

# Relativity and the Luminal Structure of Matter

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## Abstract

Special Relativity implies definite structural constraints on the massive particles. It is shown from the basic physics of luminal waves of any kind that multi-component wave systems conform to the usual relativistic mechanics for massive particles, suggesting further consideration of luminal wave soliton models. The usual length contraction and time dilation phenomena are found in an important subset of such models, leading to the conclusion that internal movements referred to the comoving frame will be luminal in any Lorentz Invariant particle model.

La relativité spéciale implique des contraintes structurelles sur les particules massives. On montre ici, à partir de la physique de base des ondes électromagnétiques de tout genre, que les systèmes ondulatoires à plusieurs composants se conforment à la mécanique relativiste habituelle des particules massives, ce qui suggère de procéder à l'examen et à l'application de modèles solitoniques. Puisque, dans un sous-ensemble important de ces modèles, l'on retrouve tous les phénomènes habituels de la contraction des longueurs et de la dilatation du temps, on est amené à conclure que les mouvements internes, dans un référentiel inertiel approprié, doivent présenter un comportement de type électromagnétique dans tous les modèles physiques des particules relativistes.

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## 1 Introduction

*“But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created.”* - Einstein & Infeld [1].

Relativistic wave equations, especially the d'Alembert, Helmholtz and Dirac [2] equations, are indispensable to Modern Physics. For example, the nonrelativistic Schroedinger wave equation is contained in the Dirac Equation as the low velocity, no spin limit. These relativistic equations either feature propagation at the characteristic velocity,  $c$ , or, in the language of the operator formalism, a velocity operator of constant modulus equal to  $c$  [3]. There are also many Lorentz invariant classical field theories in the literature, including nonlinear theories with subluminal soliton solutions that serve as candidate models for the fermions. [4] - [10] are just a few to illustrate the diverse range of approaches.

In this article we shall consider the ordinary Newtonian mechanics of multi-component luminal wave systems, i.e. systems of waves that propagate at  $c$ . We shall adapt the Newtonian momentum equation,  $p = mv$ , for use with constant speed luminal waves. The entire Newtonian paradigm, including especially the Newtonian conceptions of inertia, momentum and energy and their conservation laws, will then be consistently applied to luminal wave systems leading to a general structural analysis of solitons that is inherently relativistic without asserting any principle of relativity.

This shows that the usual relativistic mechanics of matter can be interpreted as the Newtonian mechanics of subluminally moving systems constructed entirely from luminal waves. It is also shown that this interpretation is uniquely simple. The first, necessary step towards achieving Einstein's goal for a pure field physics, is therefore to recognise that energy is a propagative phenomenon. The relativistic concepts of inertia, momentum and energy then emerge naturally from their Newtonian counterparts.

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The proposed luminal wave ontology also provides new perspectives on many issues including the Dirac velocity operator, angular momentum quantisation, the structure of Electromagnetics, gravity [11], the existence of nonlocal relations between observables, and interference phenomena in matter beams.

In Section 2 we define the basic principles and identify a simple general relationship that governs the connection between inertial frames for systems of luminal wave momenta. Section 3 shows that the usual relativistic momentum equation for particles applies to multi-component wave systems. Section 4 derives the (forward) relativistic transformation of wave components in a form that is useful for analysing wave systems as a whole. As discussed in Section 5, the results extend to any kind of wave system provided a wave vector in the direction of propagation can be defined, linear momentum is locally conserved, and propagation is luminal. In particular, linear superposition of field amplitudes is not required so the method is applicable to nonlinear wave systems where subluminal soliton solutions are often found. For luminal waves the speed of propagation is, by definition, fixed and any luminal wave model of a subluminal massive particle is immediately subject to the constraint that when the speed of the particle changes, the speed of its constituent wave components does not. Sections 6 and 7 show that length contraction and time dilation are the consequences of this kinematic constraint and all the usual relativistic phenomena are thus attributable to the proposed luminal wave structure. Section 8 considers non-luminal structures, which are readily shown from Special Relativity to be implausibly complex, and therefore our main conclusion is that the relativistic phenomena strongly imply the luminal structure.

Section 9 addresses the question how the physical phenomena of length contraction and time dilation constrain the coordinate transformations. This has been analysed in detail by Selleri [12, 13] who showed that, subject only to the use of Einstein clock synchronisation, Lorentz Transformations follow directly from length contraction and time dilation. Hence, our coordinate transformations are Lorentz Transformations and both the relativity principle and the observer independence of  $c$  are results as opposed to postulates. Finally, Section 10 outlines the reasons why Special Relativity does not preclude nonlocal relations between observables in this pure field context.

## 2 Basic Newtonian Principles

Consider a source that simultaneously emits a set of  $N$  light flashes in various directions. The development in this section can be applied to any kind of light flashes, including individual photons, short segments of laser beams, or collimated beams in general, monochromatic or not. We require only that each flash propagates at  $c$ , carrying linear momentum in a well-defined direction in space.

Let the  $i^{th}$  light flash carry linear momentum  $\mathbf{p}_i$ . According to Newtonian principles, momentum equals inertia times velocity and we therefore define the wave inertia of the  $i^{th}$  light flash as  $m_i = p_i/c$ , where  $p_i = |\mathbf{p}_i|$  is the magnitude of the momentum of the  $i^{th}$  light flash. We shall refer to  $p_i$  as the scalar momentum:

$$p_i = m_i c \quad (1)$$

This article is essentially a consistent application of Newtonian mechanics, using (1) in place of the familiar  $p = mv$ , where the speed  $v$  is a variable. We stress that, *prima facie*, the inertia,  $m_i$ , of a wave propagating in a well-defined direction in space has nothing to do with the mass of a particle. However we use the symbol  $m_i$  because, unless they ALL propagate in the same direction, the total inertia of a set of  $N$  waves will be found to correspond to the usual (relativistic) particle mass. The time differential of (1) is:

$$\frac{dp_i}{dt} = c \frac{dm_i}{dt} \quad (2)$$

Having fixed the propagation speed,  $c$ , changes of the scalar momentum are thus associated with changes of the wave inertia. Such changes may be due to a change of observer or they may be physical changes due, for example, to the application of a force. It will become clear in Section 9 that the inertia changes we will be discussing throughout the article are in fact frequency changes.

Using (2) the work integral from  $p_i = 0$  to  $p_i$  is:

$$\int_0^{p_i} \frac{d\mathbf{p}_i}{dt} \cdot d\mathbf{s} = \int_0^{m_i} c \frac{dm_i}{dt} c dt = m_i c^2 \quad (3)$$

According to Newtonian principles, the work done equals the energy change, and so the energy of the  $i^{th}$  flash is:

$$E_i = m_i c^2 = cp_i \quad (4)$$

According to Newtonian principles, momentum is conserved and the total momentum of a set of  $N$  waves is given by the vector sum over their momenta:

$$\mathbf{P} = \sum_{i=1}^N \mathbf{p}_i \quad (5)$$

Suppressing the summation range henceforth, we write the total inertia as  $m_e = \sum_i m_i$ . The total energy of the set is then:

$$E = \sum_i cp_i = m_e c^2 \quad (6)$$

According to Newtonian principles, the velocity of the centre of inertia of a system of objects is the inertia weighted average velocity,  $\mathbf{V} = \sum_i m_i \mathbf{v}_i / \sum_i m_i$  whence:

$$\mathbf{V} = \frac{\sum_i \mathbf{p}_i}{m_e} \Rightarrow \mathbf{P} = m_e \mathbf{V} \quad (7)$$

For a relativistic analysis, these Newtonian Equations (1) - (7) must of course be good for any observer, however, since we intend *inter alia* to show it, we shall not assert any principle of relativity.

Let us consider incremental changes that affect our system of light flashes as a whole. For example, an incremental change in the condition of motion of the observer would at once alter all his observations of the  $\mathbf{p}_i$ . Similarly, a single observer considering light flashes emitted by otherwise identical sources that are in different conditions of motion will find different values for the  $\mathbf{p}_i$ . Since we may not assume these two cases are equivalent, let us restrict our attention to the latter and consider, specifically, two otherwise identical sources moving at velocities  $\mathbf{v}$  and  $\mathbf{v} + d\mathbf{v}$  in the inertial frame of a single inertial observer. We may write the momenta of the light flashes as  $\mathbf{p}_i$  and  $\mathbf{p}_i + d\mathbf{p}_i$  respectively and their totals as  $\mathbf{P}$  and  $\mathbf{P} + d\mathbf{P}$ . Clearly,  $d\mathbf{P} = \sum_i d\mathbf{p}_i$ .

We are interested in how the  $d\mathbf{p}_i$  are related to  $d\mathbf{P}$  in this scenario. There is a naive analogy to the case in Newtonian Mechanics of a force,  $\mathbf{F}$ , acting on a body of total mass  $M$  composed of constituents of mass  $m_i$ . We would write the force,  $\mathbf{f}_i$ , acting on the  $i^{th}$  constituent as:

$$\mathbf{f}_i = \frac{m_i}{M} \mathbf{F} \Rightarrow \frac{d\mathbf{p}_i}{dt} = \frac{m_i}{M} \frac{d\mathbf{P}}{dt}$$

This suggests using the Newtonian ansatz  $d\mathbf{p}_i = m_i d\mathbf{P} / \sum_i m_i$ , which we shall write in the form:

$$d\mathbf{p}_i = \frac{p_i}{m_e c} d\mathbf{P} \quad (8)$$

It is shown in the Appendix that (8) is valid in Special Relativity for luminal wave systems<sup>1</sup>. This equation governs the connection between inertial frames for systems of waves of any kind that propagate at  $c$ . Incremental changes to the total momentum of a multi-component wave system are distributed amongst components in proportion to their magnitudes, independent of orientation.

In order to avoid asserting the relativity principle, we shall not associate (8) with a change of observer. It will turn out to work relativistically, but for the present purposes it has only the restricted meaning of an incremental change  $d\mathbf{v}$  in the velocity of a light source, the result of which is to add  $d\mathbf{P}$  to the total wave momentum by adding wave momentum  $d\mathbf{p}_i$  to each of the  $N$  constituent waves<sup>2</sup>. We may now analyse multi-component luminal wave systems using the basic Newtonian principles above.

### 3 The Relativistic Momentum

In this section it is shown that the relationship between the total momentum of a luminal wave system,  $\mathbf{P}$ , and the velocity of its centre of inertia,  $\mathbf{V}$ , is the same as the usual relativistic momentum equation for particles. The incremental change in the scalar momentum  $p_i$  is given by the component of  $d\mathbf{p}_i$  parallel to  $\mathbf{p}_i$ , namely:

$$dp_i = d\mathbf{p}_i \cdot \frac{\mathbf{p}_i}{p_i}$$

<sup>1</sup>It is also easily deduced from two reasonable assumptions: 1)  $d\mathbf{p}_i$  must be linear in  $p_i$ , and 2) The members of a group must transform independently of each other.

<sup>2</sup>Note that we do not need to assume that  $d\mathbf{V} = d\mathbf{v}$

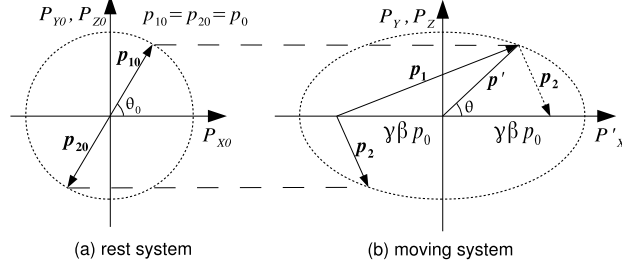


Figure 1: 2-component wave systems whose centres of inertia are (a) at rest (b) moving at speed  $V = \beta c$

Substituting (8) in this gives  $m_e c dp_i = \mathbf{p}_i \cdot d\mathbf{P}$ . Summing over the  $i$  and noting that  $\sum dp_i = c dm_e$ :

$$c^2 m_e dm_e = \mathbf{P} \cdot d\mathbf{P}$$

Integrating this we obtain the common expression for the invariance of the 4-momentum:

$$m_e^2 c^2 = P^2 + m_0^2 c^2 \quad (9)$$

Where  $m_0$  is the value of  $m_e$  for  $P = 0$ . Let  $\beta = |\mathbf{V}|/c$  as usual so  $\beta$  is a +ve real in the interval  $[0, 1]$ . The basic equations of relativistic mechanics,  $\mathbf{P} = \gamma m_0 \mathbf{V}$  and  $m_e = \gamma m_0$ , where  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ , follow immediately upon substituting (7).

## 4 Wave System Transformations in Momentum Space

In this section we show how individual wave components in a multi-component luminal wave system transform under the action of (8).

By analogy to the usual comoving frame for massive particles, let us define the rest frame of a compound wave system as the (unique) inertial frame for which the right hand side of (5) vanishes. This definition is convenient, but not essential. Given the definition, we now adopt the perspective of a single inertial observer who compares systems of light flashes emitted by two otherwise identical sources in different conditions of motion such that he considers one system's centre of inertia to be at rest, i.e.  $\mathbf{P} = 0$  in (5) and  $\mathbf{V} = 0$  in (7), and the other's to be moving at speed  $V$  in the  $x$ -direction, so that, from Section 3,  $P = \gamma m_0 V$ . We shall refer to these two systems of light flashes as the "rest system" and the "moving system" respectively. We shall use a 0 subscript to refer to rest system momenta, so  $\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$ . The analysis is expressed in momentum coordinates and it should be noted that we shall not need to say anything about spatial relations between wave components until Section 6.

The simplest case of a compound wave system where  $\mathbf{P}_0 = 0$  consists of 2 wave components of equal scalar momentum,  $p_{10} = p_{20} = p_0$ , propagating in opposite directions, as shown in Figure 1a. The moving system is shown in Figure 1b, where the wave momenta,  $\mathbf{p}_{10}$  and  $\mathbf{p}_{20}$ , have been modified in accordance with (8) so that the centre of inertia moves at speed  $V$  in the  $x$ -direction.

In Figure 1a,  $m_0 = (p_{10} + p_{20})/c = 2p_0/c$ . Recalling from Section 3 that  $m_e = \gamma m_0$ , the sum of scalar momenta in the moving system of Figure 1b is:

$$p_1 + p_2 = m_e c = 2\gamma p_0 \quad (10)$$

Whilst the total momentum,  $\mathbf{P} = m_e \mathbf{V}$  is the vector sum of momenta:

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P} = \frac{2\gamma p_0}{c} \mathbf{V} = 2\gamma\beta p_0 \hat{\mathbf{i}}$$

Consider the vector  $\mathbf{p}'$  in Figure 1b, where  $\mathbf{p}_1 = \mathbf{P}/2 + \mathbf{p}'$  and  $\mathbf{p}_2 = \mathbf{P}/2 - \mathbf{p}'$ . Using the law of cosines, its magnitude,  $p'$ , is such that:

$$p_1^2 = p'^2 + (\gamma\beta p_0)^2 + 2\gamma\beta p_0 p' \cos \theta \quad (11)$$

$$p_2^2 = p'^2 + (\gamma\beta p_0)^2 - 2\gamma\beta p_0 p' \cos \theta \quad (12)$$

Where  $\theta$  is the angle  $\mathbf{p}'$  makes with the  $X$ -axis. Upon eliminating  $p_1$  and  $p_2$  from (10)-(12) we find that  $p' = p'(\theta)$  is the ellipsoid:

$$p' = \frac{p_0}{\sqrt{1 - \beta^2 \cos^2 \theta}} \quad (13)$$

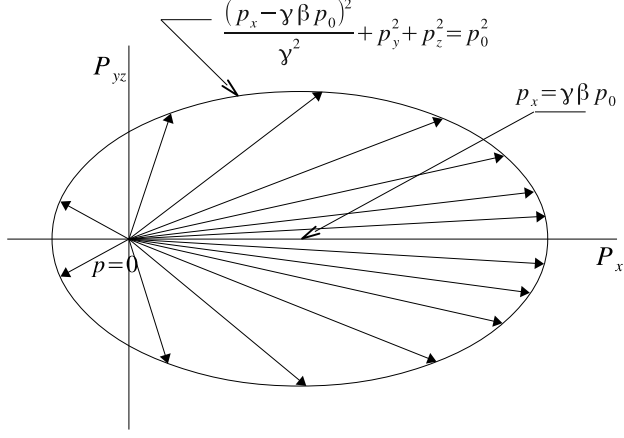


Figure 2: Individual momenta in an isotropic wave system modified such that  $V = \beta c$

Let us write the momenta in component form as  $(p_{ix}, p_{iy}, p_{iz})_{i=1,2}$ . In Cartesian coordinates (13) is then the ellipsoid  $(p'_{ix}/\gamma)^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2$ , where  $p'_{ix} = p_{ix} - \gamma\beta p_{i0}$ , so that the moving system momenta satisfy the following equation:

$$\left(\frac{p_{ix} - \gamma\beta p_{i0}}{\gamma}\right)^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2 \quad (14)$$

Which we have derived only for the case  $N=2$ , however this equation also covers the general case, as we shall now show. Let us consider as initial condition an arbitrary wave system, comprising a number  $N \geq 2$  of wave components of scalar momentum,  $p_{i0}$ , whose directions of propagation are distributed in space, such that  $\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$  and  $\sum_i p_{i0} = m_0 c$ . The rest system components are such that:

$$p_{ix0}^2 + p_{iy0}^2 + p_{iz0}^2 = p_{i0}^2 \quad (15)$$

The example for  $N = 2$  above suggests that after (8) acts on the set, bringing the total momentum to  $\mathbf{P} = \gamma m_0 V \hat{\mathbf{i}}$ , then (14) applies to the moving system momenta. Figure 2 shows the moving system momenta when all the rest system scalar momenta are the same, i.e.  $p_{i0} = p_0$  for all  $i$ . Differentiating (14) with respect to  $\beta$  using  $d\gamma/d\beta = \gamma^3\beta$  and  $d\gamma^{-1}/d\beta = -\gamma\beta$  leads to:

$$\frac{dp_{ix}}{d\beta} = \gamma(p_{i0} + \gamma\beta p_{ix}) \quad (16)$$

Expanding the first term in (14) and using  $\gamma^2\beta^2 = \gamma^2 - 1$  (twice) gives:

$$p_i = \frac{p_{i0} + \gamma\beta p_{ix}}{\gamma} \quad (17)$$

As  $P_x = \gamma m_0 V$  we also have  $dP_x = \gamma^3 m_0 dV$ , whence:

$$dp_{ix} = \frac{dp_{ix}}{dV} dV = \frac{dp_{ix}}{d\beta} \frac{dP_x}{\gamma^3 m_0 c} \quad (18)$$

Finally, substituting (16) and (17) in (18):

$$dp_{ix} = \frac{p_i}{\gamma m_0 c} dP_x$$

Which is the x-component of (8). Due to the choice of coordinates, the y and z components of momentum were unaffected, so the ellipsoidally modified distribution (14) is generated by the action of (8) on our arbitrary initial condition as expected. Comparing (14) and (15), the components of the moving system wave momenta are:

$$p_{ix} = \gamma(p_{ix0} + \beta p_{i0}) \quad , \quad p_{iy} = p_{iy0} \quad , \quad p_{iz} = p_{iz0} \quad (19)$$

It will be noted that these physical transformations due to changes in the condition of motion of a light source, are identical to Lorentz Transformations of wave momenta between different reference frames in standard configuration. However, as we are not asserting the Principle of Relativity there is no guarantee

(so far) that our analysis works relativistically, and (19) corresponds only to the forward transformations of wave momenta in relativity theory.

We can now calculate the relative velocity of the  $i^{th}$  light flash, which is to say its velocity relative to the centre of inertia of the system, which our observer considers to be moving at  $V$  in the x-direction. The total velocity of the  $i^{th}$  flash has components  $v_{ix} = cp_{ix}/p_i$ ,  $v_{iy} = cp_{iy0}/p_i$ , and  $v_{iz} = cp_{iz0}/p_i$ . Using  $\gamma^2\beta^2 = \gamma^2 - 1$  with (17) and (19), it is readily shown that the relative velocity,  $\mathbf{v}_{ri}$  has components<sup>3</sup>:

$$v_{rix} = v_{ix} - V = \frac{cp_{ix0}}{\gamma p_i} \quad ; \quad v_{riy} = v_{iy} = \frac{cp_{iy0}}{p_i} \quad ; \quad v_{riz} = v_{iz} = \frac{cp_{iz0}}{p_i}$$

Finally, if  $\mathbf{v}_{ri}$  makes the angle  $\vartheta_i$  with the X-axis, then  $\tan \vartheta_i = \sqrt{v_{riy}^2 + v_{riz}^2}/v_{rix} = \gamma \tan \vartheta_{i0}$ , where  $\vartheta_{i0}$  is the corresponding angle in the rest system.

We shall show, in Section 6, that this basic kinematic relationship leads to length contraction in “pure field” models of the massive particles where all the field energy propagates luminally. The next section discusses such models and points out that the preceding analyses are applicable regardless of any nonlinearity that may be involved.

## 5 Luminal Wave Solitons

Up to this point we have analysed the linear momentum of systems of light flashes emitted by identical sources in different conditions of motion. No functional description of the light flashes was required, neither as photons nor as solutions to any particular wave equation. Subject to a few modest conditions, the same method can be applied to any kind of waves that propagate at  $c$ , including wave models of the massive particles.

Electromagnetic waves in a vacuum, for example, obey the well known d’Alembert wave equation:

$$\{\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\} \psi = 0 \quad (20)$$

Where  $\psi(x, y, z, t)$  may be any component of either the Electric field  $\mathbf{E}$  or the Magnetic field  $\mathbf{H}$ . The individual field components are not linear momenta, but nor do they exist in isolation. Electromagnetic waves involve both Electric and Magnetic fields and there is a linear momentum density,  $\vec{\rho}_p = \mathbf{S}/c^2$ , where the Poynting vector  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$  is aligned with the wave vector,  $\mathbf{k}$  (which by definition points in the direction of propagation). The field lines of the wave vector trace out well defined trajectories at the ray velocity  $v_{ray} = c$  (in vacuo) [14, 15], and the linear momentum carried by the Electromagnetic wave propagates along these trajectories at the characteristic velocity. For any given Electromagnetic wave, we may divide the entire space into incremental regions. The product of the local momentum density,  $\mathbf{S}/c^2$ , times the incremental volume is a linear momentum propagating at  $c$  in a well defined direction in space, so that the analysis above is immediately applicable to systems of Electromagnetic waves in general.

In a linear theory such as Electromagnetics, strict superposition of field amplitudes applies, components evolve independently of each other, there are no interactions amongst the waves and any superposition must dissipate unless all the wave vectors are parallel, in which case the motion of the centre of inertia of the wave group is  $V = c$ . Electromagnetic field models of the massive particles are thus excluded.

However, the fact that compound wave systems obey the usual relativistic momentum equation for particles strongly suggests that the massive particles should be thought of as luminally propagating field systems. A collision between an electron and a positron, for example, produces almost entirely radiation, and the process is reversible. When massive particles absorb and emit light quanta, if the energy involved does not continue to propagate throughout, what is the alternative? Some magical transformation between propagative radiation and non-propagative “matter”, into which no rational insight has ever been advanced?

There are many other good reasons to consider that the division between matter and field is, as Einstein suggested, “artificial”, including the constant modulus, equal to  $c$ , of the velocity operator for the Dirac Equation [2, 3] and the existence of interference phenomena in matter beams<sup>4</sup>.

The simple fact is that the preceding results only required linear momenta to superpose. They did not require that field amplitudes superpose. As we bring together a set of initially isolated component fields,  $\psi_j(x, y, z, t)$  with total momentum  $\mathbf{P} = \sum_j \mathbf{p}_j$ , in the absence of linear superposition of field amplitudes they become distorted. They interact, exchanging momentum, with the possibility to form a persistent

<sup>3</sup>Since  $\mathbf{V}$ ,  $\mathbf{v}_i$  and  $\mathbf{v}_{ri}$  are all referred to the same observer

<sup>4</sup>In view of the de Broglie relation  $P = \hbar k$ , it is only natural to interpret the particle momentum,  $P$ , as the wave momentum added to the system in order to boost it to  $V$

bounded group with a new and different component level description whose interpenetrating components,  $\Psi_i(x, y, z, t)$ , share the same space, but linear momentum is conserved throughout.

This possibility has been extensively studied in the literature and persistent, subluminal soliton solutions are often identified. For example, Donev et al [16, 17] have found photon-like soliton solutions in their Extended Electrodynamics where Maxwell's Laws are modified by adding extra terms, introducing a non-linearity without affecting the Lorentz Invariance. As mentioned in Section 1, there is also a vast range of nonlinear Lorentz Invariant classical field theories with soliton solutions that provide candidate models for the massive particles, as in [4] - [10].

In order to apply previous results in the presence of nonlinearity, we require only that (1) momentum and energy are locally conserved within every region of space according to Equation 21 below (where  $E$  is the energy contained within the region bounded by time independent closed surface  $S$ ) and (2) energy-momentum continues to propagate at  $c$ , so that each of the new components is characterised by a linear momentum density,  $\vec{\rho}_{pi}(x, y, z, t)$ , aligned with a wave vector,  $\mathbf{k}_i(x, y, z, t)$  in the direction of propagation.

$$\sum_i \oint_S \vec{\rho}_{pi} \cdot d\mathbf{s} = -\frac{1}{c^2} \frac{dE}{dt} \quad (21)$$

The group momentum,  $\mathbf{P}$  is then still a vector sum over the new "internal momenta", and it has been conserved so  $\mathbf{P} = \sum_j \mathbf{p}_j(\psi_j) = \sum_i \mathbf{p}_i(\Psi_i)$  where the momentum of the  $i^{th}$  component is  $\mathbf{p}_i = \int \int \int_{-\infty}^{+\infty} \vec{\rho}_{pi} dx dy dz$ .

A minor extension to the notation is required. Previously, in Sections 2 to 4, we considered systems of light flashes where each individual flash propagated in a single, well-defined direction in space. This assumption cannot be applied to the components,  $\Psi_i(x, y, z, t)$ , of a wave soliton because the wave vector of the  $i^{th}$  component will generally point in different directions at different places:  $\hat{\mathbf{k}}_i = \hat{\mathbf{k}}_i(x, y, z, t)$ . Let the entire space be divided into small regions of dimension  $\delta x = \delta y = \delta z = \delta l$ , where  $\delta l$  is sufficiently small that any component's momentum density,  $\vec{\rho}_{pi}(x, y, z, t)$ , can be considered constant within each region so that  $\vec{\rho}_{pi}(x, y, z, t) \delta l^3$  is a linear momentum propagating at  $c$  in a definite direction in space. Introducing a new subscript,  $k$ , to label the regions, we write the linear momentum of the  $i^{th}$  field in the  $k^{th}$  region as  $\mathbf{p}_{ik}(t) = \vec{\rho}_{pi}(\mathbf{r}_k, t) \delta l^3$ , where  $\mathbf{r}_k$  is the position vector to the centre of the  $k^{th}$  region. The Newtonian principles of Section 2 now give:

$$d\mathbf{p}_{ik} = \frac{p_{ik}}{m_e c} d\mathbf{P} \quad (22)$$

Where  $\mathbf{P} = \sum_k \sum_i \mathbf{p}_{ik}$ ,  $m_e = \sum_k \sum_i m_{ik}$  and the rest goes through as before. The rest system is a particle that is comoving with the observer. The moving system's internal momenta,  $\mathbf{p}_{ik}$ , are related to the  $\mathbf{p}_{ik0}$ , by (19), with an additional  $k$  subscript inserted. The particle momentum is  $\mathbf{P} = \gamma m_0 \mathbf{V}$ , where the velocity of the centre of inertia of the wavegroup,  $\mathbf{V}$ , is simply the observed velocity of the particle. The relative velocity we developed at the end of the last section,  $\mathbf{v}_{rik} = \mathbf{v}_{ik} - \mathbf{V}$ , describes the internal movements of the particle as seen by an observer who considers the particle as a whole to be moving at  $\mathbf{V}$ .

Before moving onto the analysis of length contraction and time dilation in luminal wave solitons let us contrast (22) with the Newtonian concept of a force field as applied to a point-like massive particle.

There is no difficulty in principle using the force concept in the context of momentum exchanges between interacting fields. Donev and Tashkova [18] have developed this for the general case of luminally propagating bivector fields that carry linear momentum. The force acting on an interacting field is, by definition, equal to its rate of change of momentum, from which it might appear that:

$$\frac{d\mathbf{p}_{ik}}{dt} = \frac{p_{ik}}{m_e c} \frac{d\mathbf{P}}{dt} \quad (23)$$

and the left hand side of (23) should be interpreted as the force acting on the  $i^{th}$  wave component in the  $k^{th}$  region when the total externally applied force acting on the particle is  $\mathbf{F} = d\mathbf{P}/dt$ . Such a dynamic interpretation requires making unreasonable extraneous assumptions, including for example a uniform applied field. This is unnecessary for our analysis, for which (22) simply governs the relationship between particles in steady state conditions, before and after a force causes an incremental change to the particle momentum. A one to one correspondence between the wave components of rest and moving particles is assumed, but without such an assumption no inherently relativistic structure would be possible because we could never equate a boost with a change of observer.

## 6 The Lorentz-Fitzgerald Contraction

If we were given a specific set of wave trajectories for any particular kind of soliton solution at rest in an observer's inertial frame, we could use the method developed in Sections 2 to 4 to calculate wave trajectories

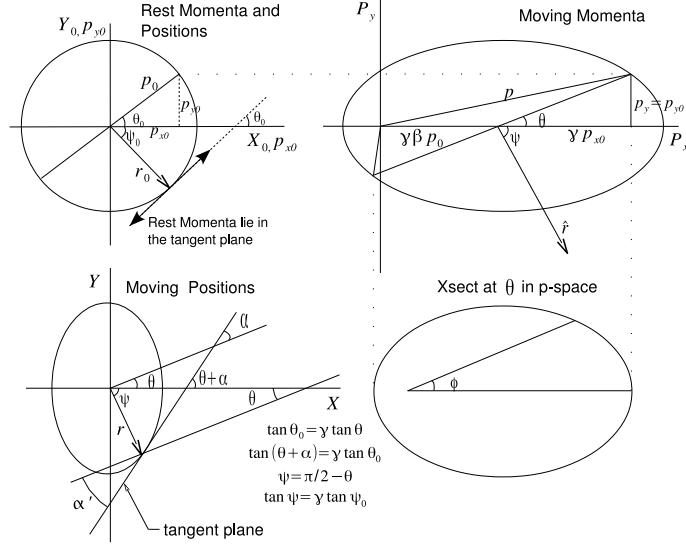


Figure 3: Momenta and Positions in Rest and Moving Luminal Wave Particle Models

for the corresponding moving solitons. Obviously, we would anticipate that they would display the usual relativistic phenomena of length contraction and time dilation.

However such a calculation would cover only one kind of soliton in one formalism whereas, as mentioned above, there is an abundance of nonlinear Lorentz Invariant classical field theories that have soliton solutions. In this section, we shall analyse the general case and by doing so we shall establish the precise mechanism of length contraction that all Lorentz Invariant soliton formalisms have in common.

We shall show here that length contraction occurs in wave solitons for any set of trajectories subject to the condition that, in the rest system, the wave vector of any component field at any point is transverse to the radial direction to the soliton's centre of inertia. Equivalently, any trajectory of the wave vector exists on the surface of a sphere in the rest system. Equivalently, the rest system evolves under the action of members of the group of spatial rotations. Such trajectory systems will be shown to be compressed by the factor  $\gamma$  in the direction of motion and if all the constituent wavegroups in a macroscopic system get compressed in the direction of motion, then so does the whole system. As is well known from Special Relativity, the group of transformations that preserves the linear momentum of a particle is the Little Group and the usual interpretation is that rest particles evolve under the action of members of the group of rotations [19], with the consequence for any Lorentz Invariant wave soliton formalism being that propagation in the rest soliton is transverse to the radius. Therefore, imposing this condition does not eliminate any Lorentz Invariant formalism and the result below applies quite generally to all relativistic wave solitons<sup>5</sup>

Consider a system of concentric spherical surfaces constructed about the rest soliton's centre of inertia, which we shall assume is at the origin. Given the abovementioned condition, all rest system wave trajectories through a given point,  $\mathbf{r}_{k0}$ , lie instantaneously in the tangent plane at that point to the sphere of radius  $r_{k0}$ . Without loss of generality, let us consider the trajectories passing through a point in the XY plane where the tangent plane makes the angle  $\theta_0$  with the X-axis, as shown in the top left of Figure 3. The wave momentum along a trajectory lying in this plane has components in the following form:

$$p_{x0} = p_0 \cos \theta_0 \cos \phi_0 \quad ; \quad p_{y0} = p_0 \sin \theta_0 \cos \phi_0 \quad ; \quad p_{z0} = p_0 \sin \phi_0$$

Where  $\phi_0$  is the angle the trajectory makes with the XY plane. Note that this is just the component form of any of the  $\mathbf{p}_{ik0}$ . The  $i$  and  $k$  subscripts can be omitted without ambiguity:  $p_{x0}$  means  $p_{ikx0}$  and so on. Using (19), the components of the corresponding wave momentum in the moving system are:

$$p_x = p_0 \gamma (\cos \theta_0 \cos \phi_0 + \beta) \quad ; \quad p_y = p_{y0} \quad ; \quad p_z = p_{z0}$$

The moving system momenta for different values of  $\phi_0$  are not coplanar, but lie on a conical surface whose vertex is at the origin of momentum coordinates, and whose base is the intersection of the plane at angle  $\theta$ ,

<sup>5</sup>With respect to compound systems comprising many such wave solitons, although the system as a whole may be in a uniform condition of motion, the linear momenta of individual solitons will not generally be invariant and compound systems will not generally evolve in accordance with the Little Group. However the arguments developed here for trajectories lying on spherical surfaces in the rest system apply equally well to any wave trajectory system with trajectories that lie on closed surfaces. Any such system undergoes the same length contraction.



where  $\tan \theta = \tan \theta_0/\gamma$ , with the moving system momentum distribution. This is shown in the top right of Figure 3. The bottom right of Figure 3 shows the elliptical cross section formed by the tips of all the moving system momentum vectors corresponding to rest system momenta,  $p_{ik0} = p_0$ , lying in the plane at  $\theta_0$ .

The (total) velocity for each of these momenta has components  $v_x = cp_x/p$ ;  $v_y = cp_y/p$ ;  $v_z = cp_z/p$  where, from (17) above:

$$p = \frac{p_0 + \gamma\beta p_x}{\gamma} = \frac{p_0}{\gamma}(1 + \gamma^2\beta(\cos \theta_0 \cos \phi_0 + \beta)) \quad (24)$$

The group velocity is  $V\hat{\mathbf{i}}$ , so using (24) the relative velocity components are:

$$v_{rx} = \frac{cp_x - pV}{p} = \frac{cp_0 \cos \theta_0 \cos \phi_0}{\gamma p} \quad ; \quad v_{ry} = \frac{cp_0 \sin \theta_0 \cos \phi_0}{p} \quad ; \quad v_{rz} = \frac{cp_0 \sin \phi_0}{p}$$

The ratio  $v_{ry}/v_{rx} = \gamma \tan \theta_0$  is independent of  $\phi_0$  (and  $\phi$ ), so the velocities that lay in a given tangent plane in the rest system transform into relative velocities lying in a corresponding moving plane, tangent to the moving trajectory system<sup>6</sup>.

The moving system tangent plane makes the angle  $\alpha + \theta$  with the X-axis, where  $\alpha$  is the angle between the plane at  $\theta$  and the tangent plane (bottom left of Figure 3). From above  $\tan(\theta + \alpha) = v_{ry}/v_{rx} = \gamma \tan \theta_0$  whilst  $\tan \theta_0 = \gamma \tan \theta$ , so using the angle sum trigonometric relations we obtain:

$$\tan \alpha = \frac{\beta^2 \sin \theta \cos \theta}{1 - \beta^2 \cos^2 \theta} \quad (25)$$

The set of all tangent planes defines the surface up to a scale factor. Due to rotational symmetry we anticipate being able to write the equation describing this surface in the form  $r = r(\psi)$ , where  $\psi$  is the angle from the position vector to the X-axis. For any function  $r(\psi)$  the angle between the tangent plane and the plane transverse to the radius vector is:

$$\tan \alpha' = \frac{1}{r} \frac{dr}{d\psi} \quad (26)$$

Consider as trial function the ellipsoid:

$$r(\psi) = \frac{\lambda}{\sqrt{1 - \beta^2 \sin^2 \psi}} \quad (27)$$

For which

$$\tan \alpha' = \frac{\beta^2 \cos \psi \sin \psi}{1 - \beta^2 \sin^2 \psi} \quad (28)$$

independent of the scale parameter  $\lambda$ . With  $\psi = \pi/2 - \theta$ , this is identical to (25), which therefore describes an ellipsoid of revolution (27), such that the plane at  $\theta$  is transverse to the radius. This is shown in the bottom left of Figure 3.

The scale factor,  $\lambda$ , is readily found by inspection. The moving system equatorial plane is the plane  $x = Vt$  and  $\psi = \pi/2$ . The tangent plane at any point in the equatorial plane is parallel to the X-axis so the  $d\mathbf{p}_{ik}$  at these points lie in the tangent plane. Therefore the equatorial tangent planes are not altered by the action of (22). Therefore the radius of a circumferential trajectory in the equatorial plane is invariant under the dimensional transformation (27), whence  $\lambda = r_0/\gamma$ , where  $r_0$  is the radius of the spherical surface in the rest system.

The result is that, for our rest observer, any wave trajectory in the moving system lies on the surface of an ellipsoid moving along the X-axis at speed  $V$  and of the form:

$$r(\psi) = \frac{r_0}{\gamma\sqrt{1 - \beta^2 \sin^2 \psi}} \quad (29)$$

The moving system wave trajectories are thus physically compressed by the factor  $\gamma$  in the direction of motion and so, when it is moving at speed  $V$  for our observer, any physical system composed of wave solitons undergoes the usual Lorentz-Fitzgerald contraction.

<sup>6</sup>Recall that we showed in Section 4 that the relative velocity of any trajectory is rotated by the kinematic relation  $\tan \vartheta = \gamma \tan \vartheta_0$ , where  $\vartheta$  is the angle the relative velocity makes with the X-axis. We now see the consequence of this for luminal wave particle models. Locally flat surfaces formed by sets of trajectories at a given point in the rest system transform into locally flat moving surfaces, rotated so that the tangent of the angle the moving surface makes with the X-axis is  $\gamma \tan \theta_0$ , where  $\theta_0$  is angle the rest system surface makes with the X-axis.

## 7 Time Dilation

In this section it is shown that all internal processes in wave solitons slow down according to  $dt/dt_0 = 1/\gamma$ . A fixed overall rate of spatiotemporal evolution can then be defined in the usual way by a 4-dimensional line element  $c^2 dt_0^2 - d\mathbf{x}_0^2 = c^2 dt^2$  where  $t_0$  and  $t$  are the rates of rest and moving clocks respectively and  $d\mathbf{x}_0$  is traversed by the moving clock. The analysis is similar to the standard analysis of a light clock, but let us first state the main idea and explain it qualitatively.

Time dilation is the direct consequence of constructing variable speed entities from fixed speed primitives.

According to (7) and as illustrated in Figure 2, the group velocity of a wave soliton is the result of spatial correlations amongst the directions of propagation  $\hat{\mathbf{k}}_i(x, y, z, t)$  of internal momenta. As the group velocity approaches the characteristic velocity, the trajectories rotate towards the group velocity:  $\hat{\mathbf{k}} \rightarrow \hat{\mathbf{V}}$ , but if all the trajectories of a wave system were exactly parallel as it moved through an observer's reference frame the spatial configuration of the system would not change and the observer would conclude that nothing happens in the inertial frame of the group. Just as spatial correlations amongst trajectories are essential for the movement of a soliton through space, internal evolution requires decorrelations. To the extent that propagation contributes to the movement of the system as a whole it is unavailable to contribute to its internal evolution. There is a direct tradeoff between the external evolution in space versus the internal evolution in time, which we shall show leads to the invariance of the line element.

The same kind of tradeoff is also found in the Dirac Equation. Consider the equation for the time dependence of the velocity operator in the Heisenberg representation of the Dirac theory [20]:

$$\vec{\alpha}(t) = (\vec{\alpha}(0) - \frac{\mathbf{p}}{H}) \exp(-2iHt) + \frac{\mathbf{p}}{H} \quad (30)$$

Where  $\mathbf{p}$  and  $H$  are both constants,  $c = 1$  and the group velocity is  $\mathbf{p}/H = \mathbf{v}_g = \text{constant}$ . The first term on the right is routinely interpreted to represent the internal movements of the electron, the "zitterbewegung". Its quantum mechanical expectation is:  $\langle \Psi | (\vec{\alpha}(0) - \mathbf{v}_g) | \Psi \rangle / \langle \Psi | \Psi \rangle$  which (noting that  $\vec{\alpha}$  has real eigenvalues) varies with  $v_g$  as  $\sqrt{1 - v_g^2}$ . The zitterbewegung slows down by a Lorentz factor as the group velocity increases.

We shall now show that internal processes in wave solitons slow down according to  $dt/dt_0 = 1/\gamma$ .

With respect to the rest soliton's trajectory system, consider any closed trajectory formed by  $n$  segments, where the  $i^{\text{th}}$  segment has length  $l_{i0}$  and makes the angle  $\theta_{i0}$  with the X-axis. The speed on all segments is  $v_0 = c$  so the period around the closed trajectory is  $T_0 = \frac{1}{c} \sum_{i=1}^n l_{i0}$ , where  $T_0$  is the time elapsed on a clock in the rest frame to traverse the trajectory in the rest system. Lengths in the rest system may be written in component form such that:

$$l_{i0}^2 = l_{ix0}^2 + l_{iy0}^2 + l_{iz0}^2$$

Let the trajectory system now move in the x-direction at speed  $V$ . Given the length contraction, x-components contract by the factor  $\gamma$  and the corresponding relationship is:

$$l_i^2 = \frac{l_{ix0}^2}{\gamma^2} + l_{iy0}^2 + l_{iz0}^2$$

It is readily shown that:

$$l_i^2 = l_{i0}^2(1 - \beta^2 \cos^2 \theta_{i0}) \quad (31)$$

The moving and rest system angles are related by  $\tan \theta_i = \gamma \tan \theta_{i0}$ , from which we get:

$$\frac{\cos \theta_i}{\cos \theta_{i0}} = \sqrt{1 - \beta^2 \sin^2 \theta_i} \quad (32)$$

The relative velocity on the  $i^{\text{th}}$  segment in the moving system,  $v_{ri}$ , is constrained by:

$$(v_{ri} \cos \theta_i + V)^2 + v_{ri}^2 \sin^2 \theta_i = c^2 \quad (33)$$

Which leads to:  $v_{ri} + V \cos \theta_i = c\sqrt{1 - \beta^2 \sin^2 \theta_i}$ . From which, using (32):

$$v_{ri} = \frac{\cos \theta_i (c - V \cos \theta_{i0})}{\cos \theta_{i0}} = \frac{l_{ix0} c (1 - \beta \cos \theta_{i0})}{\gamma l_i \cos \theta_{i0}}$$

The time taken to traverse the  $i^{\text{th}}$  segment in the moving system is  $l_i/v_{ri} = l_i^2/v_{ri}l_i$ , so, using (31), we may write the period elapsed on clocks in the rest system for traversals around the Lorentz contracted closed trajectory of the moving system as follows:

$$T_0^V = \sum_{i=1}^n l_i^2/v_{ri}l_i = \sum_{i=1}^n \frac{\gamma l_{i0}^2 \cos \theta_{i0} (1 - \beta^2 \cos^2 \theta_{i0})}{l_{ix0} c (1 - \beta \cos \theta_{i0})} = \frac{\gamma}{c} \sum_{i=1}^n l_{i0} (1 + \beta \cos \theta_{i0})$$

Since  $\sum_i l_{i0} \cos \theta_{i0} = 0$  it follows that  $T_0^V = \gamma T_0$ . It might be argued that trajectories need not form closed loops, but a path that crosses a given plane transverse to  $\mathbf{V}$  must eventually either recross the same plane or become confined to a smaller region, in which it must either routinely recross a transverse plane or become confined to an even smaller region and so on. In steady state, the trajectories can only be transverse or regularly recross a transverse plane. The analysis above also covers open paths between points in the same transverse plane, for which the condition  $\sum_i l_{i0} \cos \theta_{i0} = 0$  is also fulfilled. The time between such crossing points dilates by  $\gamma$ . We conclude that the internal processes of a wave soliton slow down by the factor  $\gamma$ . The argument from internal atomic processes to real world clocks is well established [21], and tested [22, 23, 24], so moving clocks will run slow according to the usual relation  $dt/dt_0 = 1/\gamma$ .

## 8 Non-luminal Structures

Without the concept of internal movements it would not seem possible to provide any account of internal processes (such as muon decay for example). Likewise, the fact that the massive particles possess angular momentum implies the existence of internal movements<sup>7</sup>. Let us consider internal movements at speeds other than  $c$ . To illustrate the problem, let us assume Lorentz contracted moving system trajectories.

We must still use (33), with  $v_i^2$  replacing  $c^2$  on the RHS, to connect the total and relative velocities on the  $i^{\text{th}}$  segment (as both are referred to the same observer). If  $v_i$  were the same in the moving and rest systems, then clearly, the periods would not dilate by  $\gamma$ , and yet we know that periods must dilate under Lorentz Transformations for any physical system, not just luminal systems.

The resolution is easily seen from Special Relativity. If the total velocity,  $\mathbf{v}_{i0}$ , on the  $i^{\text{th}}$  segment as seen by a comoving observer is such that  $v_{i0} \neq c$ , then for observers in other frames,  $v_i \neq v_{i0}$  and must in general be calculated according to the relativistic composition of velocities:

$$\mathbf{v}_i = \frac{\mathbf{V} + \mathbf{v}_{i0\parallel} + \sqrt{1 - \beta^2} \mathbf{v}_{i0\perp}}{1 + \frac{\mathbf{V} \cdot \mathbf{v}_{i0}}{c^2}}$$

Now as we Lorentz boost a particle in the frame of a single observer, there are two possibilities. If  $v_{i0} = c$ , then  $v_i = c$  for all  $i$  independent of the condition of motion of the particle, and structural models incorporating length contraction and the relativistic momentum are readily available. Section 9 shows that these phenomena imply Lorentz Transformations. Their elegance and simplicity therefore has a coherent explanation based on the very definition of momentum as inertia times velocity,  $p = mc$ .

Alternatively, if  $v_{i0} \neq c$  the total velocities of internal movements,  $\mathbf{v}_i$ , must depend on both the particle velocity and the orientation of individual segments in the above complicated manner, in which case the elegance and simplicity of Lorentz Transformations has at its very foundations an implausibly inelegant, complex physical structure. Ockham's razor insists that we reject nonluminal structures. Otherwise, we are left reasoning in a circle from the Lorentz Transformations to the composition of velocities to the unlikely proposition that such complex physical structures give rise to simple Lorentz Transformations.

## 9 Coordinate Transformations

We have shown length contraction and time dilation as physical effects in luminal wave particle models subject to the basic Newtonian equations (1) - (7) and (22). Our analyses were constructed from the perspective of a single observer so the principle of relativity, covariance, coordinate independence, and coordinate transformations were all irrelevant.

Let us now turn our attention to the question how these physical phenomena of length contraction and time dilation constrain the coordinate transformations. This question has been studied in some detail by Selleri [12, 13], who has shown that, subject only to the use of the Einstein clock synchronisation protocol, length contraction and time dilation directly imply Lorentz Transformations.

He considered three assumptions, namely length contraction, time dilation and constancy of the 2-way velocity of light. It was shown that any two of these assumptions both implies the third and constrains the coordinate transformations between a preferred rest frame,  $S_0 = (x_0, y_0, z_0, t_0)$  and a frame  $S = (x, y, z, t)$  in standard configuration moving with velocity  $v$  to the following form:

$$x = \frac{(x_0 - \beta ct_0)}{\sqrt{1 - \beta^2}} \quad ; \quad y = y_0 \quad ; \quad z = z_0 \quad ; \quad t = \sqrt{1 - \beta^2} t_0 + e_1(x_0 - \beta ct_0)$$

---

<sup>7</sup>The quantisation of angular momenta is also readily explicable as a wave phenomenon [11, 18].

Where  $\beta = v/c$  and  $e_1$  is a synchronisation parameter. Setting  $e_1 = -\beta/(c\sqrt{1-\beta^2})$  corresponds to the usual Einstein clock synchronisation convention and reduces this to the Lorentz Transform.

Our coordinate transformations are therefore Lorentz Transformations and the relativity principle and the constant speed of light for all observers are therefore results, not postulates. It is also now clear that the wave inertia changes we have analysed are frequency changes corresponding to the relativistic doppler shift, as opposed to, say, amplitude changes.

We conclude that, in the comoving frame, Lorentz Invariant structural models of the massive particles will have internal movements at, and only at,  $c$ .

## 9.1 Other Synchronisation Protocols

Selleri also discusses alternative clock synchronisation protocols, especially the case  $e_1 = 0$  which corresponds to using Einstein synchronisation in a preferred rest frame, and setting clocks in the moving frame to coincide with nearby clocks in the rest frame at  $t = 0$ . Both sets of observers agree that clocks in the moving system run slow, and they also agree on the simultaneity of spatially separated events. The transformations in this case, known as the inertial transformations, were first found by Tangherlini [25]. The empirical consequences of inertial transformations have been shown to comply with experimental evidence in a wide variety of situations [26]. As far as the present article is concerned, the Appendix derives (8) from the relativistic doppler shift and aberration results, which apply equally well to inertial transformations [27], and therefore so do the structural constraints developed above.

Selleri and others have advanced various arguments in favour of absolute simultaneity [28] - [33]<sup>8</sup>, but nothing that questions the Lorentz form within the domain of inertial frames. Inertial transformations do not preserve the line element,  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ , the physical laws are frame dependent, the inverse transformation is different, the relative velocity of the origin of  $S$  as seen by  $S_0$  does not equal the relative velocity of  $S_0$  as seen by  $S$  and the inertial transformations do not form a group [13].

The conventional nature of the Einstein protocol has, of course, always been stipulated in relativity theory and what Selleri has in fact shown is that, like the choice between Cartesian and Spherical coordinates, the choice of a clock synchronisation protocol really is only a matter of convenience. Provided they use it consistently, physicists solving problems on a rotating platform and engineers developing GPS satellite networks<sup>9</sup> can use whatever protocol is most effective.

The self-evident fact remains that the events that happen in the world cannot depend on the coordinate systems we use to describe them. Coordinate independence is one of the most powerful practical tools for the development of new physics. Other coordinate transformations may be empirically adequate, but special status is rightly afforded to Lorentz Transformations on the basis of symmetry and utility, not uniqueness, and what we have shown is that their “natural habitat” is field theory.

## 10 Does Local Action Imply Retarded Interaction?

Local action is the single most basic, self-evident principle in Physics - interaction requires colocation. Both Newton and Einstein agreed. In this Section we consider the logic of interaction at a distance, subject to local action, but from a pure field perspective where “mass energy” propagates luminally.

In Classical Physics it was taken for granted that matter emits field, leading to the idea that the far fields of a particle must propagate away from it at  $c$ . It then follows that long range interactions between particles are retarded and the unavoidable consequence is that there can be no causal relations between space-like separated events. On the other hand, Quantum Mechanics predicts instant causal correlations at a distance and experiments replicate these predictions [34] - [36]. However, if matter and field are one and the same, as Einstein suggested and Special Relativity implies, then the idea that matter emits field is meaningless and we need to consider whether or not the far fields propagate away from the centre of inertia in a pure field particle model.

In Section 6 we imposed the constraint that propagation in the rest particle is transverse to the radius. This constraint corresponds to the Little group, so it is fully in accordance with Special Relativity. We derived length contraction by considering how the wave trajectories lying on any given sphere in the rest system undergo an elliptical distortion in a system moving at speed  $V$ . Note that the radius of the sphere was not a relevant consideration - the analysis relates to any radius and therefore there is no good reason, neither in our analysis nor in Special Relativity, to distinguish between the near and far fields of a particle.

<sup>8</sup>Notably a simplified analysis on the rotating platform.

<sup>9</sup>Which use an inertial clock synchronisation protocol.

The distinction in Electromagnetics between the “attached” field [37] and the “body” of the particle is incompatible with Special Relativity.

Consistent with Einstein’s view that Special Relativity renders the division into matter and field “artificial”, our luminal wave structure implies that particles are unbounded, and that their far fields propagate transverse to the radius<sup>10</sup> rather than radially away from the body.

On the other hand, massive particles have finite energy and it is therefore necessary that the volume integral of the field energy density should not diverge as  $r \rightarrow \infty$ . As found in [11], the existence of  $1/r^2$  long range force fields for the charged particles implies a  $1/r^4$  energy density asymptote for luminal wave models, including both charged and neutral particles. The energy density integral does not then diverge as  $r \rightarrow \infty$  so that finite but unbounded luminal wave structures are compatible with the usual basic physics. The field energy is highly concentrated near to the centre of inertia with the result that they appear as pointlike particles. For example, with respect to a particle with the mass of an electron, the maximum energy density (at the radius  $r \sim 4 \times 10^{-13}m$ ) is  $\sim 400,000$  times greater than that at a radius of 0.1 Angstrom unit<sup>11</sup>.

As also discussed in [11], there is no good reason to presume that local action implies retarded interaction in luminal wave particle models. Local action means that the long range interactions between two particles, A and B, depend on the colocation of their respective fields, but any far fields of A that become collocated with the B particle’s centre of inertia did not travel there from A’s centre of inertia. They are part of an extended wave system that is comoving, as a whole, with the A centre of inertia so it would be more reasonable to anticipate that the direct impact of A’s far fields on the observed location of the B particle would be instantaneous, whilst only the reaction impact on the A particle would be retarded.

However, it is more apposite simply to observe that field theory problems are usually formulated and solved on whole regions that evolve subject to local action at all points in parallel. The idea of a local realist wave ontology is inherently Lorentz Invariant, but waves are inherently distributed. They run on correlations at a distance sustained by strictly local actions. Distributed interactions between distributed waves can have distributed impacts, occurring simultaneously in different places. Waves exemplify Redhead’s conclusion that ontological locality does not rule out instant relations between observables [38]. Trajectories in local realist wave systems display entanglement as shown in [15], where it was found that the Helmholtz equation contains Bohmian mechanics’ nonlocal quantum potential within it. The essential consequence is that quantum nonlocality and entanglement might be interpreted as locally realistic wave phenomena. With specific reference to the EPR paradox [39], the Bell Inequalities [40] depend on a causality analysis that uses light cones emanating from point events [41], presuming a one to one correspondence with point-like “beables” [42], but for inherently distributed systems like waves neither beables nor events can be presumed to be point-like.

## 11 Discussion

Unlike Electromagnetics, nothing prevents the simple method used here from applying to the fermions. A wide range of candidate models for the massive particles, in the form of subluminal soliton solutions found in typically nonlinear field theories, have been reported in the literature. The analysis in Sections 2 - 7 shows that Lorentz invariance is the consequence of constructing subluminally moving systems from fields that propagate luminally. The appearance of Lorentz invariance in so many disparate field models is therefore no coincidence - they are all subject to the same basic kinematic constraints.

Whilst the constraints are simple, the structures of soliton solutions are generally not simple. For example, evolution under rotations does not imply spherical symmetry and nor does it imply that the particle rotates as a whole in a simple manner, like a solid ball. Due to the kinematic constraint, trajectories at different radii necessarily evolve at different angular rates and, similarly, wave trajectories at various points on the same spherical surface in the rest system generally rotate about different axes.

## 12 Conclusions

We have considered the analogy between Special Relativity and the basic Newtonian mechanics of luminal wave systems. The fact that these systems obey the usual relativistic momentum equation prompts the suggestion that the massive particles should be thought of as generalised systems of luminal waves. We showed how length contraction and time dilation in luminal wave models result from the simple kinematic constraints imposed by constant speed propagation at  $c$ . This provides a direct physical basis for the idea

<sup>10</sup>As is also consistent with Electromagnetics’ radial force field because  $\mathbf{E}$  and  $\mathbf{H}$  are each transverse to the momentum density  $\mathbf{S}/c^2$ .  $\mathbf{H}$  fields cancel in the rest particle due to balanced movements.

<sup>11</sup>According to the energy density asymptote, which ignores nonlinearities, vacuum polarisation effects and so on.

of a combined spatiotemporal evolution, leading to the invariance of the line element, the usual spacetime symmetry, the relativity principle and the invariance of  $c$  for all observers.

The main conclusion was that Lorentz invariant structural models of the massive particles should feature internal movements at, and only at, the characteristic velocity.

The usual presumption that local action implies retarded interaction was examined from the “pure field” perspective and rejected. It cannot be presumed that Special Relativity precludes instantaneous causal relations between space-like separated events because an additional premise is involved, that matter emits field, which does not apply to pure field models.

Hopefully, this article has firmly established the equivalence of Newtonian and relativistic concepts of inertia, momentum and energy, once Einstein’s ideal of a “pure field physics” is adopted. This highlights the absence of any good reason to presume any non-propagative form of mass-energy exists. It is not the introduction of a new hypothesis, but the removal of an old one - the idea of matter as a distinct ontological class in its own right.

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## Appendix

With respect to the system of light flashes in Section 2, let us impose the condition in some inertial frame:

$$\mathbf{P}_0 = \sum_i \mathbf{p}_{i0} = 0$$

The momentum of the  $i^{th}$  light flash, referred to this frame, is then:

$$\mathbf{p}_{i0} = p_{i0}(\cos \theta_{i0} \hat{\mathbf{i}} + \sin \theta_{i0} \cos \phi_{i0} \hat{\mathbf{j}} + \sin \theta_{i0} \sin \phi_{i0} \hat{\mathbf{k}})$$

Where  $\theta_{i0}$  is the angle with the X-axis and  $\sum_i p_{i0} \cos \theta_{i0} = \sum_i p_{i0} \sin \theta_{i0} \cos \phi_{i0} = \sum_i p_{i0} \sin \theta_{i0} \sin \phi_{i0} = 0$ .

Let an observer move relative to this frame with velocity  $\mathbf{V} = -\beta c \hat{\mathbf{i}}$ . Since  $p_i/p_{i0} = f_i/f_{i0}$ , the standard relativistic doppler shift and aberration formulae (with the observer moving towards the source at speed  $v$ ) give, respectively:

$$p_i = p_{i0} \gamma \left(1 + \frac{v}{c} \cos \theta_{i0}\right) \text{ and } \cos \theta_i = \frac{\cos \theta_{i0} + \frac{v}{c}}{1 + \frac{v}{c} \cos \theta_{i0}}$$

Note that the same result also holds for non-monochromatic light flashes. The scalar momentum of the  $i^{th}$  flash in the observer frame is:

$$p_i = p_{i0} \gamma (1 + \beta \cos \theta_{i0})$$

Summing over the  $i$ , the total energy is:

$$m_e c^2 = c \sum_i p_i = \gamma c \sum_i p_{i0} = \gamma m_0 c^2$$

Where  $m_e$  and  $m_0$  are as defined in Section 2. The (vector) momentum of the  $i^{th}$  flash is:

$$\mathbf{p}_i = p_{i0}(\gamma(\beta + \cos \theta_{i0})\hat{\mathbf{i}} + \sin \theta_{i0} \cos \phi_{i0}\hat{\mathbf{j}} + \sin \theta_{i0} \sin \phi_{i0}\hat{\mathbf{k}})$$

Summing over the  $i$ , the total momentum is:

$$\mathbf{P} = \sum_i \mathbf{p}_i = \gamma\beta \sum_i p_{i0}\hat{\mathbf{i}}$$

Note that this is the relativistic momentum equation. Differentiating each of the two previous equations with respect to  $\beta$ :

$$\frac{d\mathbf{p}_i}{d\beta} = \gamma^2 p_i \hat{\mathbf{i}}; \quad \frac{d\mathbf{P}}{d\beta} = \gamma^3 \sum_i p_{i0} \hat{\mathbf{i}} = \gamma^3 m_0 c \hat{\mathbf{i}} = \gamma^2 m_e c \hat{\mathbf{i}}$$

Whence:

$$\frac{d\mathbf{p}_i}{d\beta} = \frac{d\mathbf{P}}{d\beta} \frac{p_i}{\sum_j p_j} = \frac{d\mathbf{P}}{d\beta} \frac{p_i}{m_e c}$$

Finally, since the above expressions for  $\mathbf{p}_i$  and  $\mathbf{P}$  are functions of  $\beta$  alone, we can write the incremental changes as:

$$d\mathbf{p}_i = \frac{d\mathbf{p}_i}{d\beta} d\beta, \quad d\mathbf{P} = \frac{d\mathbf{P}}{d\beta} d\beta$$

Upon which:

$$d\mathbf{p}_i = \frac{p_i}{m_e c} d\mathbf{P}$$

Therefore (8) holds for a collinear incremental boost. For transverse boosts, consider as initial condition a system whose centre of inertia is moving in the y-direction at speed  $V$ , so  $m_e = \gamma(V)m_0$ . We may repeat the above analysis for an observer moving at speed  $v_x$  in the x-direction with  $\sum_i p_{i0} \sin \theta_{i0} \cos \phi_{i0} \neq 0$  and get the result for an incremental transverse boost:

$$d\mathbf{p}_i = \frac{p_i}{\lim_{v_x \rightarrow 0} (\gamma(v_x) m_e c)} d\mathbf{P} = \frac{p_i}{m_e c} d\mathbf{P}$$

So, (8) holds for an incremental transverse boost. In Special Relativity, the general boost decomposes into a collinear boost, a transverse boost and a rotation (a Thomas precession). As the latter has no impact on linear momenta, (8) is generally valid for incremental boosts of systems of luminal wave momenta in Special Relativity.