The arithmetic of binary representations of even positive integer 2n and its application to the solution of the Goldbach's binary problem

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Abstract

One of causes why Goldbach's binary problem was unsolved over a long period is that binary representations of even integer 2n (BR2n) in the view of a sum of two odd primes(VSTOP) are considered separately from other BR2n. By purpose of this work is research of connections between different types of BR2n. For realization of this purpose by author was developed the "Arithmetic of binary representations of even positive integer 2n" (ABR2n). In ABR2n are defined four types BR2n. As shown in ABR2n all types BR2n are connected with each other by relations which represent distribution of prime and composite positive integers less than 2n between them. On the basis of this relations (axioms ABR2n) are deduced formulas for computation of the number of BR2n (NBR2n) for each types. In ABR2n also is defined and computed Average value of the number of binary sums are formed from odd prime and composite positive integers $< 2n$ (AVNBS). Separately AVNBS for prime and AVNBS for composite positive integers. We also deduced formulas for computation of deviation NBR2n from AVNBS. It was shown that if n go to infinity then NBR2n go to AVNBS that permit to apply formulas for AVNBS to computation of NBR2n. At the end is produced the proof of the Goldbach's binary problem with help of ABR2n. For it apply method of a proof by contradiction in which we make an assumption that for any 2n not exist BR2n in the VSTOP then make computations other NBR2n and for all cases we come to contradiction. Hence our assumption is false and forall $2n > 2$ exist BR2n in the VSTOP.

KEYWORDS 0.1 *Binary representations of even integer 2n;*

Goldbach's binary problem; "strong" ,"even" Goldbach's conjecture.
MSC: 11P32; 03E75 .

1 Introduction

On 7 June, 1742,the Prussian mathematician Christian Goldbach wrote a letter to Leonhard Euler in which he proposed the following conjecture: Every integer greater than 2 can be written as the sum of three primes. He considered 1 to be a prime number. A modern version of Goldbach's original conjecture is: Every integer greater than 5 can be written as the sum of three primes. Euler, becoming interested in the problem, replied by noting that this conjecture is equivalent with another version: Every even integer greater than 2 can be written as the sum of two primes. Euler's version is the form in which the conjecture is usually expressed today. It is also known as the strong,even, or binary Goldbach's conjecture.

2 General conception

Definition 2.1 *The binary representations of even positive integer 2n in ABR2n are defined as bijective mappings :*

$$f : X \to Y \tag{1}$$

$$y = 2n - x \tag{2}$$

Where:

$$X\{x|x \in N, 1 \leq x < n\}; \tag{3}$$

$$Y\{y|y \in N, n < y < 2n\}; \tag{4}$$

n- positive integer > 0

$$|X| = |Y| \tag{5}$$

Definition 2.2 *The set of binary representations of even positive integer 2n are defined as follows :*
SBR2n $\{1 + (2n - 1) = 2n$ *;*
$2 + (2n - 2) = 2n$ *;*
...
$n - 1 + (2n - (n - 1)) = 2n\}$
The last is got by represent (2)
in the view of $x + y = 2n$.

Remark 2.1 *Since n is not mapped into Y and it is mapped into itself (automorphism) therefore mapping $n \to n$ and corresponding binary representation $n + n = 2n$ do not go into SBR2n which is formed only from bijective mappings .By this reason n do not go into XUY. But it is not denoted that as a result of the exception of binary representations: $2+2 = 4$ and $3 + 3 = 6$ from SBR2n, 4 and 6 have not binary representations in the view of a sum of two noncomposite positive integers. But it is not the case.In SBR2n is in existence the binary representations $1 + 3 = 4$ and $1 + 5 = 6$. It is significant "1" with primes are took in ABR2n to noncomposite positive integers. At that special status of "1" in N is ignore in ABR2n.*

Definition 2.3 *The set XUY consist of: even positive integers , odd positive integers inclusive odd composite positive integers and noncomposite positive integers (pimes and "1").*

Proposition 2.1 $\forall\ n > 1$ *always is fulfilled the condition* $|X| = |Y|$.

Proof 2.1 *Taking into account that $2n$ do not go into the set XUY and n is excluded from the set XUY (see remark 2.1) then we have :*

$$|XUY| = 2n - 2 = 2(n - 1) \tag{6}$$

Hence $|XUY|$ is even for any n and thus $\forall\ n > 1$ always is fulfilled the condition $|X| = |Y|$. $\square$

3 The types of binary representations even positive integer 2n

In depend of that are x,y in the view of $x + y = 2n$ prime or composite it can be four types binary representations of even positive integer $2n$:

Definition 3.1 *There is Type "H" if x-odd prime positive integer or"1" and y- odd prime positive integer.*
$|H|$- the number of binary representations of Type "H".
$|H|$- positive integer $> 0\ \forall\ 2n > 2$.
This is the thesis which will be proved below (see sec.13).

Definition 3.2 *There is Type "Q" if x-odd composite positive integer; and y-odd composite positive integer .*
$|Q|$- the number of binary representations of Type "Q".

$|Q|$- *positive integer* $> 0 \ \forall \ 2n > 22$
expepting $2n = 26; 28; 32; 38;$ *in which* $|Q| = 0$ *(see subsec. 16.4) .*
The proof that $|Q| > 0 \ \forall \ 2n > 120$ *(see sec. 12) .*
$|Q| = 0 \ \ \forall \ 2 < 2n < 24.$

Definition 3.3 *There is Type "L" if x- odd prime positive integer or"1" and y-odd composite positive integer or x-odd composite positive integer and y- odd prime positive integer.*
$|L|$ *- the number of binary representations of Type "L".*
$|L|$*- positive integer* $> 0 \ \forall \ 2n > 8.$ *(see corollary 8.1)*
$|L| = 0 \ \forall \ 2 < 2n < 10.$

Definition 3.4 *There is Type "E" if x-even positive integer and y-even positive integer .*
$|E|$*- the number of binary representations of Type"E".*
$|E|$*- positive integer* $> 0 \ \forall \ 2n > 4.$ *(see sec.5 prop.5.2)*

4 The axioms of ABR2n

Definition 4.1 *p- the number of noncomposite integers (primes and "1")* $< 2n$.
$p = round(\pi(2n) + 1);$
round - round -up to the nearest integer.
$p - positive\ integer \ > \ 1 \ \forall \ 2n \ > \ 2.$ *(see proposition 5.5)*

Definition 4.2 $\pi(2n) - the\ number\ of\ primes \ < \ 2n$

Axiom 4.1 *The number of the binary representations type H (NBRH) is connected with the number of the binary representations type L (NBRL) as follows:*

$$2|H| + |L| = p - 1 \tag{7}$$

$\forall \ 2n > 2.$
The expression (7) insists that odd noncomposite positive integers less than $2n$ *are allotted to types "H"," L" in compliance with balance (7). In (7) the* (-1) *takes into account that "2"is not odd noncomposite positive integer.*

Definition 4.3 s_0 *- the number of odd composite positive integers* $< 2n$.
s_0 *- positive integer* $> 0 \ \forall \ 2n > 8$ *(see proposition 5.4)*
$s_0 = 0 \ \ \forall \ 2 < 2n < 10.$

Axiom 4.2 *The number of the binary representations Type Q (NBRQ) is connected with the number of the binary representations Type "L" (NBRL) as follows:*

$$2|Q| + |L| = s_0 \tag{8}$$

$\forall\ 2n > 2$.

The expression(8) insists that odd composite positive integers less than $2n$ *are allotted to types "Q","L" in compliance with balance (8).*

Definition 4.4 G *- the general number of binary representations in SBR2n.*

G *- positive integer* $\forall\ 2n > 2$ *(see proposition 5.1).*

Axiom 4.3

$$|Q| + |L| + |H| + |E| = G \tag{9}$$

Definition 4.5 F*-the general number of binary representations with odd positive integers .*

$F -$ *positive integer* $\forall\ 2n\ > 2$ *(see proposition 5.3).*

Axiom 4.4

$$|Q| + |L| + |H| = F \tag{10}$$

5 The computation of $G, F, S_0, p, |E|$

Proposition 5.1

$$G = n - 1 \tag{11}$$

$\forall n > 1$

Proof 5.1 *The general number of elements in the set* XUY *by (6) equals* $2(n-1)$*. Taking into account that in forming of each binary representation participate with two elements from the set* XUY *then we have:* $G = n-1$. $\forall n > 1$ $\square$

Proposition 5.2

$$|E| = [(n-1)/2] \tag{12}$$

$\forall\ n > 2$

Proof 5.2 *By definitions 2.1 ; 2.2 by elements of the set XUY are numbers of the natural scale. Half of them are even integers. Then the number of even integers in the set XUY equals:*
$1/2|XUY| = 2(1/2)(n-1) = n-1$.
Taking into account that in forming of each binary representation participate by two elements from the set XUY we have: $|E| = (n-1)/2$.
Taking into account that for n - even $|E|$ is not integer that breaks the status of $|E|$ since $|E|$ is positive integer > 0 then $|E| = [(n-1)/2]$ is aliquot of $(n-1)/2$ then we get : $|E| = [(n-1)/2]\ \forall\ n > 2$ $\square$

Proposition 5.3

$$F = G - |E| = (n-1) - [(n-1)/2] \tag{13}$$

$\forall\ n > 1$

Proof 5.3 *Subtracting (10) from (9) we get: $|E| = G - F$ whence $F = G - |E|$. Taking into account (11), (12) finally we get:*
$F = (n-1) - [(n-1)/2]\ \forall\ n > 1$ $\square$

Proposition 5.4

$$S_0 = 2(n-1) - (2[(n-1)/2] - 1) - p \tag{14}$$

$\forall\ n > 1$.

Proof 5.4 *By definition 2.3 for computation S_0 it needs to subtract from $|XUY|$ the number of even composite positive integers ($2|E| - 1$) (here (-1) takes into accounts that "2" is even prime integer) and also the number of noncomposite positive integers- p then we get:*
$s_0 = 2(n-1) - (2[(n-1)/2] - 1) - p.\ \forall\ n > 1$ $\square$

Definition 5.1 $p_1 - the\ first\ approximation\ of\ p.$
$p_2 - the\ second\ approximation\ of\ p.$
$p_3 - the\ third\ approximation\ of\ p.$

Proposition 5.5

$$p_1 = round(2n/ln2n + 1) \tag{15}$$

$$p_2 = round(2n/ln2n + 2n/(ln2n)^2 + 1) \tag{16}$$

$$p_3 = round(2n/ln2n + 2n/(ln2n)^2 + 4n/(ln2n)^3 + 1) \tag{17}$$

Proof 5.5 *As everybody knows [1] that the number of the primes less than* $2n$ *is expressed as follows:*
$\pi(2n) = (2n/ln2n)\int_0^1(1-(lny/ln2n)+(ln^2y/ln^22n)+\ldots)dy$
We are limited to three of the first terms of the series.
Integrating in parts then we get:
$\pi(2n) = (2n/ln2n)(1+1/ln2n+2/ln^22n)$
Whence taking into account def 4.1 then we get :
$p = round(2n/ln2n+2n/ln^22n+4n/ln^32n+1)$
Whence we get:
$p_1 = round(2n/ln2n+1)$;
$p_2 = round(2n/ln2n+2n/ln^22n+1)$;
$p_3 = round(2n/ln2+2n/ln^22n+4n/ln^32n+1)$; $\square$

6 Corollaries of the axioms of ABR2n

Definition 6.1 $|Q|-|H|>0, \forall 2n>120.$
$|Q|-|H|<0, \forall 2<2n<120.$
Excepting $2n = 4; 94; 96; 100; 106; 118$
(see (58) and definitions 8.9; 3.2).

Remark 6.1 *Only* $|E|$ *can be computed directly by (12) others* $(|Q|,|L|,|H|)$ *if is given one of them.*

Corollary 6.1 *Let are given* $|Q|, p$ *then by axiom 4.2 we get:*

$$|L| = s_0 - 2|Q| \tag{18}$$

Taking into account (14) then we get:

$$|L| = 2(n-1)-(2[(n-1)/2]-1)-p-2|Q| \tag{19}$$

Next subtracting (7) from (8) we get:

$$2|Q|-2|H| = s_0 - p + 1 \tag{20}$$

Whence we get:

$$|H| = (2|Q|-s_0+p-1)/2 \tag{21}$$

Taking into account (14) then we get:

$$|H| = (2|Q|-2(n-1)+(2[(n-1)/2]-1)+2p-1)/2 \tag{22}$$

Corollary 6.2 *Let are given $|L|, p$ then by axiom 4.2 we get:*

$$|Q| = (s_0 - |L|)/2 \tag{23}$$

Taking into account (14) then we get:

$$|Q| = (2(n-1) - (2[(n-1)/2] - 1) - p - |L|)/2 \tag{24}$$

Next by axiom 4.1 we get:

$$|H| = (p - |L| - 1)/2 \tag{25}$$

Corollary 6.3 *Let are given $|H|, p$ then by axiom 4.1 we get:*

$$|L| = p - 2|H| - 1 \tag{26}$$

Next by axiom 4.2 we get:

$$|Q| = (s_0 - |L|)/2 \tag{27}$$

Taking into account (26) then we get:

$$|Q| = (s_0 - p + 2|H| + 1)/2 \tag{28}$$

Taking into account (14) then we get:

$$|Q| = (2(n-1) - (2[(n-1)/2] - 1) - 2p + 2|H| + 1)/2 \tag{29}$$

Corollary 6.4 *subtract (7) from (8) then we get:*

$$|Q| - |H| = (S_0 - p + 1)/2 \tag{30}$$

Taking into account (14) then we get:

$$|Q| - |H| = (2(n-1) - (2[(n-1)/2] - 1) - 2p + 1)/2 \tag{31}$$

Corollary 6.5 *With halp of the axioms 4.3; 4.4 we can control an accuracy of computations $|Q|, |L|, |H|, |E|$.*

Remark 6.2 *In the formulas (20) , (21) ,(22), (23) ,(24) , (25), (27) , (28) ,(29),(30) ,(31)*
numerator is always even positive integer (The examples of the proof of parity of numerator for the some formulas see subsection 16.1) then for corresponding n the division by "2" without residue is always possible and status $|Q|, |L|, |H|$(positive integer) is not broken.

7 The simplified formulas

The formulas (12),(13),(14), (19),(22),(24) ,(29) , (31) can be simplified if divide its for

$$n = 2i + 1, i \in N \tag{32}$$

and

$$n = 2i, i \in N; \tag{33}$$

Substituting (32) , (33) to (12)...(31) and making simplifications then returning to $n : (i = (n-1)/2;\ i = n/2)$ (The examples of simplification for the some formulas see subsection 16.2) we get :

$$|E| = (n-1)/2 \tag{34}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|E| = n/2 - 1 \tag{35}$$

$\forall\ n = 2i, i \in N; n > 2$

$$F = (n-1)/2 \tag{36}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$F = n/2 \tag{37}$$

$\forall\ n = 2i, i \in N; n > 1$

$$S_0 = n - p \tag{38}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$S_0 = n - p + 1 \tag{39}$$

$\forall\ n = 2i, i \in N; n > 1$

$$|L| = n - p - 2|Q| \tag{40}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|L| = n - p + 1 - 2|Q| \tag{41}$$

$\forall\ n = 2i, i \in N; n > 1$

$$|H| = (2|Q| + 2p - n - 1)/2 \tag{42}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|H| = (2|Q| + 2p - n - 2)/2 \tag{43}$$

$\forall\ n = 2i, i \in N; n > 1$

$$|Q| = (n - p - |L|)/2 \tag{44}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|Q| = (n - p - |L| + 1)/2 \tag{45}$$

$\forall\ n = 2i, i \in N; n > 1$

$$|Q| = (n - 2p + 2|H| + 1)/2 \tag{46}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|Q| = (n - 2p + 2|H| + 2)/2 \tag{47}$$

$\forall\ n = 2i, i \in N; n > 1$

$$|Q| - |H| = (n - 2p + 1)/2 \tag{48}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

$$|Q| - |H| = (n - 2p + 2)/2 \tag{49}$$

$\forall\ n = 2i, i \in N; n > 1$

Remark 7.1 *The examples of use of ABR2n see below (subsection 16.5).*

8 The limited values of possible range of $|Q|, |L|, |H|$

Definition 8.1 $|H|_b$ *-lower limit of possible range of* $|H|$*.*

Axiom 8.1

$$|H|_b = 0 \tag{50}$$

Definition 8.2 *p(n) - the number of noncomposite integers (primes and "1") < n.*
p(n)- positive integer > 1 *forall* $n > 1$*.*

Definition 8.3 *(p - 1) - the number of odd noncomposite integers in the set* XUY*.*
(-1) takes into account that "2" is not odd noncomposite integers.

Definition 8.4 *(p(n) - 1) - the number of odd noncomposite integers in X .*

Definition 8.5 $(p-1)-(p(n)-1)$ *- the number of odd noncomposite integers in Y .*

Definition 8.6 $|H|_c$*-upper limit of possible range of* $|H|$.

Proposition 8.1

$$|H|_c = p - p(n) \tag{51}$$

Proof 8.1 *By Low of distribution of primes the number of odd noncomposite integers in X is greater than the number of odd noncomposite integers in Y : (p(n) - 1) > (p - 1) - (p(n) - 1) .*
Since the number of primes decreases with increase of n. Hence maximal number of pair of odd noncomposite integers in the set XUY equals the number of odd noncomposite integers in Y : (p - 1) - (p(n) - 1)
then $|H|_c = p - p(n)$ *.* □

Corollary 8.1 *The number of unpaired odd noncomposite positive integers in X equals :* $2p(n) - p - 1$ *and are allotted to type "L". Then* $|L| > 0\ \forall 2n > 8$

Proof 8.2 *The number of unpaired odd noncomposite positive integers in X by definitions 8.4, 8.5 equals :*
(p(n) - 1) - ((p - 1) - (p(n) - 1)) = 2p(n) - p - 1
and are allotted to type "L". □

Definition 8.7 $|L|_b$ *- lower limit of possible range of* $|L|$.

Proposition 8.2

$$|L|_b = 2p(n) - p - 1 \tag{52}$$

Proof 8.3 *Substituting upper limit of* $|H|$ *by (51) to (7)(axiom 4.1) then we get lower limit for* $|L|$ *:* $|L|_b = 2p(n) - p - 1$ □

Definition 8.8 $|L|_c$ *-upper limit of possible range of* $|L|$.

Proposition 8.3

$$|L|_c = p - 1 \tag{53}$$

Proof 8.4 *Substituting lower limit of* $|H|$ *by (50) to (7)(axiom 4.1) then we get upper limit for* $|L|$ *:* $|L|_c = p - 1$ □

Definition 8.9 $|Q|_b$ - *lower limit of possible range of* $|Q|$.
$|Q|_b$- *positive integer* $> 0 \ \ \forall\ 2n > 120$.
$|Q|_b$ - *negative integer* $< 0 \ \ \forall\ 2 < 2n < 120$.
excepting $2n = 4; 94; 96; 100; 106; 118;$ *for which* $|Q|b = 0$ *(see subsec.16.3).*
At that $2n = 120; (|Q|_b = 0)$ *is border point.*

Proposition 8.4

$$|Q|_b = (n - 2p + 1)/2 \tag{54}$$

$\forall\ n = 2i + 1, i \in N; n > 1$

Proof 8.5 *Substituting upper limit of* $|L|$ *by (53) to (8)(axiom 4.2) then we get lower limit for* $|Q|$*:* $|Q|_b = (S_0 - p + 1)/2$ *Substituting* S_0 *by (38), then we get:* $|Q|_b = (n - 2p + 1)/2\ \forall\ n = 2i + 1, i \in N; n > 1 \ \ \square$

Proposition 8.5

$$|Q|_b = (n - 2p + 2)/2 \tag{55}$$

$\forall\ n = 2i, i \in N; n > 1$

Proof 8.6 *Substituting upper limit of* $|L|$ *by (53) to (8)(axiom 4.2) then we get lower limit for* $|Q|$*:* $|Q|_b = (S_0 - p + 1)/2$ *Substituting* S_0 *by (39), finally we get:* $|Q|_b = (n - 2p + 2)/2;\ \forall\ n = 2i, i \in N; n > 1 \ \ \square$

Definition 8.10 $|Q|_c$ - *upper limit of possible range of* $|Q|$.

Proposition 8.6

$$|Q|_c = (n - 2p(n) + 1)/2 \tag{56}$$

$\forall\ n = 2i + 1, i \in N;$

Proof 8.7 *Substituting lower limit of* $|L|$ *by (52) to (8)(axiom 4.2) then we get upper limit for* $|Q|$*:* $|Q|_c = (S_0 - (2p(n) - p - 1))/2$ *Substituting* S_0 *by (38) finally we get:* $|Q|_c = (n - 2p(n) + 1)/2;\ \forall\ n = 2i + 1, i \in N;\ \ \square$

Proposition 8.7

$$|Q|_c = (n - 2p(n) + 2)/2 \tag{57}$$

$\forall\ n = 2i, i \in N;$

Proof 8.8 *Substituting lower limit of* $|L|$ *by (52) to (8)(axiom 4.2) then we get upper limit for* $|Q|$*:* $|Q|_c = (S_0 - (2p(n) - p - 1))/2$ *Substituting* S_0 *by (39) finally we get:* $|Q|_c = (n - 2p(n) + 2)/2;\ forall\ n = 2i, i \in N;\ \ \square$

Proposition 8.8

$$|Q| - |H| = |Q|_b \tag{58}$$

Proof 8.9 *By (48), (49) we have:* $|Q| - |H| = (n - 2p + 1)/2$*;* $\forall\ n = 2i+1, i \in N$; $|Q| - |H| = (n-2p+2)/2\ \forall\ n = 2i, i \in N$, *By (54), (55) we have:* $|Q|_b = (n - 2p + 1)/2$*;* $\forall\ n = 2i + 1, i \in N$ *;* $|Q|_b = (n - 2p + 2)/2$*;* $\forall\ n = 2i, i \in N$ *; Whence we get:* $|Q| - |H| = |Q|_b$ $\square$

9 Average value of the number of binary sums are formed from odd composite positive integers $< 2n$

Definition 9.1 $S-ordered\ set\ of\ odd\ composite\ positive\ integers\ < 2n$
$s - element\ of\ S$
$|S|_0 - power\ of\ S$
$s_i - vary\ over\ all\ s$
$s_j - vary\ over\ all\ s$

Definition 9.2 $V\ \{v_k | v_k \in N, v_k = s_i + s_j\}$
is a set by elements of which are every possible binary sums of odd composite integers $< 2n$*. (each with all the rest)*
Since $s_i < 2n; s_j < 2n$ *then max* $v_k < 4n$*.*
$|V|$ *- the power of set* V*.*

Definition 9.3 $W\{w | w \in N, w = 2k, 1 \leq k \leq 2n\}$
is a set of even composite positive integers $< 4n + 2$
$(inf\ W = 2; sup\ W = 4n)$.
$|W| - the\ power\ of\ set\ W; |W| = 2n.$

Definition 9.4 $|Q|_m$ *- mean quantity of binary sums are formed of odd composite positive integers* $< 2n$ *which are mapped into* W *at surjective mapping :*
$f :\ V \Rightarrow\ W$
$s_i + s_j = 2k$
where: k *- positive integer situated in the range:* $11 < k < 2n$

$$|Q|_m = |V|/|W| \tag{59}$$

i.e. uniform mapping regardless of real.
$|Q|_m\ - positive\ rational\ number\ > 0\ \forall\ 2n\ >\ 22.$

Proposition 9.1

$$|V| = SXS = S_0^2 \tag{60}$$

Proof 9.1 *Since $|V|$ - is the number of every possible binary sums are formed of odd composite positive integers $< 2n$ in the view of $v_k = s_i + s_j$ which as well known equals Cartesian product: $|V| = SXS = S_0^2$* $\square$

Proposition 9.2

$$|Q|_m = s_0^2/2n \tag{61}$$

Proof 9.2 *By (59), (60), def.9.3 we have : $|Q|_m = |V|/|W| = s_0^2/2n$* $\square$

Proposition 9.3

$$|Q|_m = (n-p)^2/2n \tag{62}$$

$\forall\ n = 2i+1, i \in N; n > 6$

Proof 9.3 *Substituting s_0 by (38) to (61) then we get :*
$|Q|_m = (n-p)^2/2n;$
$\forall\ n = 2i+1, i \in N; n > 7$ $\square$

Proposition 9.4

$$|Q|_m = (n-p+1)^2/2n \tag{63}$$

$\forall\ n = 2i, i \in N; n > 4$

Proof 9.4 *Substituting S_0 by (39) to (61) then we get :*
$|Q|_m = (n-p+1)^2/2n;\ \forall\ n = 2i, i \in N; n > 4$ $\square$

10 Average value of the number of binary sums are formed from odd noncomposite positive integers $< 2n$

Definition 10.1 *P - ordered set of odd noncomposite positive integers* $< 2n$;
p - elements of P ;
$|P| = (p-1)$ - power of P ;
p_i- vary over all p ;
p_j - vary over all p ;

Definition 10.2 $T\{t_k|t_k \in N, t_k = p_i + p_j\}$;
is a set by elements of which are every possible binary sums of odd non-composite integers $< 2n$. *(each with all the rest) Since* $p_i < 2n; p_j < 2n$ *then max* $t_k < 4n$.
$|T|$ *- the power of set T.*

Definition 10.3 $|H|_m$ *is mean quantity of binary sums are formed of odd noncomposite positive integers* $< 2n$ *which are mapped into* W *at surjective mapping :*
$f: T \Rightarrow W$
$p_i + p_j = 2k$
where: k *- positive integer situated in the range:* $1 < k < 2n$

$$|H|_m = |T|/|W| \tag{64}$$

i.e. uniform mapping regardless of real.
$|H|_m$*-positive rational number* $> 0 \quad \forall 2n > 2$.

Proposition 10.1

$$|T| = PXP = (p-1)^2 \tag{65}$$

Proof 10.1 *Since* $|T|$ *is the number of every possible binary sums are formed of odd noncomposite positive integers* $<$ $2n$ *in the view of* $p_i+p_j = 2k$. *Which as well known equals Cartesian product. Taking into account definition 8.3 we get:* $|T| = P\ X\ P = (p-1)^2$ $\square$

Proposition 10.2

$$|H|_m = (p-1)^2/2n \tag{66}$$

$\forall\ 2n > 2$

Proof 10.2 *By (64),(65),def.9.3 we have :* $|H|_m = (p-1)^2/2n\ \forall\ 2n > 2$ $\square$

11 The deviation of $|Q|, |H|$ $from$ $|Q|_m, |H|_m$

Definition 11.1 *The deviation of* $|Q|$ $from$ $|Q|_m$ *is:*
$\Delta|Q| = |Q|_m - |Q|$
$\Delta|Q| > 0\ if\ |Q|_m > |Q|$
$\Delta|Q| < 0\ if\ |Q|_m < |Q|$

Definition 11.2 *The deviation of* $|H|$ *from* $|H|_m$ *is:*
$\Delta|H| \ = \ |H|_m - \ |H|$;
$\Delta|H| \ > \ 0 \ if \ |H|_m > \ |H|$;
$\Delta|H| \ < \ 0 \ if \ |H|_m < \ |H|$

Proposition 11.1 $\forall \ n = 2i, i \in N$;

$$\Delta|Q| = \Delta|H| \tag{67}$$

FOR $\Delta|Q| > 0; \Delta|H| > 0$
OR $\Delta|Q| < 0; \Delta|H| < 0$
exepting the cases:
$\Delta|Q| < 0; \Delta|H| > 0$
$\Delta|Q| > 0; \Delta|H| < 0$

Proof 11.1 ***Case 11.1*** *We computate* $|Q|_m - |H|_m; \forall \ n = 2i, i \in N$;
By (63), (66) we have:
$(n-p+1)^2/2n - (p-1)^2/2n = (n-2p+2)/2$
Taking into account (55) we get:
$|Q|_m - |H|_m = |Q|_b$
Next by definitions 11.1 , 11.2 we have;
FOR $\Delta|Q| > 0; \Delta|H| > 0$
$(|Q| + \Delta|Q|) - (|H| + \Delta|H|) = |Q|_b$
or
$|Q| - |H| + \Delta|Q| - \Delta|H| = |Q|_b$
Taking into account (58) finally we get:
$\Delta|Q| - \Delta|H| = 0$
or
$\Delta|Q| = \Delta|H|$

Case 11.2 *FOR* $\Delta|Q| < 0; \Delta|H| < 0$ *we have:*
$(|Q| - \Delta|Q|) - (|H| - \Delta|H|) = |Q|_b$
$|Q| - |H| - \Delta|Q| + \Delta|H| = |Q|_b$
Taking into account (58) finally we get:
$-\Delta|Q| + \Delta|H| = 0$
or
$\Delta|Q| = \Delta|H|$

Case 11.3 *FOR the case of* $\Delta|Q| > 0; \Delta|H| < 0$ *We have:*
$(|Q| + \Delta|Q|) - (|H| - \Delta|H|) = |Q|_b$
$|Q| - |H| + \Delta|Q| + \Delta|H| = |Q|_b$
Taking into account (58) finally we get:

$\Delta|Q| + \Delta|H| = 0$
whence $\Delta|Q| = 0; \Delta|H| = 0;$
We come to contradiction since by definitions 11.1,11.2
$\Delta|Q| > 0;\ \Delta|H| > 0; \Delta|Q| < 0; \Delta|H| < 0$
By this reason the combination: $\Delta|Q| > 0; \Delta|H| < 0$ *is excluded.*
Analogously the combination: $\Delta|Q| < 0; \Delta|H| > 0$ *is also excluded.* □

Proposition 11.2 $\forall\ n = 2i + 1, i \in N$ *;*
FOR $\Delta|Q| > 0; \Delta|H| > 0$

$$\Delta|Q| = \Delta|H| - K \tag{68}$$

FOR $\Delta|Q| < 0; \Delta|H| < 0$

$$\Delta|Q| = \Delta|H| + K \tag{69}$$

FOR $\Delta|Q| < 0; \Delta|H| > 0$

$$\Delta|Q| = -\Delta|H| + K \tag{70}$$

Where:$K = (n - 2p + 1)/2n$

Proof 11.2 ***Case 11.4*** *We computate* $(|Q|_m - |H|_m)\ \forall\ n = 2i+1, i \in N;$
By (62), (66) we have:
$(n - p)^2/2n - (p - 1)^2/2n = (n^2 - 2np + 2p - 1)/2n = (n^2 - 2np + n + 2p - 1 - n)/2n = (n - 2p + 1)/2 + (2p/n - (1/n) - 1)/2 = (n - 2p + 1)/2 - (n - 2p + 1)/2n$
Taking into account (54) we get:
$|Q|_m - |H|_m = |Q|_b - K$
$\forall\ n = 2i + 1, i \in N;$
Where:$K = (n - 2p + 1)/2n$
Next by definitions 11.1 , 11.2 we have:
FOR $\Delta|Q| > 0; \Delta|H| > 0$
$(|Q| + \Delta|Q|) - (|H| + \Delta|H|) = |Q|_b - K$
$|Q| - |H| + \Delta|Q| - \Delta|H| = |Q|_b - K$
Taking into account (58) we get:
$\Delta|Q| - \Delta|H| = -K;$
or
$\Delta|Q| = \Delta|H| - K$

Case 11.5 *FOR* $\Delta|Q| < 0; \Delta|H| < 0$ *we have:*
$(|Q| - \Delta|Q|) - (|H| - \Delta|H|) = |Q|_b - K$
or

$|Q| - |H| - \Delta|Q| + \Delta|H| = |Q|_b - K$
Taking into account (58) we get:
$-\Delta|Q| + \Delta|H| = -K$;
or
$\Delta|Q| = \Delta|H| + K$
$\forall\ n = 2i + 1, iN$.

Case 11.6 *FOR* $\Delta|Q| > 0; \Delta|H| < 0$ *we have:*
$(|Q| + \Delta|Q|) - (|H| - \Delta|H|) = |Q|_b - K$
or
$|Q| - |H| + \Delta|Q| + \Delta|H| = |Q|b - K$
Taking into account (58) we get:
$\Delta|Q| + \Delta|H| = -K$
We come tu contradiction since sum of positive numbers can not be equal to negative number.
By this reason the combination: $\Delta|Q| > 0; \Delta|H| < 0;$ *is excluded.*

Case 11.7 *FOR the case of* $\Delta|Q| < 0; \Delta|H| > 0$ *we have:*
$(|Q| - \Delta|Q|) - (|H| + \Delta|H|) = |Q|_b - K$
or
$|Q| - |H| - \Delta|Q| - \Delta|H| = |Q|_b - K$
Taking into account (58) we get:
$-\Delta|Q| - \Delta|H| = -K$
$\Delta|Q| + \Delta|H| = K$
$\Delta|Q| = -\Delta|H| + K$ $\square$

12 The existence of minimal value of "Q"

Theorem 12.1 $\forall 2n > 120$ *there is no less than* $|Q|_b$ *of representations of the type "Q" .*

Proof 12.1 *We need to prove that* $|Q|_b > 0\ \ \forall 2n > 120$. *By (54) we have* $|Q|_b > 0$ *if* $(n + 1) > 2p$. *And by (55) we have* $|Q|_b > 0$ *if* $(n + 2) > 2p$. *Let* $F_1(2n) = n - 2p + 1$ *and* $F_2(2n) = n - 2p + 2$. *Substituting for p its the second- order approximation by (16). Then we get:*
$F_3(2n) = n-2(2n/ln2n+2n/ln^2 2n+1)+1 = n-4n/ln2n-4n/ln^2 2n-1.$
$F_4(2n) = n-2(2n/ln2n+2n/ln^2 2n+1)+2 = n-4n/ln2n-4n/ln^2 2n$. *We compute* $F_3'(2n) = (ln^4 2n - 4ln^3 2n + 8ln2n)/ln^4 2n$. $F_3'(2n) > 0\ \forall 2n > 2$ *then* $F_3(2n)$ *increase* $\forall 2n > 2$.
$F_4'(2n) = (ln^4 2n - 4ln^3 2n + 8ln2n)/ln^4 2n$. $F_4'(2n) > 0\ \forall 2n > 2$ *then* $F_4(2n)$ *increase* $\forall 2n > 2$. *Hence* $|Q|_b$ *increases* $\forall 2n > 2$. *Next we shall*

find the point of intersection of $|Q|_b(2n)$ *with abscissa axis. For it we need to test of fulfillment of conditions:* ($F_1(2n) = 0$; $F_1(2n+2) > 0$) *and* ($F_2(2n) = 0; F_2(2n+2) > 0$). *As follows from numerical solution (see subsection 16.3). The point of intersection is 2n = 120. Since conditions are fulfilled only for its. Thus* $|Q|_b > 0 \ \ \forall 2n > 120$. □

Corollary 12.1 $\forall\ 2 < 2n < 120; |Q|_b$ *is negative integer excepting 2n = 4; 94; 96; 100; 106; 118. In which* $|Q|_b = 0$.

Proof 12.2 *From the beginning we need to prove that* $|Q|_b$ *is integer* $\forall 2n > 2$. *By (54)* $|Q|_b = (n-2p+1)/2$ *for* n *is odd.*
By (55) $|Q|_b = (n-2p+2)/2$ *for* n *is even .*
By definition 2.1 n*- positive integer* > 0.
By definition 4.1 p *-positive integer* > 1.
Then $(n-2p+1)$ *is integer and* $(n-2p+2)$ *is integer .*
then for n *is odd -* $(n-2p+1)$ *is even*
and for n *is even -* $(n-2p+2)$ *is even .*
And for any n *the division by "2" in formulas (54),(55) without residue is always possible . And status* $|Q|_b$ *is not broken.* $|Q|_b$ *is integer* $\forall\ 2n > 2$. *By proof 12.1 follows* $\forall 2 < 2n < 120\ 2p > n+1;\ 2p > n+2$ *then* $|Q|_b < 0$. *By direct computation we find* $2n$ *for which* $|Q|_b = 0$(*see subsection* 16.3) :. *Which are excluded from the stated range.*
Thus $|Q|_b$ *is negative integer in the range* $2 < 2n < 120$.
Excepting $2n = 4; 94; 96; 100; 106; 118;$ *in which* $|Q|_b = 0$. □

Proposition 12.1 *In the range of* $2 < 2n < 120$ *if fulfilled the condition* $|H| = |Q|_b$ *then* $|Q| = 0$.

Proof 12.3 *By (58) and corollary 12.1 we have for* $2 < 2n < 120$:
$|H| = |Q| + |Q|_b$.
Let $|Q| = 0$; *then* $|H| = |Q|_b$.
Thus if it is fulfilled the condition $|H| = |Q|_b$ *then* $|Q| = 0$. □

13 The solution of the Goldbach's binary problem

Lemma 13.1 $\forall\ 2 < 2n < 120$ *exists at least one representation of type "H".*

Proof 13.1 *Taking into account (58) and corollary 12.1 we have :*
$|H| = |Q| + |Q|_b$. *Since in the range of* $2 < 2n < 120$ *by corollary*12.1

$|Q|_b \neq 0$ then $|H| > 0$ excepting $2n = 4; 94; 96; 100; 106; 118;$ in which $|Q|_b = 0$. For this $2n$ the truth of lemma follows thereout for $2n$ in which $|Q|_b = 0$ then $|Q| > 0$ in this points since the points of exclusion for $|Q|$ don't coincidence with $|Q|_b$. Excepting point $2n = 4$ for which the truth of lemma is controled directly. Thus $|H| > 0 \ \forall \ 2 < 2n < 120$ $\square$

Theorem 13.1 $\forall \ 2n > 2$ *exists at least one representation of even positive integer $2n$ in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.*

Proof 13.2 *For the proof of the theorem we need to prove that $|H| > 0 \ \forall \ 2n > 2$ for the five cases:*

The condition 13.2.1
$n = 2i, i \in N, n > 60, \ \Delta|Q| > 0, \ \Delta|H| > 0, \ \Delta|Q| = \Delta|H|$.
The condition 13.2.2
$n = 2i, i \in N; n > 60, \ \Delta|Q| < 0, \ \Delta|H| < 0, \ \Delta|Q| = \Delta|H|$
The condition 13.2.3
$n = 2i + 1, i \in N, n > 60, \ \Delta|Q| > 0, \ \Delta|H| > 0, \ \Delta|Q| = \Delta|H| - K$
The condition 13.2.4
$n = 2i + 1, i \in N, n > 60, \ \Delta|Q| < 0, \ \Delta|H| < 0, \ \Delta|Q| = \Delta|H| + K$
The condition 13.2.5
$n = 2i + 1, i \in N, n > 60, \ \Delta|Q| < 0, \ \Delta|H| > 0, \ \Delta|Q| = -\Delta|H| + K$

The case 1 of 5: The proof with condition 13.2.1
Let for any value of $2n > 120, |H| = 0$. Then by definition 11.2 $\Delta|H| = |H|_m;$ for $|H| = 0$. Taking into account (66) $\Delta|H| = (p-1)^2/2n$. Taking into account (67)

$$\Delta|Q| = (p-1)^2/2n \tag{71}$$

By definition 11.1 and condition 13 .2.1 we have : $|Q| = |Q|_m - \Delta|Q|$. Taking into account (55), (63), (71) and $|Q| = |Q|_b;$ for $|H| = 0$ we get: $|Q|_b = |Q|_m - \Delta|Q|$ or $(n - 2p + 2)/2 = (n - p + 1)^2/2n - (p - 1)^2/2n$. Then we get identity: $n - 2p + 2 = n - 2p + 2$ We come to contradiction : as follows from identity each n correspond to more than one p . The last is impossible since each n correspond to one p by (15) . (16), (17).

The case 2 of 5: The proof with condition 13.2.2
Let for any value of $2n > 120, |H| = 0$. Then by definition 11.2 $\Delta|H| = |H|_m;$ for $|H| = 0$ Taking into account (66) $\Delta|H| = (p-1)^2/2n$. Taking into account (67) $\Delta|Q| = (p - 1)^2/2n$ By definition 11.1 and condition 13.2.2 we have :

$$\Delta|Q| = (p-1)^2/2n \tag{72}$$

Taking into account (55), (63), (72) and $|Q| = |Q|_b$; for $|H| = 0$ *we get:* $|Q|_b = |Q|_m + \Delta|Q|$ *or* $(n-2p+2)/2 = (n-p+1)^2/2n + (p-1)^2/2n$ *Then we get :* $p^2 - 2p + 1 = 0$ *This quadratic equation has one solution in positive integers:* $p = 1$ *We come to contradiction since* $p > 1$ *by definition 4.1*

The case 3 of 5: The proof with condition 13.2.3
Let for any value of $2n > 120, |H| = 0$ *Then by definition 11.2* $\Delta|H| = |H|_m$; for $|H| = 0$ *Taking into account (66)* $\Delta|H| = (p-1)^2/2n$ *Taking into account (68),*

$$\Delta|Q| = (p-1)^2/2n - (n-2p+1)/2n \tag{73}$$

By definition 11.1 and condition 13.2.3 we have : $|Q| = |Q|_m - \Delta|Q|$ *. Taking into account (54), (62), (73) and* $|Q| = |Q|_b$; for $|H| = 0$ *we get:* $|Q|_b = |Q|_m - \Delta|Q|$ *or* $(n-2p+1)/2 = (n-p)^2/2n - ((p-1)^2/2n - (n-2p+1)/2n)$ *Then we get identity:* $n - 2p + 1 = n - 2p + 1$ *We come to contradiction : as follows from identity each* n *correspond to more than one p . The last is impossible since each* n *correspond to one p by (15) .(16), (17).*

The case 4 of 5: The proof with condition 13.2.4
Let for any value of $2n > 120, |H| = 0$ *Then by definition 11.2* $\Delta|H| = |H|_m$; for $|H| = 0$. *Taking into account (66)* $\Delta|H| = (p-1)^2/2n$ *Taking into account (69)*

$$\Delta|Q| = (p-1)^2/2n + (n-2p+1)/2n \tag{74}$$

By definition 11.1 and condition 13.2.4 we have : $|Q| = |Q|_m + \Delta|Q|$ *. Taking into account (54), (62), (74) and* $|Q| = |Q|_b$ *; for* $|H| = 0$ *we get:*$|Q|_b = |Q|_m + \Delta|Q|$ *or* $(n-2p+1)/2 = (n-p)^2/2n + ((p-1)^2/2n + (n-2p+1)/2n)$ *Then we get :* $p^2 - 2p + 1 = 0$ *This quadratic equation has one solution in positive integers:*$p = 1$ *We come to contradiction since* $p > 1$ *by definition 4.1.*

The case 5 of 5: The proof with condition 13.2.5
Let for any value of $2n > 120, |H| = 0$. *Then by definition 11.2* $\Delta|H| = |H|_m$; for $|H| = 0$. *Taking into account (66)* $\Delta|H| = (p-1)^2/2n$ *Taking into account (70)*

$$\Delta|Q| = -(p-1)^2/2n + (n-2p+1)/2n \tag{75}$$

By definition 11.1 and condition 13.2. 5 we have : $|Q| = |Q|_m + \Delta|Q|$ *. Taking into account (54), (62), (75) and* $|Q| = |Q|_b$ *; for-* $|H| = 0$ *we get:*$|Q|_b = |Q|_m + \Delta|Q|$ *or* $(n-2p+1)/2 = (n-p)^2/2n - (p-1)^2/2n +$

$(n-2p+1)/2n$
Then we get identity: $(n-2p+1) = (n-2p+1)$ *We come to contradiction : as follows from identity each* n *correspond to more than one* p *. The last is impossible since each* n *correspond to one* p *by (15) .(16), (17).*

We come to contradiction for all cases. Hence our assumption that $|H| = 0$ *is false and* $|H| > 0$ $\forall 2n > 120$ *.*

Earlier by Lemma we proved that $|H| > 0$ $\forall$ $2 < 2n < 120$.

Thus we proved that $|H| > 0$ $\forall$ $2n > 2$ *.*
Hence $\forall$ $2n > 2$ *exists at least one representation of even positive integer* $2n$ *in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.* $\square$

Remark 13.1 *Thereby it is proved thesis formulated in def.3.1 .*

14 The computation of the real values of $|Q|, |H|$

14.1 The relative accuracy of computation of $|Q|, |H|$

Definition 14.1 *The relative accuracy of computation of* $|Q|$ *as follows below :*

$$\delta_Q = \left(\frac{\Delta|Q|}{|Q|_m} 100 \right) \% \tag{76}$$

Definition 14.2 *The relative accuracy of computation of* $|H|$ *as follows below:*

$$\delta_H = \left(\frac{\Delta|H|}{|H|_m} 100 \right) \% \tag{77}$$

Proposition 14.1

$$\frac{100(p^2 - n)}{(n-p)^2} \ > \ \delta_Q \tag{78}$$

$\forall$ $n = 2i + 1, i \in N, n > 60;$

Proof 14.1 *By definition 11.1 we have* $|Q| = |Q|_m - \Delta|Q|$*. By defin. 14.1*
we have: $\Delta|Q| = \frac{\delta_Q}{100}|Q|_m$ *then we get :* $|Q| = |Q|_m - \frac{\delta_Q}{100}|Q|_m$.
By definition 8.9 $|Q|_m - \frac{\delta_Q}{100}|Q|_m > |Q|_b$, $\forall$ $2n > 60$ *.*
Whence it follows that $\frac{100(|Q|_m - |Q|_b)}{|Q|_m} > \delta_Q$.

Taking into account (62) and (54). Then we get: $\frac{100\frac{(n-p)^2}{2n}-100\frac{(n-2p+1)}{2}}{\frac{(n-p)^2}{2n}} > \delta_Q$.

Hence $\frac{100(p^2-n)}{(n-p)^2} > \delta_Q . \forall\ n = 2i+1, i \in N, n > 60;$ $\square$

Proposition 14.2

$$\frac{100(p-1)^2}{(n+1-p)^2} \ > \ \delta_Q \tag{79}$$

$\forall\ n = 2i, i \in N, n > 60;$

Proof 14.2 *By definition 11.1 we have* $|Q| = |Q|_m - \Delta|Q|$. *By defin 14.1 we have :*
$\Delta|Q| = \frac{\delta_Q}{100}|Q|_m$ *then we get:* $|Q| = |Q|_m - \frac{\delta_Q}{100}|Q|_m$.
By definition 8.9 $|Q|_m - \frac{\delta_Q}{100}|Q|_m > |Q|_b$, $\forall\ n > 60$.
Whence it follows that $\frac{100(|Q|_m-|Q|_b)}{|Q|_m} > \delta_Q$.
Taking into account (63) and (55) then we get:
$\frac{100\frac{(n+1-p)^2}{2n}-100\frac{(n-2p+2)}{2}}{\frac{(n+1-p)^2}{2n}} > \delta_Q$.
Hence $\frac{100(p-1)^2}{(n+1-p)^2} > \delta_Q . \forall\ n = 2i, i \in N, n > 60;$ $\square$

14.2 The character of dependence of $\delta(2n)$

Theorem 14.1 *If* $n \to \infty$ *then* $\delta_Q \to 0$.

Proof 14.3 *We replace* δ_Q *with its estimation by (78)* $\delta_Q = 100\frac{(p^2-n)}{(n-p)^2}$. *Replacing* p *with its first order approximation by (15) we get:* $\delta_Q = \frac{4n^2-n\ln^2 2n}{(n\ln 2n-2n)^2}$*; We represent it in the view of* $\delta_Q = \frac{(4-\frac{\ln^2 2n}{n})}{\ln^2 2n-4\ln 2n+4}$. *The numerator of this expression* $\to 4$ *if* $n \to \infty$ *and the denominator* $\to \infty$ *if* $n \to \infty$. *Whence follows that if* $n \to \infty$ *then* $\delta_Q \to 0$. $\forall\ n = 2i+1, i \in N, n > 60;$ $\square$

Remark 14.1 *For* $\forall\ n = 2i, i \in N, n > 60$ *the proof by analogy with proof 14.3.*

Theorem 14.2 *If* $n \to \infty$ *then* $\delta_H \to 0$.

Proof 14.4 *By theorem 14.1* $\delta_Q \to 0$ *if* $n \to \infty$ *then by definition 14.1* $\Delta|Q| \to 0$ *if* $n \to \infty$ *then by proposition 11.1* $\Delta|H| \to 0$ *if* $n \to \infty$ *then by definition 14.2* $\delta_H \to 0$ *if* $n \to \infty$. $\forall\ n = 2i, i \in N, n > 60;$ $\square$

14.3 The character of dependence of $|Q|(2n), |H|(2n)$

Theorem 14.3 *If $n \to \infty$ then $|Q| \to |Q|_m$.*

Proof 14.5 *By Theorem 14.1 $\delta_Q \to 0$ if $n \to \infty$ then by Defenition 14.1 $\Delta|Q| \to 0$. Whence by Definition 11.1 we have: If $n \to \infty$ then $|Q| \to |Q|_m$.* $\square$

Theorem 14.4 *If $n \to \infty$ then $|H| \to |H|_m$.*

Proof 14.6 *By Theorem 14.2 $\delta_H; \to 0$ if $n \to \infty$ then by Defenition 14.2 $\Delta|H| \to 0$. Whence by Definition 11.2 we have : If $n \to \infty$ then $|H| \to |H|_m$;* $\square$

14.4 The formulas for computation of the real values of $|Q|, |H|$

$$|Q| = round(\frac{(n-p)^2}{2n}) \tag{80}$$

$\forall\ n = 2i+1, i \in N, n > 19$

$$|Q| = round(\frac{(n-p+1)^2}{2n}) \tag{81}$$

$\forall\ n = 2i, i \in N, n > 19$

Where $p = round(\frac{2n}{ln 2n} + \frac{2n}{\ln^2 2n} + 1)$. By (16)
In this formulas $|Q| = |Q|_m$. An estimation of such replace has a big value at the beginning of region of values of $2n$. But it decreaces by theorem 14.1.

The computed value of $|Q|$ it can be used for computation of $|H|$ by (42) (43):
$|H| = (2|Q| + 2p - n - 1)/2\ \forall\ n = 2i+1, i \in N, n > 19$
$|H| = (2|Q| + 2p - n - 2)/2\ \forall\ n = 2i, i \in N, n > 19$

15 RESUME

With help of the "Arithmetic of binary representations of even integer 2n"(ABR2n) it is got one of possible solutions of the Goldbach's binary problem.
With help of the ABR2n it can also be solved other arithmetical problems.
In ABR2n are given formulas for computation of the number of binary representations even integer 2n for basic types of BR2n. Particularly for values inaccessible for computer programs.

16 Appendix

16.1 The examples of the proof of parity of numerator for the some formulas

Example 16.1 *For formulas (21),(22) we have:*
$|H| = (2|Q| - s_0 + p - 1)/2 = (2|Q| - (2(n-1) - (2[(n-1)/2] - 1) - p) + p - 1)/2.$
if $n = 2i + 1, i \in N$ *then :*
$|H| = (2|Q| - (4i - (2i - 1) - p) + p - 1)/2 = (2|Q| - 2i + 2p - 2)/2$
if $n = 2i, i \in N$ *then:*
$|H| = (2|Q| - (4i - 2 - (2i - 3) - p) + p - 1)/2 = (2|Q| - 2i + 2p - 2)/2$

Example 16.2 *For formulas (23),(24) we have:*
$|Q| = (2(n-1) - (2[(n-1)/2] - 1) - p - |L|)/2$
if $n = 2i + 1, i \in N$ *then :*
$|Q| = (4i - (2i - 1) - (p + |L|)/2 = (2i - (p + |L|) + 1)/2$ *is even since* $(p + |L|)$ *is odd by (40) and* $-(p + |L|) + 1$ *is even .*
if $n = 2i, i \in N$ *then :*
$|Q| = (4i - 2 - (2i - 3) - (p + |L|)/2 = (2i - (p + |L|) + 1)/2$ *is even since* $(p + |L|)$ *is odd by (41) and* $-(p + |L|) + 1$ *is even .*

Example 16.3 *For formula (25) we have:*
$|H| = (p - |L| - 1)/2$
if $n = 2i + 1, i \in N$ *then :*
$(p - |L| - 1)$ *is even since* $(p - |L|)$ *is odd by (40) and* $p - |L| - 1$ *is even .* *if* $n = 2i, i \in N$ *then :*
$(p - |L| - 1)$ *is even since* $(p - |L|)$ *is odd by (41) and* $p - |L| - 1$ *is even .*

Example 16.4 *For formulas (27),(28),(29) we have:*
$|Q| = (2(n-1) - (2[(n-1)/2] - 1) - 2p + 2|H| + 1)/2$
if $n = 2i + 1, i \in N$ *then :*
$|Q| = (4i - (2i - 1) - 2p + 2|H| + 1)/2 = (2i - 2p + 2|H| + 2)/2$
if $n = 2i, i \in N$ *then :*
$|Q| = (4i - 2 - (2i - 3) - 2p + 2|H| + 1)/2 = (2i - 2p + 2|H| + 2)/2$

16.2 The examples of simplification for the some formulas

Example 16.5 *If* $n = 2i + 1, i \in N$ *then by (29) at* $|H| = 0$ *we get:*
$|Q|_b = \frac{2(2i+1-1)-(2\left[\frac{2i+1-1}{2}\right]-1)-2p+1}{2} = \frac{4i-(2i-1)-2p+1}{2} = \frac{n-2p+1}{2}$

Example 16.6 *If $n = 2i, i \in N$ then by (29) at $|H| = 0$ we get:*
$|Q|_b = \frac{2(2i-1)-(2\left[\frac{2i-1}{2}\right]-1)-2p+1}{2} = \frac{4i-2-(2i-3)-2p+1}{2} = \frac{n-2p+2}{2}$

Example 16.7 *If $n = 2i+1, i \in N$ then by (29) we get:*
$|Q| = \frac{2(2i+1-1)-(2\left[\frac{2i+1-1}{2}\right]-1)-2p+2|H|+1}{2} = \frac{4i-(2i-1)-2p+2|H|+1}{2} = \frac{n-2p+2|H|+1}{2}$

Example 16.8 *If $n = 2i, i \in N$ then by (29) we get:*
$|Q| = \frac{2(2i-1)-(2\left[\frac{2i-1}{2}\right]-1)-2p+2|H|+1}{2} = \frac{4i-2-(2i-3)-2p+2|H|+1}{2} = \frac{n-2p+2|H|+2}{2}$

Example 16.9 *If $n = 2i+1, i \in N$ then by (13) we get:*
$F = 2i+1-1-\left[\frac{2i+1-1}{2}\right] = i = \frac{n-1}{2}$

Example 16.10 *If $n = 2i, i \in N$ then by (13) we get:*
$F = 2i-1-\left[\frac{2i-1}{2}\right] = i = \frac{n}{2}$

Example 16.11 *If $n = 2i+1, i \in N$ then by (14) we get:*
$S_0 = 2(2i+1-1)-(2[\frac{2i+1-1}{2}]-1)-p = 4i-(2i-1)-p = 2i+1-p = n-p$

Example 16.12 *If $n = 2i, i \in N$ then by (14) we get:*
$S_0 = 2(2i-1)-(2[\frac{2i-1}{2}]-1)-p = 4i-2-(2i-3)-p = 2i+1-p = n-p+1$

16.3 The numerical solution of $|Q|_b = 0$ in the range $2 < 2n < 134$

Remark 16.1 *If n is prime then value of p is decreased per 1. $p^* = p-1$ since n excluded from XUY (see remark 2.1).*

$2n = 4; n = 2; p^* = 2; |Q|_b = 0$ by (55).
$2n = 6; n = 3; p^* = 3; |Q|_b = -1$ by (54).
$2n = 8; n = 4; p = 5; |Q|_b = -2$ by (55).
$2n = 10; n = 5; p^* = 4; |Q|_b = -1$ by (54).
$2n = 12; n = 6; p = 6; |Q|_b = -2$ by (55).
$2n = 14; n = 7; p^* = 6; |Q|_b = -2$ by (54).
$2n = 16; n = 8; p = 7; |Q|_b = -2$ by (55).
$2n = 18; n = 9; p = 8; |Q|_b = -3$ by (54).
$2n = 20; n = 10; p = 9; |Q|_b = -3$ by (55).
$2n = 22; n = 11; p^* = 8; |Q|_b = -2$ by (54).
$2n = 24; n = 12; p = 10; |Q|_b = -3$ by (55).
$2n = 26; n = 13; p^* = 9; |Q|_b = -2$ by (54).
$2n = 28; n = 14; p = 10; |Q|_b = -2$ by (55).
$2n = 30; n = 15; p = 11; |Q|_b = -3$ by (54).

$2n = 32; n = 16; p = 12; |Q|_b = -3$ by (55).
$2n = 34; n = 17; p^* = 11; |Q|_b = -2$ by (54).
$2n = 36; n = 18; p = 12; |Q|_b = -2$ by (55).
$2n = 38; n = 19; p^* = 12; |Q|_b = -2$ by (54).
$2n = 40; n = 20; p = 13; |Q|_b = -2$ by (55).
$2n = 42; n = 21; p = 14; |Q|_b = -3$ by (54).
$2n = 44; n = 22; p = 15; |Q|_b = -3$ by (55).
$2n = 46; n = 23; p^* = 14; |Q|_b = -2$ by (54).
$2n = 48; n = 24; p = 16; |Q|_b = -3$ by (55).
$2n = 50; n = 25; p = 16; |Q|_b = -3$ by (54).
$2n = 52; n = 26; p = 16; |Q|_b = -2$ by (55).
$2n = 54; n = 27; p = 17; |Q|_b = -3$ by (54).
$2n = 56; n = 28; p = 17; |Q|_b = -2$ by (55).
$2n = 58; n = 29; p^* = 16; |Q|_b = -1$ by (54).
$2n = 60; n = 30; p = 18; |Q|_b = -3$ by (55).
$2n = 62; n = 31; p^* = 18; |Q|_b = -2$ by (54).
$2n = 64; n = 32; p = 19; |Q|_b = -2$ by (55).
$2n = 66; n = 33; p = 19; |Q|_b = -2$ by (54).
$2n = 68; n = 34; p = 20; |Q|_b = -2$ by (55).
$2n = 70; n = 35; p = 20; |Q|_b = -2$ by (54).
$2n = 72; n = 36; p = 21; |Q|_b = -2$ by (55).
$2n = 74; n = 37; p^* = 21; |Q|_b = -2$ by (54).
$2n = 76; n = 38; p = 22; |Q|_b = -2$ by (55).
$2n = 78; n = 39; p = 22; |Q|_b = -2$ by (54).
$2n = 80; n = 40; p = 23; |Q|_b = -2$ by (55).
$2n = 82; n = 41; p^* = 22; |Q|_b = -1$ by (54).
$2n = 84; n = 42; p = 24; |Q|_b = -2$ by (55).
$2n = 86; n = 43; p^* = 23; |Q|_b = -1$ by (54).
$2n = 88; n = 44; p = 24; |Q|_b = -1$ by (55).
$2n = 90; n = 45; p = 25; |Q|_b = -2$ by (54).
$2n = 92; n = 46; p = 25; |Q|_b = -1$ by (55).
$2n = 94; n = 47; p^* = 24; |Q|_b = 0$ by (54).
$2n = 96; n = 48; p = 25; |Q|_b = 0$ by (55).
$2n = 98; n = 49; p = 26; |Q|_b = -1$ by (54).
$2n = 100; n = 50; p = 26; |Q|_b = 0$ by (55).
$2n = 102; n = 51; p = 27; |Q|_b = -1$ by (54).
$2n = 104; n = 52; p = 28; |Q|_b = -1$ by (55).
$2n = 106; n = 53; p^* = 27; |Q|_b = 0$ by (54).
$2n = 108; n = 54; p = 29; |Q|_b = -1$ by (55).
$2n = 110; n = 55; p = 30; |Q|_b = -2$ by (54).
$2n = 112; n = 56; p = 30; |Q|_b = -1$ by (55).

$2n = 114; n = 57; p = 31; |Q|_b = -2$ by (54).
$2n = 116; n = 58; p = 31; |Q|_b = -1$ by (55).
$2n = 118; n = 59; p^* = 30; |Q|_b = 0$ by (54).
$2n = 120; n = 60; p = 31; |Q|_b = 0$ by (55).
$2n = 122; n = 61; p^* = 30; |Q|_b = +1$ by (54).
$2n = 124; n = 62; p = 31; |Q|_b = +1$ by (55).
$2n = 126; n = 63; p = 31; |Q|_b = +1$ by (54).
$2n = 128; n = 64; p = 32; |Q|_b = +1$ by (55).
$2n = 130; n = 65; p = 32; |Q|_b = +1$ by (54).
$2n = 132; n = 66; p = 33; |Q|_b = +1$ by (55).

16.4 The numerical solution of $|Q| = 0$ in the range $8 < 2n < 134$

$2n = 10; |Q| = 0; (10 - 9 = 1)$.
$2n = 12; |Q| = 0; (12 - 9 = 3)$.
$2n = 14; |Q| = 0; (14 - 9 = 5)$.
$2n = 16; |Q| = 0; (16 - 9 = 7)$.
$2n = 18; |Q| = 0; (18 - 9 = 9)$ excluded as automorphism.
$2n = 20; |Q| = 0; (20 - 9 = 11)$.
$2n = 22; |Q| = 0; (22 - 9 = 13)$.
$2n = 24; |Q| > 0; (24 - 9 = 15)$.
$2n = 26; |Q| = 0; (26 - 9 = 17)$.
$2n = 28; |Q| = 0; (28 - 9 = 19)$.
$2n = 30; |Q| > 0; (30 - 9 = 21)$.
$2n = 32; |Q| = 0; (32 - 9 = 23)$.
$2n = 34; |Q| > 0; (34 - 9 = 25)$.
$2n = 36; |Q| > 0; (30 - 9 = 21)$.
$2n = 38; |Q| = 0; (38 - 9 = 29)$.
$2n = 40; |Q| > 0; (40 - 15 = 25)$.
$2n = 42; |Q| > 0; (42 - 9 = 33)$.
$2n = 44; |Q| > 0; (44 - 9 = 35)$.
$2n = 46; |Q| > 0; (46 - 21 = 25)$.
$2n = 48; |Q| > 0; (48 - 9 = 39)$.
$2n = 50; |Q| > 0; (50 - 15 = 35)$.
$2n = 52; |Q| > 0; (52 - 25 = 27)$.
$2n = 54; |Q| > 0; (54 - 9 = 45)$.
$2n = 56; |Q| > 0; (56 - 21 = 35)$.
$2n = 58; |Q| > 0; (58 - 9 = 49)$.
$2n = 60; |Q| > 0; (60 - 9 = 51)$.

$2n = 62; |Q| > 0; (62 - 9 = 21)$.
$2n = 64; |Q| > 0; (64 - 9 = 55)$.
$2n = 66; |Q| > 0; (66 - 9 = 57)$.
$2n = 68; |Q| > 0; (68 - 33 = 35)$.
$2n = 70; |Q| > 0; (70 - 15 = 55)$.
$2n = 72; |Q| > 0; (72 - 9 = 63)$.
$2n = 74; |Q| > 0; (74 - 9 = 21)$.
$2n = 76; |Q| > 0; (76 - 21 = 55)$.
$2n = 78; |Q| > 0; (78 - 9 = 69)$.
$2n = 80; |Q| > 0; (80 - 15 = 65)$.
$2n = 82; |Q| > 0; (82 - 25 = 57)$.
$2n = 84; |Q| > 0; (84 - 9 = 75)$.
$2n = 86; |Q| > 0; (86 - 9 = 77)$.
$2n = 88; |Q| > 0; (88 - 25 = 63)$.
$2n = 90; |Q| > 0; (90 - 9 = 81)$.
$2n = 92; |Q| > 0; (92 - 15 = 77)$.
$2n = 94; |Q| > 0; (94 - 9 = 85)$.
$2n = 96; |Q| > 0; (96 - 9 = 87)$.
$2n = 98; |Q| > 0; (98 - 21 = 77)$.
$2n = 100; |Q| > 0; (100 - 15 = 85)$.
$2n = 102; |Q| > 0; (102 - 9 = 93)$.
$2n = 104; |Q| > 0; (104 - 9 = 95)$.
$2n = 106; |Q| > 0; (106 - 15 = 91)$.
$2n = 108; |Q| > 0; (108 - 9 = 99)$.
$2n = 110; |Q| > 0; (110 - 15 = 95)$.
$2n = 112; |Q| > 0; (112 - 21 = 91)$.
$2n = 114; |Q| > 0; (114 - 9 = 105)$.
$2n = 116; |Q| > 0; (116 - 21 = 95)$.
$2n = 118; |Q| > 0; (118 - 25 = 93)$.
$2n = 120; |Q| > 0; (120 - 9 = 111)$.
$2n = 122; |Q| > 0; (122 - 35 = 87)$.
$2n = 124; |Q| > 0; (124 - 9 = 115)$.
$2n = 126; |Q| > 0; (126 - 9 = 117)$.
$2n = 128; |Q| > 0; (128 - 9 = 119)$.
$2n = 130; |Q| > 0; (130 - 9 = 121)$.
$2n = 132; |Q| > 0; (132 - 9 = 123)$.

16.5 The examples of use of ariphmetic of binary representations of positive even integer 2n

The rules of direct computation of ariphmetic parameters
of binary representations of positive integer 2n . "1" and "2" - are positive noncomposite integers. The representation $(1 + (2n - 1))$ is of type L if $(2n - 1)$ -composite positive integer and of type H if $(2n - 1)$-prime positive integer. $\oplus$ -mark positive prime integers or "1"; $\otimes$-mark positive odd composite integers; $\oslash$-mark positive even integers.

Example 16.13 $2n = 24$
$\oplus$ *1+23* $\oplus$ $=$ *24* (H)
$\oplus$ *2+22* $\oslash$ $=$ *24* (E)
$\oplus$ *3+21* $\otimes$ $=$ *24* (L)
$\oslash$ *4+20* $\oslash$ $=$ *24* (E)
$\oplus$ *5+19* $\oplus$ $=$ *24* (H)
$\oslash$ *6+18* $\oslash$ $=$ *24* (E)
$\oplus$ *7+17* $\oplus$ $=$ *24* (H)
$\oslash$ *8+16* $\oslash$ $=$ *24* (E)
$\otimes$ *9+15* $\otimes$ $=$ *24* (Q)
$\oslash$ *10+14*$\oslash$ $=$ *24* (E)
$\oplus$ *11+13*$\oplus$ $=$ *24* (H)
Data of direct computations :
$P = 10;\ S_0 = 3;\ G = 11;\ F = 6;\ |E| = 5;\ |Q| = 1;\ |L| = 1;\ |H| = 4;$
The computations of parameters of binary representations of the positive integer $2n = 24$ *with help of arithmetic stated above*
$n = \frac{24}{2} = 12;$
$G = n - 1 = 12 - 1 = 11;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{12-1}{2}\right] = 5;$
$F = G - |E| = 11 - 5 = 6;$
$F = \frac{n}{2} = \frac{12}{2} = 6;$
$S_0 = n - p + 1 = 12 - 10 + 1 = 3;$
We take $|Q| = 1$ *from direct computations then*
$|L| = n - p + 1 - 2|Q| = 12 - 10 + 1 - 2 = 1;$
$|H| = \frac{2|Q|+2p-n-2}{2} = \frac{2+20-12-2}{2} = 4;$
We take $|L| = 1$ *from direct computations then*
$|Q| = \frac{n-p-|L|+1}{2} = \frac{12-10-1+1}{2} = 1;$
$|H| = \frac{p-|L|-1}{2} = \frac{10-1-1}{2} = 4;$
We take $|H| = 4$ *from direct computations then*
$|Q| = \frac{n-2p+1+2|H|+1}{2} = \frac{12-20+1+8+1}{2} = 1;$
$|L| = p - 2|H| - 1 = 10 - 8 - 1 = 1;$

The control :
$F = |Q| + |L| + |H| = 1 + 1 + 4 = 6;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.14 $2n = 26$
$\oplus$ *1+25* $\otimes$ $=$ *26* (L)
$\oplus$ *2+24* $\oslash$ $=$ *26* (E)
$\oplus$ *3+23* $\oplus$ $=$ *26* (H)
$\oslash$ *4+22* $\oslash$ $=$ *26* (E)
$\oplus$ *5+21* $\otimes$ $=$ *26* (L)
$\oslash$ *6+20* $\oslash$ $=$ *26* (E)
$\oplus$ *7+19* $\oplus$ $=$ *26* (H)
$\oslash$ *8+18* $\oslash$ $=$ *26* (E)
$\otimes$ *9+17* $\oplus$ $=$ *26* (L)
$\oslash$ *10+16*$\oslash$ $=$ *26* (E)
$\oplus$ *11+15*$\otimes$ $=$ *26* (L)
$\oslash$ *12+14*$\oslash$ $=$ *26* (E)
Data of direct computatios :
$P = 9;$ (*since* $n = 13$ *is vikolotaya point*) $S_0 = 4;\ G = 12;$
$F = 6;\ |E| = 6;\ |Q| = 0;\ |L| = 4;\ |H| = 2;$ *The computations of parameters of binary representations of the positive integer* $2n = 26$ *with help of arithmetic stated above:*
$n = \frac{26}{2} = 13;$
$G = n - 1 = 13 - 1 = 12;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{13-1}{2}\right] = 6;$
$F = G - |E| = 12 - 6 = 6;$
$F = \frac{n-1}{2} = \frac{13-1}{2} = 6;$
$S_0 = n - p = 13 - 9 = 4;$
We take $|Q| = 0$ *from direct computations then*
$|L| = n - p - 2|Q| = 13 - 9 - 0 = 4;$
$|H| = \frac{2|Q|+2p-n-1}{2} = \frac{0+18-13-1}{2} = 2;$
We take $|L| = 4$ *from direct computations then*
$|Q| = \frac{n-p-|L|}{2} = \frac{13-9-4}{2} = 0;$
$|H| = \frac{p-|L|-1}{2} = \frac{9-4-1}{2} = 2;$
We take $|H| = 2$ *from direct computations then*
$|Q| = \frac{n-2p+2|H|+1}{2} = \frac{13-18+4+1}{2} = 0;$
$|L| = p - 2|H| - 1 = 9 - 4 - 1 = 4;$
The control:
$F = |Q| + |L| + |H| = 0 + 4 + 2 = 6;$
The compare of data of direct computations with computed values shows

full coincidence of results.

Example 16.15 $2n = 28$
$\oplus$ *1+27* $\otimes$ = *28* (L)
$\oplus$ *2+26* $\oslash$ = *28* (E)
$\oplus$ *3+25* $\otimes$ = *28* (L)
$\oslash$ *4+24* $\oslash$ = *28* (E)
$\oplus$ *5+23* $\oplus$ = *28* (H)
$\oslash$ *6+22* $\oslash$ = *28* (E)
$\oplus$ *7+21* $\otimes$ = *28* (L)
$\oslash$ *8+20* $\oslash$ = *28* (E)
$\otimes$ *9+19* $\oplus$ = *28* (L)
$\oslash$ *10+18*$\oslash$ = *28* (E)
$\oplus$ *11+17*$\oplus$ = *28* (H)
$\oslash$ *12+16*$\oslash$ = *28* (E)
$\oplus$ *13+15*$\otimes$ = *28* (L)
Data of direct computations :
$P = 10;\ S_0 = 5;\ G = 13;\ F = 7;\ |E| = 6;\ |Q| = 0;\ |L| = 5;\ |H| = 2;$
The computations of parameters of binary representations of the positive integer $2n = 28$ *with help of arithmetic stated above:*
$n = \frac{28}{2} = 14;$
$G = n - 1 = 14 - 1 = 13;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{14-1}{2}\right] = 6;$
$F = G - |E| = 13 - 6 = 7;$
$F = \frac{n}{2} = \frac{14}{2} = 7;$
$S_0 = n - p + 1 = 14 - 10 + 1 = 5;$
We take $|Q| = 0$ *from direct computations then*
$|L| = n - p + 1 - 2|Q| = 14 - 10 + 1 - 0 = 5;$
$|H| = \frac{2|Q|+2p-n-2}{2} = \frac{0+20-14-2}{2} = 2;$
We take $|L| = 5$ *from direct computations then*
$|Q| = \frac{n-p-|L|+1}{2} = \frac{14-10-5+1}{2} = 0;$
$|H| = \frac{p-|L|-1}{2} = \frac{10-5-1}{2} = 2;$
We take $|H| = 2$ *from direct computations then*
$|Q| = \frac{n-2p+1+2|H|+1}{2} = \frac{14-20+1+4+1}{2} = 0;$
$|L| = p - 2|H| - 1 = 10 - 4 - 1 = 5;$
The control:
$F = |Q| + |L| + |H| = 0 + 5 + 2 = 7;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.16 $2n = 30$

$\oplus$ *1+29* $\oplus$ $=$ *30* (H)
$\oplus$ *2+28* $\oslash$ $=$ *30* (E)
$\oplus$ *3+27* $\otimes$ $=$ *30* (L)
$\oslash$ *4+26* $\oslash$ $=$ *30* (E)
$\oplus$ *5+25* $\otimes$ $=$ *30* (L)
$\oslash$ *6+24* $\oslash$ $=$ *30* (E)
$\oplus$ *7+23* $\oplus$ $=$ *30* (H)
$\oslash$ *8+22* $\oslash$ $=$ *30* (E)
$\otimes$ *9+21* $\otimes$ $=$ *30* (Q)
$\oslash$ *10+20*$\oslash$ $=$ *30* (E)
$\oplus$ *11+19*$\oplus$ $=$ *30* (H)
$\oslash$ *12+18*$\oslash$ $=$ *30* (E)
$\oplus$ *13+17*$\oplus$ $=$ *30* (H)
$\oslash$ *14+16*$\oslash$ $=$ *30* (E)
Data of direct computations :
$P = 11;\ S_0 = 4;\ G = 14;\ F = 7;\ |E| = 7;\ |Q| = 1;\ |L| = 2;\ |H| = 4;$
The computations of parameters of binary representations of the positive integer $2n = 30$ *with help of arithmetic stated above:*
$n = \frac{30}{2} = 15;$
$G = n - 1 = 15 - 1 = 14;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{15-1}{2}\right] = 7;$
$F = G - |E| = 14 - 7 = 7;$
$F = \frac{n-1}{2} = \frac{15-1}{2} = 7;$
$S_0 = n - p = 15 - 11 = 4;$
We take $|Q| = 1$ *from direct computations then*
$|L| = n - p - 2|Q| = 15 - 11 - 2 = 2;$
$|H| = \frac{2|Q|+2p-n-1}{2} = \frac{2+22-15-1}{2} = 4;$
We take $|L| = 2$ *from direct computations then*
$|Q| = \frac{n-p-|L|}{2} = \frac{15-11-2}{2} = 1;$
$|H| = \frac{p-|L|-1}{2} = \frac{11-2-1}{2} = 4;$
We take $|H| = 4$ *from direct computations then*
$|Q| = \frac{n-2p+2|H|+1}{2} = \frac{15-22+8+1}{2} = 1;$
$|L| = p - 2|H| - 1 = 11 - 8 - 1 = 2;$
The control:
$F = |Q| + |L| + |H| = 1 + 2 + 4 = 7;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.17 $2n = 32$
$\oplus$ *1+31* $\oplus$ $=$ *32* (H)
$\oplus$ *2+30* $\oslash$ $=$ *32* (E)

$\oplus$ *3+29* $\oplus$ $=$ *32* (H)
$\oslash$ *4+28* $\oslash$ $=$ *32* (E)
$\oplus$ *5+27* $\otimes$ $=$ *32* (L)
$\oslash$ *6+26* $\oslash$ $=$ *32* (E)
$\oplus$ *7+25* $\otimes$ $=$ *32* (L)
$\oslash$ *8+24* $\oslash$ $=$ *32* (E)
$\otimes$ *9+23* $\oplus$ $=$ *32* (L)
$\oslash$ *10+22*$\oslash$ $=$ *32* (E)
$\oplus$ *11+21*$\otimes$ $=$ *32* (L)
$\oslash$ *12+20*$\oslash$ $=$ *32* (E)
$\oplus$ *13+19*$\oplus$ $=$ *32* (H)
$\oslash$ *14+18*$\oslash$ $=$ *32* (E)
$\otimes$ *15+17*$\oplus$ $=$ *32* (L)
Data of direct computations :
$P = 12;\ S_0 = 5;\ G = 15;\ F = 8;\ |E| = 7;\ |Q| = 0;\ |L| = 5;\ |H| = 3;$
The computations of parameters of binary representations of the positive integer $2n = 28$ *with help of arithmetic stated above:*
$n = \frac{32}{2} = 16;$
$G = n - 1 = 16 - 1 = 15;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{16-1}{2}\right] = 7;$
$F = G - |E| = 15 - 7 = 8;$
$F = \frac{n}{2} = \frac{16}{2} = 8;$
$S_0 = n - p + 1 = 16 - 12 + 1 = 5;$
We take $|Q| = 0$ *from direct computations then*
$|L| = n - p + 1 - 2|Q| = 16 - 12 + 1 - 0 = 5;$
$|H| = \frac{2|Q|+2p-n-2}{2} = \frac{0+24-16-2}{2} = 3;$
We take $|L| = 5$ *from direct computations then*
$|Q| = \frac{n-p-|L|+1}{2} = \frac{16-12-5+1}{2} = 0;$
$|H| = \frac{p-|L|-1}{2} = \frac{12-5-1}{2} = 3;$
We take $|H| = 3$ *from direct computations then*
$|Q| = \frac{n-2p+1+2|H|+1}{2} = \frac{16-24+1+6+1}{2} = 0;$
$|L| = p - 2|H| - 1 = 12 - 6 - 1 = 5;$
The control:
$F = |Q| + |L| + |H| = 0 + 5 + 3 = 8;$
The compare of data of direct computations with computed values shows full coincidence of results.

Remark 16.2 *The following data of direct computations was given with help of program "Goldbach" developed by the author . The program realizes of binary representations of even integer 2n. It identies type of each representation and calculates* $P, E, F, G, S_0, |Q|, |L|, |H|, |E|$ *.*

Example 16.18 $2n = 10026$;
Data of direct computations :
$P = 1232;\ S_0 = 3781;\ G = 5012;\ F = 2506;$
$|E| = 2506;\ |Q| = 1469;\ |L| = 843;\ |H| = 194;$
The computations of parameters of binary representations of the positive integer $2n = 10026$ *with help of arithmetic stated above:*
$n = \frac{10026}{2} = 5013;$
$G = n - 1 = 5013 - 1 = 5012;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{5013-1}{2}\right] = 2506;$
$F = G - |E| = 5012 - 2506 = 2506;$
$F = \frac{n-1}{2} = \frac{5013-1}{2} = 2506;$
$S_0 = n - p = 5013 - 1232 = 3781;$
We take $|Q| = 1469$ *from direct computations then*
$|L| = n - p - 2|Q| = 5013 - 1232 - 2938 = 843;$
$|H| = \frac{2|Q|+2p-n-1}{2} = \frac{2938+2464-5013-1}{2} = 194;$
We take $|L| = 843$ *from direct computations then*
$|Q| = \frac{n-p-|L|}{2} = \frac{5013-1232-843}{2} = 1469;$
$|H| = \frac{p-|L|-1}{2} = \frac{1232-843-1}{2} = 194;$
We take $|H| = 194$ *from direct computations then*
$|Q| = \frac{n-2p+2|H|+1}{2} = \frac{5013-2464+388+1}{2} = 1469;$
$|L| = p - 2|H| - 1 = 1232 - 388 - 1 = 843;$
The control:
$F = |Q| + |L| + |H| = 1469 + 843 + 194 = 2506;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.19 $2n = 20028$;
Data of direct computations :
$P = 2266;\ S_0 = 7749;\ G = 10013;\ F = 5007;$
$|E| = 5006;\ |Q| = 3078;\ |L| = 1593;\ |H| = 336;$
The computation of parameters of binary representations of the positive integer $2n = 20028$ *with help of arithmetic stated above:*
$n = \frac{20028}{2} = 10014;$
$G = n - 1 = 10014 - 1 = 10013;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{10014-1}{2}\right] = 5006;$
$F = G - |E| = 10013 - 5006 = 5007;$
$F = \frac{n}{2} = \frac{10014}{2} = 5007;$
$S_0 = n - p + 1 = 10014 - 2266 + 1 = 7749;$
We take $|Q| = 3078$ *from direct computating then*
$|L| = n - p + 1 - 2|Q| = 10014 - 2266 + 1 - 6156 = 1593;$
$|H| = \frac{2|Q|+2P-n-2}{2} = \frac{6156+4532-10014-2}{2} = 336;$

We take $|L| = 1593$ *from direct computating then*
$|Q| = \frac{n-p-|L|+1}{2} = \frac{10014-2266-1593+1}{2} = 3078;$
$|H| = \frac{p-|L|-1}{2} = \frac{2266-1593-1}{2} = 336;$
We take $|H| = 336$ *from direct computating then*
$|Q| = \frac{n-2p+1+2|H|+1}{2} = \frac{10014-4532+1+672+1}{2} = 3078;$
$|L| = p - 2|H| - 1 = 2266 - 672 - 1 = 1593;$
The control:
$F = |Q| + |L| + |H| = 3078 + 1593 + 336 = 5007;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.20 $2n = 100030;$
Data of direct computations :
$P = 9595;\ S_0 = 40420;\ G = 50014;\ F = 25007;$
$|E| = 25007;\ |Q| = 16385;\ |L| = 7650;\ |H| = 972;$
The computations of parameters of binary representations of the positive integer $2n = 100030$ *with help of arithmetic stated above:*
$n = \frac{100030}{2} = 50015;$
$G = n - 1 = 50015 - 1 = 50014;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{50015-1}{2}\right] = 25007;$
$F = G - |E| = 50014 - 25007 = 25007;$
$F = \frac{n-1}{2} = \frac{50015-1}{2} = 25007;$
$S_0 = n - p = 50015 - 9595 = 40420;$
We take $|Q| = 16385$ *from direct computations then*
$|L| = n - p - 2|Q| = 50015 - 9595 - 32770 = 7650;$
$|H| = \frac{2|Q|+2p-n-1}{2} = \frac{32770+19190-50015-1}{2} = 972;$
We take $|L| = 7650$ *from direct computations then*
$|Q| = \frac{n-p-|L|}{2} = \frac{50015-9595-7650}{2} = 16385;$
$|H| = \frac{p-|L|-1}{2} = \frac{9595-7650-1}{2} = 972;$
We take $|H| = 972$ *from direct computations then*
$|Q| = \frac{n-2p+2|H|+1}{2} = \frac{50015-19190+1944+1}{2} = 16385;$
$|L| = p - 2|H| - 1 = 9595 - 1944 - 1 = 7650;$
The control:
$F = |Q| + |L| + |H| = 16385 + 7650 + 972 = 25007;$
The compare of data of direct computations with computed values shows full coincidence of results.

Example 16.21 $2n = 200032;$
Data of direct computations :
$p = 17990;\ S_0 = 82027;\ G = 100015;\ F = 50008;$

$|E| = 50007; |Q| = 33386;\ |L| = 15255;\ |H| = 1367;$
The computations of parameters of binary representations of the positive integer $2n = 200032$ *with help of arithmetic stated above:*
$n = \frac{200032}{2} = 100016;$
$G = n - 1 = 100016 - 1 = 100015;$
$|E| = \left[\frac{n-1}{2}\right] = \left[\frac{100016-1}{2}\right] = \ 50007;$
$F = G - |E| = 100015 - 50007 = 50008;$
$F = \frac{n}{2} = \frac{100016}{2} = 500008;$
$S_0 = n - p + 1 = 100016 - 17990 + 1 = 82027;$
We take $|Q| = 33386$ *from direct computations then*
$|L| = n - p + 1 - 2|Q| = 100016 - 17990 + 1 - 66772 = 15255;$
$|H| = \frac{2|Q|+2p-n-2}{2} = \frac{66772+35980-100016-2}{2} = 1367;$
We take $|L| = 15255$ *from direct computations then*
$|Q| = \frac{n-p-|L|+1}{2} = \frac{100016-17990-15255+1}{2} = 33386;$
$|H| = \frac{p-|L|-1}{2} = \frac{17990-15255-1}{2} = 1367;$
We take $|H| = 1367$ *from direct computations then*
$|Q| = \frac{n-2P+1+2|H|+1}{2} = \frac{100016-35980+1+2734+1}{2} = 33386;$
$|L| = p - 2|H| - 1 = 17990 - 2734 - 1 = 15255;$
The control:
$F = |Q| + |L| + |H| = 33386 + 15255 + 1367 = 50008;$
The compare of data of direct computations with computed values shows full coincidence of results.

References

[1] J.B.Zeldovich , A.D.Myshkis: *The elements of applied mathematics*, pgs: 562-567, publishers "Nauka",Moscow (1965)