathbf{v}_{rik} = \mathbf{v}_{ik} - \mathbf{V}$, describes the internal movements of the system as seen by an observer who considers it to be moving at $\mathbf{V}$.

Since internal movements obviously change in response to changes in the observed velocity, neither the shape nor the internal evolution of a subluminally moving wave system can be assumed to be velocity independent so that, in order to determine coordinate transformations, we must first calculate the impacts this has on rulers and clocks constructed from luminal wave energy.

However, before moving onto the analysis of length contraction and time dilation in luminal wave models let us contrast (22) with the Newtonian concept of a force field as applied to a point-like massive particle. According to the fourth basic principle, the force acting on an interacting field is, by definition, equal to its rate of change of momentum. Donev and Tashkova [20] have also developed this within a field variables approach to luminally propagating bivector fields. It might appear at first blush that:

$$\frac{d\mathbf{p}_{ik}}{dt} = \frac{p_{ik}}{m_e c} \frac{d\mathbf{P}}{dt} \tag{23}$$

and the left hand side of (23) should be interpreted as the force acting on the i^{th} wave object in the k^{th} region when the total externally applied force acting on the particle is $\mathbf{F} = d\mathbf{P}/dt$. Such a dynamic interpretation requires making unreasonable extraneous assumptions, including not least a uniform applied field. This is unnecessary for our analysis, for which (22) simply governs the relationship between systems in steady state conditions, before and after (but not necessarily during) some physical process that results in an incremental boost to the system's momentum. A one to one correspondence between the momentum densities of rest and moving systems is assumed, but without such an assumption no inherently relativistic structure would be possible because we could never equate a boost with a change of observer.

6 The Lorentz-Fitzgerald Contraction

This section shows that closed wave trajectory systems contract in the direction of motion. This is easily understood by considering the special case of a rest system where the wave vector is transverse to the direction to the centre of inertia so that the system evolves under rotations and any wave trajectory exists on the surface of a sphere. Such systems are of particular interest because the group of transformations in Special Relativity that preserves the linear momentum of a particle is the Little Group and the usual interpretation is that rest particles evolve under the action of members of the group of rotations [21].

Consider a system of concentric spherical surfaces constructed about the rest system's centre of inertia, which we shall assume is at the origin. Given the abovementioned condition, all rest system wave trajectories through a given point, $\mathbf{r}_{k0}$, lie instantaneously in the tangent plane at that point to the sphere of radius r_{k0}. Without loss of generality, let us consider the trajectories passing through a point in the xy plane where the tangent plane makes the angle θ_0 with the x-axis, as shown in the top left of Fig. 3. The wave momentum along a trajectory lying in this plane has components in the following form:

$$p_{x0} = p_0 \cos\theta_0 \cos\phi_0 \;;\; p_{y0} = p_0 \sin\theta_0 \cos\phi_0 \;;\; p_{z0} = p_0 \sin\phi_0$$

Where ϕ_0 is the angle the trajectory makes with the xy plane. Note that this is just the component form of any of the $\mathbf{p}_{ik0}$. The i and k subscripts can be omitted without ambiguity: p_{x0} means p_{ikx0} and so on. Using (19), the components of the corresponding wave momentum in the moving system are:

$$p_x = p_0 \gamma(\cos\theta_0 \cos\phi_0 + \beta) \;;\; p_y = p_{y0} \;;\; p_z = p_{z0}$$

The moving system momenta for different values of ϕ_0 are not coplanar. As shown in the top right of Figure 3, they lie on a conical surface whose vertex is at the origin of momentum coordinates, and whose base is the intersection of the plane at angle θ, where $\tan\theta = \tan\theta_0/\gamma$, with the moving system momentum distribution. This elliptical intersection is shown in the bottom right of Figure 3 .

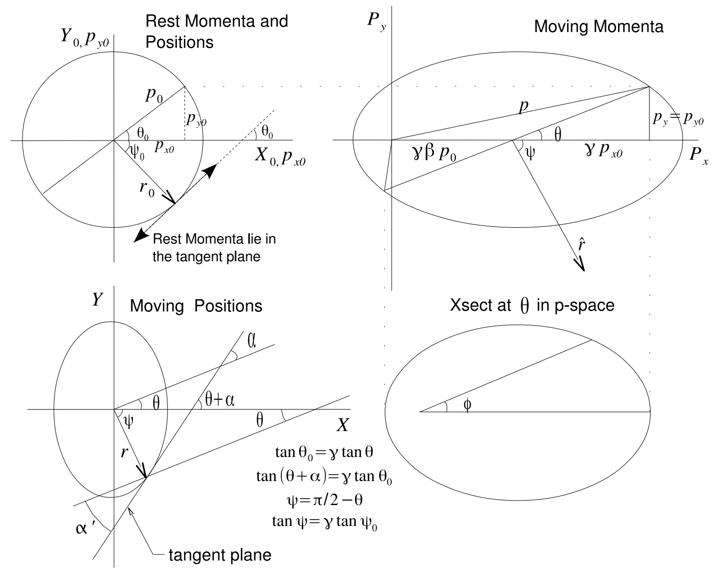

Figure 3: Momenta and Positions in Rest and Moving Luminal Wave Particle Models

The (total) velocity for each of these momenta has components $v_x = cp_x/p$; $v_y = cp_y/p$; $v_z = cp_z/p$ where, from (17) above, the moving system scalar momenta are:

$$p = \frac{p_0 + \gamma\beta p_x}{\gamma} = \frac{p_0}{\gamma}(1 + \gamma^2\beta(\cos\theta_0\cos\phi_0 + \beta)) \tag{24}$$

The group velocity is $V\hat{\mathbf{i}}$, so using (24) the relative velocity components are:

$$v_{\mathrm{r}x} = \frac{cp_x - pV}{p} = \frac{cp_0\cos\theta_0\cos\phi_0}{\gamma p}$$

$$v_{\mathrm{r}y} = \frac{cp_0\sin\theta_0\cos\phi_0}{p} \quad ; \quad v_{\mathrm{r}z} = \frac{cp_0\sin\phi_0}{p}$$

The ratio $v_{\mathrm{r}y}/v_{\mathrm{r}x} = \gamma\tan\theta_0$ is independent of ϕ_0 (and ϕ), so the velocities that lay in a given tangent plane in the rest system transform into relative velocities lying in a corresponding moving plane, tangent to the moving trajectory system[11]. Let α be the angle between the plane at θ and the tangent plane, as shown in the bottom left of Figure 3. The moving system tangent plane makes the angle $\alpha + \theta$ with the x-axis, where $\tan(\theta + \alpha) = v_{\mathrm{r}y}/v_{\mathrm{r}x} = \gamma\tan\theta_0 = \gamma^2\tan\theta$. Using the angle sum trigonometric relations we obtain:

$$\tan\alpha = \frac{\beta^2\sin\theta\cos\theta}{1 - \beta^2\cos^2\theta} \tag{25}$$

The set of all tangent planes defines the surface up to a scale factor. Due to rotational symmetry we can anticipate being able to write the equation describing this surface in the form $r = r(\psi)$, where ψ is the angle from the position vector to the x-axis. For any function $r(\psi)$ the angle between the tangent plane and the plane transverse to the radius vector is:

$$\tan\alpha' = \frac{1}{r}\frac{dr}{d\psi} \tag{26}$$

Consider as trial function the ellipsoid:

$$r(\psi) = \frac{\lambda}{\sqrt{1 - \beta^2\sin^2\psi}} \tag{27}$$

for which

$$\tan\alpha' = \frac{\beta^2\cos\psi\sin\psi}{1 - \beta^2\sin^2\psi} \tag{28}$$

independent of the scale parameter λ. With $\psi = \pi/2 - \theta$, this is identical to (25), which therefore describes an ellipsoid of revolution (27), such that the plane at θ is transverse to the position vector, $\mathbf{r}$, shown in the bottom left of Figure 3.

The scale factor, λ, is readily found by inspection. The moving system equatorial plane is the plane $x = Vt$ and $\psi = \pi/2$. The tangent plane at any point in the equatorial plane is parallel to the x-axis so the $d\mathbf{p}_{ik}$ at these points lie in the tangent plane. Therefore the equatorial tangent planes are not altered by the action of (22). Therefore the radius of a circumferential trajectory in the equatorial plane is invariant under the dimensional transformation (27), and $\lambda = r_0/\gamma$, where r_0 is the radius of the spherical surface in the rest system.

The result is that, for our rest observer, any wave trajectory in the moving system lies on the surface of an ellipsoid moving along the x-axis at speed V and of the form:

$$r(\psi) = \frac{r_0}{\gamma\sqrt{1 - \beta^2\sin^2\psi}} \tag{29}$$

The moving system wave trajectories are thus physically compressed by the factor γ in the direction of motion. Let us

[11] Recall that we showed in Section 4 that the relative velocity of any trajectory is rotated by the kinematic relation $\tan\vartheta = \gamma\tan\vartheta_0$, where ϑ is the angle the relative velocity makes with the x-axis. We now see the consequence of the Little Group for luminal wave particle models. Locally flat surfaces formed by sets of trajectories at a given point in the rest system transform into locally flat moving surfaces, rotated so that the tangent of the angle the moving surface makes with the x-axis is $\gamma\tan\theta_0$, where θ_0 is the angle the rest system surface makes with the x-axis.

now consider general wave trajectories that are not confined to the surfaces of spheres in the rest system. The analysis above shows that any short segment of the general trajectory is rotated so that the ratio of its dimensions parallel and transverse to V is suppressed by γ. Since this applies to every segment it applies to entire trajectories and since we have already identified specific trajectories whose transverse dimensions are invariant, the same scale factor applies to the general case.

Therefore, closed luminal wave trajectory systems are physically compressed by the factor γ in the direction of motion with the result that any macroscopic physical objects, including rulers, that are constructed entirely from luminal wave energy undergo the usual Lorentz-Fitzgerald length contraction.

7 Time Dilation

In this section it is shown that all internal processes in closed wave systems slow down according to $dt/dt_0 = 1/\gamma$. A fixed overall rate of spatiotemporal evolution can then be defined in the usual way by a 4-dimensional line element $c^2dt_0^2 - d\mathbf{x}_0^2 = c^2dt^2$ where t_0 and t are the rates of rest and moving clocks respectively and $d\mathbf{x}_0$ is traversed by the moving clock. The analysis is similar to the standard analysis of a light clock, but let us first explain the main idea qualitatively.

According to (7) and as illustrated in Fig. 2, the group velocity of a wave system is the result of spatial correlations amongst the directions of propagation $\hat{\mathbf{k}}_i(x, y, z, t)$ of internal momenta. As the group velocity approaches the characteristic velocity, the trajectories rotate towards the group velocity: $\hat{\mathbf{k}} \to \hat{\mathbf{V}}$. But if all the trajectories of a wave system were exactly parallel as it moved through an observer's reference frame the spatial configuration of the system would not change and the observer would conclude that nothing happens in the inertial frame of the group. While spatial correlations amongst trajectories are essential for the movement through space, internal evolution requires decorrelations. There is a direct tradeoff between external evolution in space and internal evolution in time, so time dilation is the direct consequence of constructing variable speed entities from fixed speed primitives.

The same kind of tradeoff is also found in the Dirac Equation. Consider the equation for the time dependence of the velocity operator in the Heisenberg representation of the Dirac theory [22]:

$$\overrightarrow{\alpha}(\mathrm{t}) = (\overrightarrow{\alpha}(0) - \frac{\mathbf{p}}{\mathrm{H}}) \exp(-2i\mathrm{H}t) + \frac{\mathbf{p}}{\mathrm{H}} \tag{30}$$

Where $\mathbf{p}$ and H are both constants, $c = 1$ and the group velocity is $\mathbf{p}/\mathrm{H} = \mathbf{v}_g = const.$. The first term on the right is routinely interpreted to represent the internal movements of the electron, the 'Zitterbewegung'. Its quantum mechanical expectation is:

$$< \Psi \mid (\overrightarrow{\alpha}(0) - \mathbf{v}_g) \mid \Psi > / < \Psi \mid \Psi >$$

which (noting that $\overrightarrow{\alpha}$ has real eigenvalues) varies with v_g as $\sqrt{1 - v_g^2}$. The zitterbewegung slows down by a Lorentz factor as the group velocity increases.

We shall now show that internal processes in luminal wave systems slow down according to $dt/dt_0 = 1/\gamma$.

With respect to the rest system's wave trajectory system, consider any closed trajectory formed by n segments, where the i^{th} segment has length l_{i0} and makes the angle θ_{i0} with the x-axis. The speed on all segments is $v_0 = c$ so the period around the closed trajectory is $T_0 = \frac{1}{c}\sum_{i=1}^{n} l_{i0}$, where T_0 is the time elapsed on a clock in the rest frame to traverse the trajectory in the rest system. Lengths in the rest system may be written in component form such that:

$$l_{i0}^2 = l_{ix0}^2 + l_{iy0}^2 + l_{iz0}^2$$

Let the trajectory system now move in the x-direction at speed V. Given the length contraction, x-components contract by the factor γ and the corresponding relationship is:

$$l_i^2 = \frac{l_{ix0}^2}{\gamma^2} + l_{iy0}^2 + l_{iz0}^2$$

It is readily shown that:

$$l_i^2 = l_{i0}^2(1 - \beta^2 \cos^2 \theta_{i0}) \tag{31}$$

The moving and rest system angles are related by $\tan \theta_i = \gamma \tan \theta_{i0}$, from which we get:

$$\frac{\cos \theta_i}{\cos \theta_{i0}} = \sqrt{1 - \beta^2 \sin^2 \theta_i} \tag{32}$$

The relative velocity on the i^{th} segment in the moving system, $v_{\mathrm{r}i}$, is constrained by:

$$(v_{\mathrm{r}i} \cos \theta_i + V)^2 + v_{\mathrm{r}i}^2 \sin^2 \theta_i = c^2 \tag{33}$$

Which leads to: $v_{\mathrm{r}i} + V \cos \theta_i = c\sqrt{1 - \beta^2 \sin^2 \theta_i}$. From which, using (32):

$$v_{\mathrm{r}i} = \frac{\cos \theta_i (c - V \cos \theta_{i0})}{\cos \theta_{i0}} = \frac{l_{ix0} c (1 - \beta \cos \theta_{i0})}{\gamma l_i \cos \theta_{i0}}$$

The time taken to traverse the i^{th} segment in the moving system is $l_i/v_{\mathrm{r}i} = l_i^2/v_{\mathrm{r}i} l_i$, so, using (31), we may write the period elapsed on clocks in the rest system for traversals around the Lorentz contracted closed trajectory of the moving system as follows:

$$T_0^V = \sum_{i=1}^{n} l_i^2/v_{\mathrm{r}i} l_i = \sum_{i=1}^{n} \frac{\gamma l_{i0}^2 \cos \theta_{i0} (1 - \beta^2 \cos^2 \theta_{i0})}{l_{ix0} c (1 - \beta \cos \theta_{i0})}$$

$$= \frac{\gamma}{c} \sum_{i=1}^{n} l_{i0}(1 + \beta \cos \theta_{i0})$$

Since $\sum_i l_{i0} \cos \theta_{i0} = 0$ it follows that $T_0^V = \gamma T_0$. It might be argued that trajectories need not form closed loops, but a path that crosses a given plane transverse to $\mathbf{V}$ must eventually either recross the same plane or become confined to a smaller region, in which it must either routinely recross a transverse plane or become confined to an even smaller region and so on. In steady state, the trajectories can only be transverse or regularly recross a transverse plane. The analysis above also covers open paths between points in the same transverse plane, for which the condition $\sum_i l_{i0} \cos\theta_{i0} = 0$ is also fullfilled. The time between such crossing points dilates by γ. We conclude that the internal processes of a luminal wave system slow down by the factor γ. The argument from internal processes to real world clocks is well established [23], and tested [24, 25, 26], so moving clocks will run slow according to the usual relation $dt/dt_0 = 1/\gamma$.

8 Coordinate Transformations

We have shown length contraction and time dilation as physical effects in luminal wave models subject to the basic mechanics Eqs. (1) - (7) and the incremental momentum boost generator (22). The analyses were constructed from the perspective of a single observer so the principle of relativity, covariance, coordinate independence, and coordinate transformations were *all* irrelevant.

Let us now focus on the question of how these physical phenomena of length contraction and time dilation constrain the coordinate transformations. Selleri has studied this question in some detail [13, 14]. He considered three assumptions, namely: length contraction, time dilation and constancy of the 2-way velocity of light. He showed that any two of these assumptions both implies the third and constrains the coordinate transformations between a preferred rest frame, $S_0 = (x_0, y_0, z_0, t_0)$ and a frame $S = (x, y, z, t)$ in standard configuration moving with speed v to the following form:

$$x = \frac{(x_0 - \beta c t_0)}{\sqrt{1-\beta^2}}$$

$$y = y_0 \qquad ; \qquad z = z_0$$

$$t = \sqrt{1-\beta^2}\; t_0 + e_1(x_0 - \beta c t_0)$$

Where $\beta = v/c$ and e_1 is a synchronisation parameter. Setting $e_1 = -\beta/(c\sqrt{1-\beta^2})$ corresponds to the usual Einstein clock synchronisation convention and reduces this to the Lorentz Transformation. Our coordinate transformations are therefore Lorentz Transformations and the relativity principle and the constant speed of light for all observers are therefore results, not postulates. It is also now finally clear that the wave inertia changes we have analysed are frequency changes corresponding to the relativistic Doppler shift, as opposed to, say, amplitude changes.

8.1 Other Synchronisation Protocols

Selleri also discusses alternative clock synchronisation protocols, especially the case $e_1 = 0$ which corresponds to using Einstein synchronisation in a preferred rest frame, and setting clocks in the moving frame to coincide with nearby clocks in the rest frame at $t = 0$. Both sets of observers agree that clocks in the moving system run slow, and they also agree on the simultaneity of spatially separated events. The transformations in this case, known as the inertial transformations, were first found by Tangherlini [27]. The empirical consequences of inertial transformations have been shown to comply with experimental evidence in a wide variety of situations [28]. As far as the present article is concerned, Appendix 2 derives (8) from the relativistic Doppler shift and aberration results, which apply equally well to inertial transformations [15], and therefore so do the structural consequences developed above.

Selleri and others have advanced various arguments in favour of absolute simultaneity [29] - [34] (notably a simplified analysis on the rotating platform), but nothing that questions the Lorentz form within the domain of inertial frames. Inertial transformations do not preserve the line element, $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$, the physical laws are frame dependent, the inverse transformation is different, the relative velocity of the origin of S as seen by S_0 does not equal the relative velocity of S_0 as seen by S and the inertial transformations do not form a group [14]. In short, they fail to deliver elegant and simple analysis in most physical situations.

The conventional nature of the Einstein protocol has, of course, always been stipulated in relativity theory and what Selleri has in fact shown is that, like the choice between Cartesian and Spherical coordinates, the choice of a clock synchronisation protocol really is only a matter of convenience. Provided they use it consistently, physicists solving problems on a rotating platform and engineers developing GPS satellite networks (which use an inertial clock synchronisation protocol) can use whatever protocol is most effective.

The self-evident fact remains that the events that happen in the world cannot depend on the coordinate systems we use to describe them. Coordinate independence is one of the most powerful practical tools for the development of new physics. Other coordinate transformations may be empirically adequate, but special status is rightly afforded to Lorentz Transformations on the basis of symmetry and utility, not uniqueness, and what we have shown is that their 'natural habitat' is field theory.

8.2 Objective Simultaneity and the Preferred Frame

An immediate consequence of the Einstein synchronisation protocol is that observers in relative motion find themselves in disagreement over intrinsically objective facts such as the rates of their respective clocks and the temporal ordering of spacelike separated events.

Philosophical relativism sought to leave these conflicts unresolved on the basis, ultimately, that a preferred frame cannot be observed. This approach induces numerous paradoxes that have been criticised for over a century [35]. More recently, Hardy [32] and Percival [33, 34] have each shown that relativity of simultaneity when combined with quantum nonlocality leads to more than just conflicts between observers. It leads to manifest contradictions for individual observers.

Percival's double Bell paradox, for example, considers two EPR/Bell experiments in relative motion. According to relativity of simultaneity, a temporal loop can be constructed by using the measurement results in one arm of each experiment to select the measurement axis in the corresponding arm of the other experiment. Given the quantum predictions for individual EPR/Bell experiments, he showed that an observable measurement result is, on at least some occasions, inverted by the loop becoming equal to its own opposite which is a manifest contradiction. Therefore, either the quantum predictions are incorrect or relativity of simultaneity is invalid.

The long standing loopholes [36] in EPR experiments are rapidly closing, for example [37], but the evidence is perhaps still not quite crystal clear and it may still be possible to assert that the quantum predictions are "wrong" when they correctly predict these experimental outcomes.

Alternatively, we can simply admit what good sense always demanded: When two observers disagree regarding the rates of clocks or the temporal order of spacelike separated events, only one of them can be right. We then require two concepts of simultaneity, apparent and objective. Apparent simultaneity is what appears to observers using a given clock synchronisation protocol. Provided the protocol corresponds to a definite value for the synchronisation parameter, e_1, apparent simultaneity is sufficient for making predictions. Relativistic simultaneity is just the apparent simultaneity for observers using the Einstein protocol but there is no need to assert the truth value of this clock synchronisation protocol.

As far as objective simultaneity is concerned, the analysis above shows that motion induces objective changes in clocks and rulers that are constructed entirely from luminal waves, whilst we shall see in the next Section that nonluminal structures can effectively be ruled out. The luminal wave interpretation of Lorentz Invariance also allows us to determine an observer's velocity relative either to the medium in which the wave energy propagates or to the universe as a whole, which in turn allows us to define simultaneity objectively.

In a wave theory, the nett observed Doppler shift for a given source and detector depends only on the relative velocity and the direction to the source. We cannot isolate the detector velocity. However, with a large number of sources lying in different directions whose individual masses and conditions of motion are independent of the direction in space, we can determine the detector velocity relative to the centre of mass of the group as a whole. Similarly, measurements on an *a priori* isotropic radiation bath are sufficient [38] to determine the detector velocity relative to the rest frame of the bath, as defined in Section 4.

As discussed in [12] and references therein, two important cases have already been studied, namely the anisotropies of (1) the Cosmic Microwave Background Radiation (CMBR) [38, 39], which gives the Earth's velocity relative to the medium and (2) the angular number density of observable astronomical objects [40], which gives the Earth's velocity relative to the rest of the universe. In both cases, an identical velocity dipole of magnitude ~ 350 Km/Sec is observed! It is anticipated that future observations on other isotropic radiation baths will show the same anisotropy and the same velocity dipole. Variations in the average red shift of distant galaxies as a function of the direction in space constitute a further example that can be tested in the future to confirm this prediction. Note that these results are at odds with the relativist interpretation.

On the wave interpretation, the momentum density distribution of any wave system whose centre of inertia has zero velocity relative to the medium "really" has no bias in any given direction, so that we can safely state that clocks in this condition of motion "really" do run faster, rulers "really" are longer and so on. This preferred frame is a necessary consequence of the analysis and it provides the essential empirical basis for asserting at last that absolute simultaneity coincides with the Einstein simultaneity of observers at rest in the CMBR frame. The wave interpretation presented here has therefore eliminated all the paradoxes associated with Special Relativity without sacrificing any of the practical benefits of Lorentz symmetry, whilst also covering a wider range of observables.

9 Non-luminal Structures

It is of course possible that wave propagation slows down or stops altogether under interaction, so the wave energy is transformed into some ill-defined notion of 'substance'. Nothing prevents applying the same basic mechanics principles to such non-luminal structures, however once we introduce entities that do not move at c, an immediate casualty is the work integral connection between momentum and energy. We would have no choice but to re-define inertia as being fundamentally velocity dependent.

Such a flexible approach to so pivotal a definition might raise eyebrows if it were not for the fact that this particular step is an integral part of Special Relativity. So, let us assume that we could somehow make sense of the relativistic inertia in its own right, as we have done in this Article but on some other grounds that are also independent of Special Relativity.

As far as the structure of particles is concerned, without the concept of internal movements it would not seem possible to provide any account of internal processes (such as muon decay for example). Likewise, the fact that the massive particles possess angular momentum implies the existence of internal movements[12]. Let us consider internal movements at speeds other than c. To illustrate the difficulties this causes, we shall also assume that we can somehow produce Lorentz contracted moving system trajectories on other grounds that are also independent of Special Relativity.

We must still use (33), with v_i^2 replacing c^2 on the RHS, to connect the total and relative velocities on the i^{th} segment (as both are referred to the same observer). If v_i were the same in the moving and rest systems, then clearly the periods would not dilate by γ, and yet we know that for any physical system, not just luminal systems, periods must dilate by γ under Lorentz Transformations.

The resolution is most easily seen from Special Relativity. If the total speed, v_{i0}, on the i^{th} segment as seen by a comoving observer is such that $v_{i0} \neq c$, then for observers in other frames, $v_i \neq v_{i0}$ and must in general be calculated according to the relativistic composition of velocities:

$$\mathbf{v}_i = \frac{\mathbf{V} + \mathbf{v}_{i0\parallel} + \sqrt{1-\beta^2}\mathbf{v}_{i0\perp}}{1 + \frac{\mathbf{V}\cdot\mathbf{v}_{i0}}{c^2}}$$

[12] The quantisation of angular momenta is also readily explicable as a wave phenomenon [12, 20].

Now as we Lorentz boost a particle in the frame of a single observer, there are two possibilities. If $v_{i0} = c$, then $v_i = c$ for all i independent of the condition of motion of the particle, and structural models incorporating length contraction and the relativistic momentum are readily available. Sect. 8 showed that these phenomena imply Lorentz Transformations, whose elegance and simplicity therefore has a coherent explanation based on the very definition of momentum as inertia times velocity, $p = mc$.

Alternatively, if $v_{i0} \neq c$ the total velocities of internal movements, $\mathbf{v}_i$, must depend on both the particle velocity and the orientation of individual segments in the above complicated manner. Why? The elegance and simplicity of Lorentz Transformations then has at its very foundations an implausibly inelegant, complex structure. We are left reasoning in a circle from Lorentz Transformations to the composition of velocities to the proposition that such complex structures are necessary as the basis for our simple coordinate transformations and we have no physical basis for either length contraction or the relativistic momentum. Ockham's razor insists that we reject nonluminal structures. We therefore conclude that, in the comoving frame, Lorentz invariant structural models of the massive particles will have internal movements at, and only at, c.

10 Does Local Action Imply Retarded Interaction?

Local action is the single most basic, self-evident principle in Physics - interaction requires colocation. Both Newton and Einstein agreed. This section considers the logic of interaction at a distance, subject to local action, but from a pure field perspective where 'mass energy' propagates luminally.

In Classical Physics it was taken for granted that matter emits field, leading to the idea that the far fields of a particle must propagate away from it at c. It then follows that long-range interactions between particles are retarded and the unavoidable consequence is that there can be no causal relations between space-like separated events. On the other hand, Quantum Mechanics predicts instant causal correlations at a distance and experiments replicate these predictions [41] - [43]. However, if matter and field are one and the same, as Einstein suggested, then the idea that matter emits field is meaningless and we need to consider whether or not the far fields propagate away from the centre of inertia in a pure field particle model.

Section 6 considered a rest system that evolves under rotations, corresponding to SRT's Little group. Note that the radius of the rest system sphere was not relevant - the analysis applies to any radius, and there is no good reason, neither in our analysis nor in Special Relativity, to distinguish between the near and far fields of a particle. The distinction in Electromagnetics between the 'attached' field [44] and the 'body' of the particle is arguably incompatible with Special Relativity because it implicitly introduces (radial) field movements that contravene the Little Group.

Consistent with Einstein's view that Special Relativity renders the division into matter and field 'artificial', our luminal wave structure implies that particles are unbounded with far fields that propagate transverse to the radius[13] rather than radially away from a 'body'[14]. There is then no good reason to presume that local action implies retarded interaction.

The long range interaction between two particles, A and B, depends on the colocation of their respective fields. It is an integral over all space that is dominated by terms close to the two centres, but any far fields of A that become colocated with the B particle's centre of inertia did not travel there from A's centre of inertia. They are part of an extended wave system that is comoving, as a whole, with the A centre of inertia so one might anticipate that the direct impact of A's far fields on the observed location of the B particle would be instantaneous, whilst only the reaction impact on the observed location of the A particle would be retarded.

However, it is more apposite simply to observe that field theory problems are usually formulated and solved on whole regions evolving subject to local action at all points in parallel. The idea of a local realist wave ontology is inherently Lorentz invariant, but waves are inherently distributed. They run on correlations at a distance sustained by strictly local actions. Distributed interactions between distributed waves can have distributed impacts, occurring simultaneously in different places. Waves exemplify Redhead's conclusion that ontological locality does not rule out instant relations between observables [45]. Trajectories in local realist wave systems display entanglement as shown in [16], where it was found that the Helmholtz equation contains Bohmian mechanics' nonlocal quantum potential within it. The essential consequence is that quantum nonlocality and entanglement might be interpreted as locally realistic wave phenomena. With specific reference to the EPR paradox [46], the Bell Inequalities [47] depend on a causality analysis that uses light cones emanating from point events [48], presuming a one to one correspondence with point-like 'beables' [49], but for inherently distributed systems like waves neither beables nor events can be presumed to be point-like.

11 Discussion

Unlike Electromagnetics, nothing prevents the simple method used here from applying to the fermions. A wide range of candidate models for the massive particles, in the form of subluminal soliton solutions found in typically nonlinear field

[13] As is also consistent with Electromagnetics' radial force field because $\mathbf{E}$ and $\mathbf{H}$ are each transverse to the momentum density $\mathbf{S}/c^2$, whilst $\mathbf{H}$ fields cancel in the rest particle due to balanced movements.

[14] Note that since massive particles have finite energy, the volume integral of the field energy density must not diverge as $r \to \infty$. The $1/r^2$ long range force fields for the charged particles imply a $1/r^4$ energy density asymptote for both charged and neutral particles in luminal wave models [12]. The energy density integral does not then diverge as $r \to \infty$ so finite but unbounded luminal wave structures are compatible with the usual basic physics. They appear as pointlike particles because the field energy is highly concentrated near the centre. For example, according to a $1/r^4$ energy density asymptote the maximum energy density for a particle with the mass of an electron, at the radius $r \sim 4 \times 10^{-13} m$, is $\sim 400,000$ times greater than that at a radius of 0.1 Angstrom unit.

theories, have been reported in the literature. The analyses in Sects. 2 - 7 show that Lorentz invariance is the consequence of constructing subluminally moving systems from fields that are constrained to propagate luminally. The appearance of Lorentz invariance in so many disparate field models is therefore no coincidence as they are all subject to the same basic kinematic constraints.

While the constraints are simple, the structures of soliton solutions are generally not simple. For example, evolution under rotations does not imply spherical symmetry and nor does it imply that the particle rotates as a whole in a simple manner, like a solid ball. Due to the kinematic constraint, trajectories at different radii necessarily evolve at different angular rates and, similarly, wave trajectories at various points on the same spherical surface in the rest system generally rotate about different axes.

12 Conclusions

This Article has developed a particularly simple hypothesis: Energy-momentum propagates at c. It has shown why subluminally moving physical systems, including observers' measuring devices, then display time dilation and length contraction, so that an underlying luminal wave reality, although objective, presents a Lorentz covariant "spacetime" to its observers. Neither the Relativity Principle nor the invariance of the observed speed of light were assumed. These two cornerstones of STR were shown as results, not put in as postulates.

This 3D+t reality also comes equipped with an observable preferred frame that has been observed in practice in at least two independent ways, providing a natural definition of objective simultaneity. All the paradoxes formerly associated with STR's subjective notions of reality are thus removed, and, unlike STR, the proposed luminal wave interpretation of Lorentz invariance is consistent with all the relevant facts.

Although the Lorentz invariance of luminal wave systems was perhaps already familiar, the basic mechanics underlying Lorentz symmetry remained unnoticed for over a century. The discovery of this direct link between wave systems and Lorentz Invariance has wide ranging implications for the interpretation and unification of modern physics.

Rather than replacing Newtonian Mechanics, Einstein's relativistic mechanics is the natural step accompanying the shift in our founding physical ideas from particle to wave concepts. The wave packet is reformed by giving explicit recognition to the conservation of momentum between wave components and particles, which can now be seen as widely distributed systems with instantly correlated far fields. Quantum nonlocality can be understood within this framework whilst general covariance is readily incorporated, conceptually and analytically, with a refractive medium approach to gravity [12] that produces the relevant phenomena without the raft of problems flowing from the usual field equations.

Hopefully, this article has highlighted the absence of any good reason to presume that any non-propagative form of mass-energy exists. It's not so much the introduction of a new hypothesis, as the removal of an old one - the idea of matter as a distinct ontological class in its own right.

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Appendix 1

Consider a constant momentum density $\vec{\rho}_{p\mathrm{i}}$ in a region of transverse crossectional area A and length l_i. The total momentum is $\mathbf{p}_\mathrm{i} = Al_\mathrm{i}\rho_{p\mathrm{i}}\hat{\mathbf{k}}$. Let this be normally incident on a mirror that is moving with velocity $\mathbf{v} = -v\hat{\mathbf{k}}$. Let the reflection begin at $t = 0$. It then ends at $\Delta t = l_\mathrm{i}/(c+v)$, after which there is a reflected wave with momentum density $\vec{\rho}_{p\mathrm{r}}$ that occupies a region of length $l_\mathrm{r} = (c-v)\Delta t$ and crossectional area A, so the momentum of the reflected light flash is $\mathbf{p}_\mathrm{r} = -Al_\mathrm{r}\rho_{p\mathrm{r}}\hat{\mathbf{k}}$.

During the reflection, the rates of change of momentum for the incident and reflected waves are $\dot{\mathbf{p}}_i = -(c+v)A\vec{\rho}_{p\mathrm{i}}$ and $\dot{\mathbf{p}}_\mathrm{r} = (c-v)A\vec{\rho}_{p\mathrm{r}}$ respectively, where a dot over a variable indicates the time differential. The total rate of change of momentum is:

$$\dot{\mathbf{p}} = \dot{\mathbf{p}}_\mathrm{i} + \dot{\mathbf{p}}_\mathrm{r} = -A((c+v)\rho_{p\mathrm{i}} + (c-v)\rho_{pr})\hat{\mathbf{k}}$$

where $\rho_{p\mathrm{i}} = |\vec{\rho_{p\mathrm{i}}}|$ and $\rho_{p\mathrm{r}} = |\vec{\rho_{p\mathrm{r}}}|$. As far as scalar momentum is concerned, for the incident wave $\dot{p}_\mathrm{i} = c\dot{m}_\mathrm{i} = -A(c+v)\rho_{p\mathrm{i}}$, for the reflected wave $\dot{p}_\mathrm{r} = c\dot{m}_\mathrm{r} = A(c-v)\rho_{pr}$ and the total is:

$$\dot{p} = c\dot{m} = c\dot{m}_\mathrm{r} + c\dot{m}_\mathrm{i} = A((c-v)\rho_{pr} - (c+v)\rho_{p\mathrm{i}})$$

The work done by the mirror on the incident and reflected waves is: $\int \dot{\mathbf{p}}_\mathrm{i} \cdot d\mathbf{s}_\mathrm{i} = -\int_0^{\Delta t} A(c+v)\rho_{p\mathrm{i}} \; cdt$ and $\int \dot{\mathbf{p}}_\mathrm{r} \cdot d\mathbf{s}_\mathrm{r} = \int_0^{\Delta t} A(c-v)\rho_{p\mathrm{r}} \; cdt$ respectively, where $d\mathbf{s}_\mathrm{i}$ and $d\mathbf{s}_\mathrm{r}$ are the incremental movements of the incident and reflected waves, in the directions $\hat{\mathbf{k}}$ and $-\hat{\mathbf{k}}$ respectively. The total work done is just $W = \int_0^{\Delta t} c\dot{m}cdt = (m_\mathrm{r} - m_\mathrm{i})c^2$.

The energy change of the light flash is of course equal and opposite to the work done by the radiation pressure force on the mirror, so $(m_\mathrm{r} - m_\mathrm{i})c^2 = -(-\dot{\mathbf{p}})(-v)\Delta t$, and it is easily shown that $p_\mathrm{r}/p_\mathrm{i} = (c+v)/(c-v)$, from which we may infer the momentum shift factor for light emitted by a source moving towards an observer as $\sqrt{(c+v)/(c-v)}$, in agreement with the usual relativistic doppler shift.

Appendix 2

With respect to the system of light flashes in Subsect. 2.2, let us impose the condition in some inertial frame:

$$\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$$

The momentum of the i^{th} light flash, referred to this frame, is then:

$$\mathbf{p}_{i0} = p_{i0}(\cos\theta_{i0}\hat{\mathbf{i}} + \sin\theta_{i0}\cos\phi_{i0}\hat{\mathbf{j}} + \sin\theta_{i0}\sin\phi_{i0}\hat{\mathbf{k}})$$

Where θ_{i0} is the angle with the x-axis and $\sum_i p_{i0}\cos\theta_{i0} = \sum_i p_{i0}\sin\theta_{i0}\cos\phi_{i0} = \sum_i p_{i0}\sin\theta_{i0}\sin\phi_{i0} = 0$

Let an observer move relative to this frame with velocity $\mathbf{v} = -\beta c\hat{\mathbf{i}}$. Since $p_i/p_{i0} = f_i/f_{i0}$, the standard relativistic doppler shift and aberration formulae (with the observer moving towards the source at speed v) give, respectively:

$$p_i = p_{i0}\gamma(1 + \frac{v}{c}\cos\theta_{i0})$$

and

$$\cos\theta_i = \frac{\cos\theta_{i0} + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta_{i0}}$$

Note that the same result also holds for non-monochromatic light flashes. The scalar momentum of the i^{th} flash in the observer frame is:

$$p_i = p_{i0}\gamma(1 + \beta\cos\theta_{i0})$$

Summing over i, the total energy is:

$$m_e c^2 = c\sum_i p_i = \gamma c \sum_i p_{i0} = \gamma m_0 c^2$$

Where m_e and m_0 are as defined in subsection 2.2 and Section 3 respectively. Noting that $p_{xi} = p_i\cos\theta_i$, the (vector) momentum of the i^{th} flash is:

$$\mathbf{p}_i = p_{i0}(\gamma(\beta + \cos\theta_{i0})\hat{\mathbf{i}} + \sin\theta_{i0}\cos\phi_{i0}\hat{\mathbf{j}} + \sin\theta_{i0}\sin\phi_{i0}\hat{\mathbf{k}})$$

Summing over i, the total momentum is:

$$\mathbf{P} = \sum_i \mathbf{p}_i = \gamma\beta\sum_i p_{i0}\hat{\mathbf{i}}$$

Note that this is the relativistic momentum equation. Differentiating each of the two previous equations with respect to β:

$$\frac{d\mathbf{p}_i}{d\beta} = \gamma^2 p_i\hat{\mathbf{i}} \qquad ; \qquad \frac{d\mathbf{P}}{d\beta} = \gamma^3\sum_i p_{i0}\hat{\mathbf{i}} = \gamma^2 m_e c\hat{\mathbf{i}}$$

So that:

$$\frac{d\mathbf{p}_i}{d\beta} = \frac{d\mathbf{P}}{d\beta}\frac{p_i}{\sum_j p_j} = \frac{d\mathbf{P}}{d\beta}\frac{p_i}{m_e c}$$

Finally, since the above expressions for $\mathbf{p}_i$ and $\mathbf{P}$ are functions of β alone, the incremental changes can be written as:

$$d\mathbf{p}_i = \frac{d\mathbf{p}_i}{d\beta}d\beta \qquad ; \qquad d\mathbf{P} = \frac{d\mathbf{P}}{d\beta}d\beta$$

Upon which:

$$d\mathbf{p}_i = \frac{p_i}{m_e c}d\mathbf{P}$$

Therefore (8) holds for a collinear incremental boost. For transverse boosts, consider as initial condition a system whose centre of inertia is moving in the y-direction at speed V, so $m_e = \gamma(V)m_0$. We may repeat the above analysis for an observer moving at speed v_x in the x-direction with $\sum_i p_{i0}\sin\theta_{i0}\cos\phi_{i0} \neq 0$ and get the result for an incremental transverse boost:

$$d\mathbf{p}_i = \frac{p_i}{\lim_{v_x\to 0}(\gamma(v_x)m_e c)}d\mathbf{P} = \frac{p_i}{m_e c}d\mathbf{P}$$

So, (8) holds for an incremental transverse boost. In SRT, the general boost decomposes into a collinear boost, a transverse boost and a rotation (a Thomas precession). As the latter has no impact on linear momenta,

(8) is generally valid for incremental boosts of systems of luminal wave momenta in SRT.