The mass effect of gravitational potential (MEGP)

Mahgoub Salih

mahgoubasalih@hotmail.com

Address: Department of Physics, Faculty of Science and Art at al-Muznab, Al Qassim University,KSA

P.O. Box 931,Al Muznub 51931, Saudi Arabia.

Abstract

We showed that the effect of gravitational potential is equivalent to the dark matter that should be added to Newtonian dynamic to solve the cosmological problems. The amount of mass effect that could be generated by the gravity was found to be 34% of the universe's mass. We proposed Modified Newtonian dynamics (MOND) as consequence of a mass effect of gravitational potential.

Key words: Einstein equation, dark matter, modified Newtonian dynamics.

Introduction

Modified Newtonian dynamics (MOND), an empirical low, was proposed by Mordehai Milgrom in 1983.it is a hypothesis that proposes a modification of Newton's law of gravity to solve the galaxy rotation problem. This problem began with the discovery of Oort. In 1932 he noticed a shortage of mass required to describe the velocity of stars in the solar neighborhood (Oort, J. H. 1932) . In 1933, Zwicky found the same problem by applying the virial theorem to the Coma cluster, based on the radial velocities of a few galaxies(Zwicky. F,1933).

Later, many authors showed that a large fraction of mass, in addition to the observed luminous mass, was necessary to describe the dynamics of galaxies (JAAN EINASTO et al,1974),(Diaferio, A, 2008),(Ostriker, et al, 1973)
Milgrom noted that Newton's law for gravitational force has been verified only where gravitational acceleration is large, and suggested that for extremely low accelerations the theory may not hold. MOND theory posits that acceleration is not linearly proportional to force at low values. The basis of the modification is the assumption that, in the limit of small acceleration a very low characteristic acceleration, a_0, the acceleration of a particle at distance *r* from a mass *M* satisfies approximately the relation $a_N = a\,\mu\left({a}/{a_0}\right)$, where a_N is the Newtonian acceleration,a is the MOND acceleration of gravity, a_0 is constant acceleration and $\mu\left({a}/{a_0}\right)$ is the interpolation function (M. Milgrom,1983,2001),(Stacy McGaugh,2005).
MOND contradicts the theory of dark matter (Lars Bergström,2000),(Gianfranco Bertone, et al,2004). Dark matter theory suggests that each galaxy contains a halo of an unidentified type of matter that provides an overall mass distribution different from the observed distribution of normal matter. This dark matter modifies gravity so as to cause the uniform rotation velocity data.

Most astrophysicists and cosmologists do not believe that Modified Newtonian Dynamics fits the evidence. The mainstream scientific community supports the theory of dark matter. Because, Modified Newtonian Dynamics proposes fundamental changes to our understanding of the way gravity works. In contrary, dark matter theory, which supposes that our understanding of gravity is fine, but we're missing some physical substances that need to be included in the calculations.

In this work we constructed mass formulae in concept of gravitational effects on the mass, which resolves some part of the missing matter. This was done by using the stress energy momentum tensor of a perfect fluid in Background of cosmic fluid with negative pressure. After that, we compare the mass term with the equivalent term in the relativistic Klein – Gordon equation.

The mass effect of gravitational potential (MEGP)

Stress energy momentum tensor of a perfect fluid, in Background cosmic fluid with negative pressure $p = -\rho c^2$ is given by $T_{\mu\nu} = g_{\mu\sigma} g_{\nu\zeta}\left(\rho + \frac{p}{c^2}\right)c^2 - p g_{\mu\nu} \equiv \rho g_{\mu\nu}$ and Einstein equation is $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$.

We start by writing R.H.S of Einstein equation in terms of Schwarzschild radius and rational stress energy momentum tensor as

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -4\pi R_s \widetilde{T_{\mu\nu}} \qquad (1)$$

Where $R_s = \frac{2GM}{c^2}$ and $\widetilde{T_{\mu\nu}} = {}^{T_{\mu\nu}}/_{Mc^2}$.

Then, replacing Schwarzschild radius with Compton length (λ_c) $R_s = \alpha^2 \lambda_c$ in Einstein equation (1). That gives

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\frac{4\pi\alpha^2\hbar}{mc} \widetilde{T_{\mu\nu}} \qquad (2)$$

Now Einstein equation in Background of cosmic fluid with negative pressure, can be written as the following

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\frac{4\pi\alpha^2\hbar}{mc} \tilde{\rho} g_{\mu\nu} \qquad (3)$$

Where $\tilde{\rho} = {}^{\rho}/_{Mc^2} \propto r^{-3}$, a is the scale (radius) of the system.

Solving Einstein equation in conformal flat space time with metric tensor,

$g_{\mu\nu} = e^{\psi} \eta_{\mu\nu}$, where ψ is coordinate function. Here, we recourse the result in (John.A, et al,1997) to write Einstein equation as

$$\frac{\partial^2 \psi}{\partial x^\mu \partial x_\nu} + \tfrac{1}{2}\delta^\mu_\nu \frac{\partial^2 \psi}{\partial x^\alpha \partial x_\alpha} - \tfrac{3}{2}\delta^\mu_\nu \psi = -\frac{4\pi\alpha^2\hbar}{mc} \tilde{\rho}(1 + 2\psi)\delta^\mu_\nu \qquad (4)$$

Equation (4) implies that $\Box\psi - \frac{32\pi\alpha^2\hbar}{3mc} \tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc} \tilde{\rho}$.

In static case, one has

$$\nabla^2\psi - \frac{32\pi\alpha^2\hbar}{3mc} \tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc} \tilde{\rho} \qquad (5)$$

By comparing equation (5) with Klein – Gordon equation

$$\nabla^2\emptyset - \frac{m_\emptyset^2 c^2}{\hbar^2}\emptyset = \frac{1}{c^2}\frac{\partial^2\emptyset}{\partial t^2} \qquad (6)$$

Where, $m_\emptyset$ is the relativistic mass. The equivalent mass expression is given as

$$m_\emptyset = \sqrt{\frac{16\pi\alpha^2\hbar^3}{3mc^3}\tilde{\rho}} \qquad (7)$$

To calculate the equivalent mass, we assumed that, the system is a sphere of radius r, where $r \geq R_s$. at this end one finds

$$m_\emptyset = \sqrt{\frac{4\alpha^2\hbar^3}{mc^3} r^{-3}} \quad = \frac{2\hbar}{c\,r}\sqrt{\frac{R_s}{r}} \qquad (9)$$

We try to write equation (9) in two cases, microscopic scale ($m < m_p$) and macroscopic scale ($m > m_p$). Before this we should get the transformation between microscopic and macroscopic scale, this transformation can be done according to(Mahgoub Salih, et al,2012) $GM \to \frac{\hbar c}{m}$, where M refers to macroscopic mass and m is the microscopic mass.

First, we write Planck length in form of combination of two lengths, ${l_p}^2 = \frac{2Gm}{c^2} \; \frac{\hbar}{mc}$. As we know the transformation keeps the relation invariant. Thus, the corresponding transformation will be,

$${l_p}^2 \to {R_s}^2 \to {\lambda_c}^2 \qquad (12)$$

According to equation (9) and the transformation (12) one has in microscopic scale

$$m_\emptyset = \frac{2\hbar}{c\,r}\sqrt{\frac{\lambda_c}{r}} \qquad (13)$$

When $r \to \lambda_c$, one finds $m_\emptyset = 2m$.

On the other hand, in macroscopic scale, one has

$$m_\emptyset = \frac{2mR_s}{r}\sqrt{\frac{R_s}{r}} \qquad (14)$$

At the limit when, $r \to R_s$, one finds $m_\emptyset = 2m$.

Equations (13) and (14) confirm that the mass $m_\emptyset = 2m$ at the critical radius and vanished when $r \to \infty$ .therefore, $m_\emptyset$ couldn't be a real mass (baryonic matter) its existence depends on the gravitational potential, it is a dark matter. Thus, this mass effect appears as an external mass interact gravitationally in the same manner of baryonic mass. The additional mass that could be added to the interaction at the critical radius is twice the baryonic mass. The standard interpretation of the observations of the cosmic structure, from galactic scales to the CMB, suggests that only 17% of the matter in the Universe is baryonic. If the baryonic matter distributed uniformly at the critical radius of the universe (Schwarzschild's radius) and interact gravitationally, thus one finds that, the universe has 34% mass due gravity in addition to the 17% baryonic mass. Hence, this shed light on 51% of the total mass of the universe.

MOND is a consequence of MEGP

The acceleration according to equation (9) and Newton law, is given by

$$a = -\frac{Gm_\emptyset}{r^2} = -\left(\frac{l_p}{r}\right)^2 \frac{\sqrt{2GMa_0}}{r} \qquad (15)$$

Where $a_0 = \frac{c^2}{r}$ and $l_p = \sqrt{\frac{2\hbar G}{c^3}}$ is the Planck length.

Equation (15) is the same equation which was proposed by Milgrom, in modified Newtonian Dynamics, with specific value of interpolation function $\mu\left({}^{a}/_{a_0}\right)$ (M. Milgrom,1983).

MOND weak acceleration limit of gravity is $a = -\frac{\sqrt{GMa_0}}{r}$. This means that, according to equation (15) the weak limit of gravity is obtained when the interaction radius is equal to the

critical radius, in this case $r = l_p$ requires the mass M to be equal to Planck mass (threshold mass between macroscopic and microscopic world).accordingly, the weak limit of gravity is obtained when the interaction radius is equal to Schwarzschild radius or Compton length in macroscopic scale and microscopic scale respectively.
To this end, equation (15) should be written in macroscopic scale and microscopic scale. First in macroscopic scale, by substituting equation (14) in the weak equivalence principle, one has

$$a = -\left(\frac{R_s}{r}\right)^2 \frac{\sqrt{2GMa_0}}{r} \qquad (16)$$

Second in microscopic scale, equation (15) reads

$$a = -\left(\frac{\lambda_c}{r}\right)^2 \frac{c\sqrt{\lambda_c a_0}}{r} \qquad (17)$$

Equations (16) and (17) satisfy that, the acceleration approaches a_0 at the limit when $r \to R_s$ and $r \to \lambda_c$ respectively. At this limit the transition occurs smoothly. This agrees with MOND (The transition occurs smoothly near the distance where the acceleration falls to $\sim a_0$, a constant $\sim 10^{-10}\ m/s^2$). On the other hand, $a_0 = \frac{c^2}{r}$ is constant due to the system e.g. for the universe where $r \sim 10^{26}\, m$ one finds $a_0 \sim 10^{-10}\ m/s^2$.

Conclusion

Equations (13) and (14) explain the existence of dark matter in microscopic and macroscopic scales. The amount of 34% of missed mass in the universe is related to MEGP If the baryonic matter distributed uniformly at Schwarzschild's radius of the universe and interacts gravitationally.
The result that MOND is not a modification of Newtonian Dynamics rather than modification of non-baryonic mass relation (MEGP) had shown by equation (15).
Equations (16) and (17) describe the acceleration in both macroscopic and microscopic scales which provide full treatment to MEGP and its applications.

References:

[1]Oort, J. H. 1932, The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems,Bulletin of the Astronomical Institutes of the Netherlands, 6, 249
[2]Zwicky, F. The redshift of extragalactic nebulae 1933, Helv. Phys. Acta, 6, 110
[3]JAAN EINASTO, ENN SAAR, ANTS KAASIK , ARTHUR D. CHERNIN,1974, Nature 252, 111 - 113 (08 November 1974); doi:10.1038/252111a0
[4]Diaferio, A, 2008, the evidence for unusual gravity from the large-scale structure of the Universe, arXiv:0802.2532.
[5]Ostriker, J. P.; Peebles, P. J. E., Astrophysical Journal, Vol. 186, pp. 467-480 (1973) A Numerical Study of the Stability of Flattened Galaxies: or, can Cold Galaxies Survive?)
[6]Lars Bergström 2000 Rep. Prog. Phys. 63 793 doi:10.1088/0034-4885/63/5/2r3, Non-baryonic dark matter, observational evidence and detection methods.
[7]Gianfranco Bertone, Dan Hooper, Joseph Silk, Particle Dark Matter: Evidence, Candidates and Constraints,2004,arXiv:hep-ph/0404175.
[8]John.A; Corneliu.C, Massive Gravitons in General Relativity, Aust.j.Phys.,1997,50,879-91
[9]Mahgoub Salih;Tayssir.M.Al Mahdi (Micro-Particle Model and Black Hole; 2012) viXra:1206.0011
[10]M. Milgrom, A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis, Astrophys. J. 270, 365-370, 1983.
[11]M. Milgrom, MOND – A Pedagogical Review, Acta Phys. Polon. B 32, 3613 (2001). ArXiv:astro-ph/0112069.
[12]Stacy McGaugh , The Balance of Dark and Luminous Mass in Rotating Galaxies,2005. ArXiv: astro-ph/0509305.